On the Houde Damper for a Damped Vibration System*

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This paper treats analytically the vibration prevention effect of the Houde damper which is attached to a main vibration system with positive or negative damping.

Analyses about the damper of this type have been done in the past only in the cases of a main system without damping. In this paper, the analyses are done both in the case of the main system with negative damping, that is a self-excited vibration system, and in the case of the main system with positive damping which is subjected to various alternate disturbing external forces.

It is shown that better effect can be obtained in each case, if the optimum value of damping is given to the Houde damper with consideration of the damping of the main system.

1. Introduction

For a vibration system disturbed by an alternate exciting force, the dynamic damper has been used as a means to prevent the system from vibration. The analysis of a system in which the damper consisting of spring-mass is added to the main system composed of spring-mass was studied by Frahm and the trials to insert a suitable damping factor into the dynamic damper were made by Brock⁽¹⁾. These analyses have been made on the condition that the main vibration system has no damping. The analyses of a system (normal type of the dynamic damper) in which a dynamic damper consisting of spring-mass and positive damping is added to the main system with damping, positive or negative, were reported by authors^{(2),(3)}. However, analysis of the system (Fig.1) in which a Houde damper is added to the vibration system with damping has not yet been tried elsewhere. The above analysis has been done in the past only in the cases of the main system without damping which is subjected to a constant steady alternate exciting force.

In this paper a vibration system with two degrees of freedom is considered in which a Houde damper is attached to the main vibration system with damping, positive or negative (the latter has a feature of self-excited vibration)^{(4),(6)}. The cases analysed in this paper as the alternate disturbing forces of forced vibration system are the following three cases: case I (a constant alternate exciting force is acting on the mass of the main system), case II (an alternate exciting force, of which the magnitude is proportional to the square of its frequency, is acting on the mass of the main system), and case III (an alternate displacement is given to the foundation of the

main system).

The Houde damper is applied mainly to the torsional vibration system. The objective in a forced vibration system of this paper is to get new knowledge on how to prevent the system from vibration by finding an optimum damping factor of the Houde damper and the corresponding value of maximum amplitude of the main vibration system. The objective of this paper which treats the self-excited vibration system is to get an optimum damping factor of the Houde damper and a critical value of the negative damping factor of the main system.

Nomenclature

- μ : mass ratio ($=m_2/m_1$)
- g : forced frequency ratio ($=\omega/\Omega_n$)
- H : ratio of damping factor to critical damping factor of the main vibration system ($=c_1/c_{c1}$)
- h : ratio of damping factor of Houde damper to the critical damping factor of the main vibration system multiplied by mass ratio ($=c_2/\mu c_{c1}$)
- η : magnitude of amplitude ratio of main vibration system ($=|X_1|/X_{st}$)
- ξ : magnitude of relative amplitude ratio between main vibration system and foundation ($=|X_1 - a_0|/a_0$)
- Ω_n : natural frequency of main vibration system ($=\sqrt{k_1/m_1}$)
- c_{c1} : critical damping factor of main vibration system ($=2\sqrt{m_1 k_1}$)
- X_{st} : statical deflection ($=p_0/k_1$)
- m_1 : mass of main system
- m_2 : mass of Houde damper
- c_1 : damping factor of main system
- c_2 : damping factor of Houde damper
- k_1 : spring constant of main system
- X_1 : amplitude of main system
- a_0 : amplitude of foundation
- ω : alternate disturbing external frequency
- p_0 : amplitude of disturbing force

* Received 3rd February, 1977.

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2. Investigation of a vibration system in which the main system has positive damping.

2.1 Case I (constant steady al-

ternate exciting force is acting on the mass of main system.)

2.1.1 Equations of motion and their analyses

To begin with, a vibration system with two degrees of freedom in which the main vibration system has positive damping will be treated. Fig.1 shows a vibration model. Equations of motion are given, where the damping force is assumed to be proportional to the velocity of vibration, and $j^2 = -1$.

$$\begin{cases} m_1 \ddot{x}_1 + (c_1 + c_2) \dot{x}_1 + k_1 x_1 - c_2 \dot{x}_2 = p_0 e^{j\omega t} \\ m_2 \ddot{x}_2 + c_2 \dot{x}_2 - c_2 \dot{x}_1 = 0 \end{cases} \quad (1)$$

Assuming furthermore that the solutions are of exponential forms expressed by $x_1 = X_1 e^{j\omega t}$, $x_2 = X_2 e^{j\omega t}$, the magnitude of amplitude ratio of the main vibration system is obtained.

$\eta = |X_1|/X_{st} = (a_1/b_1)^{1/2} \quad (2)$
where, $a_1 = 4h^2 g^2 + g^4$, $b_1 = 4g^2 [4H^2 g^2 + \{1 - (1+\mu)g^2\}^2] h^2 + 8Hg^6 \mu h + g^4 (g^2 - 1)^2 + 4H^2 g^6$, η being expressed by a function of μ, H, h and g . Fig.2 shows, as an example, the response curve (solid line) of the magnitude of amplitude ratio of the main system for $\mu=0.1$, $H=0.0625$ and $h = \{2(2+\mu)(1+\mu)\}^{-1/2}$. The value of h used in Fig.2 (solid line) is the optimum damping factor^{(1),(5)} of the Houde damper regarding the main vibration system with no damping. The response curve of the main vibration system with Houde damper has one peak pertaining to the exciting frequency and the feature of this curve differs from the peak of amplitude of the main system with a normal type absorber. If the height of this peak is expressed by η_1 , the value of h that can minimize the value of η_1 will have to be determined. Namely, for the present system, the optimum value of h is thus obtained such as to satisfy the following equation.

$$\partial \eta_1 / \partial h = 0 \quad (3)$$

2.1.2 Numerical analysis

Equating, $F = \partial \eta_1 / \partial h$, iterative procedure by Newton's method can be used to obtain the optimum value of h , for which $F=0$ is satisfied with sufficient accuracy. As the initial values of this procedure, the optimum value⁽⁵⁾ of h which is found in the case of $H=0$ can be used. This value is expressed by $h_1 = \{2(2+\mu)(1+\mu)\}^{-1/2}$. In this case, convergency speed of iterative calculation is found to be rather high, which is shown by $h_{n+1} = h_n + \Delta h_n \quad (4)$, which is derived from $F(h) + (\partial F / \partial h) \Delta h = 0$.

Fig.1 Vibration model

2.1.3 Analytical results and their investigation

Correction terms to be added to the optimum value of the vibration system without damping are given by $p(H, \mu)$ and $r(H, \mu)$. They correspond respectively to the damping coefficients of Houde damper of the main vibration system with damping and to the maximum response ratio of amplitude of the main vibration system.

$$h_{opt} = \{2(2+\mu)(1+\mu)\}^{-1/2} - p(H, \mu) \quad (5)$$

$$\eta_{max} = \{\mu/(2+\mu) + r(H, \mu)\}^{-1} \quad (6)$$

wherein, $\mu/(2+\mu)$ shows the inverse of the maximum magnitude of amplitude ratio of the vibration system for $H=0$. Practical formulae are obtained by use of correction terms expressed by the following.

$$p = p_1(\mu)H + p_2(\mu)H^2 \quad (7)$$

$$r = r_1(\mu)H + r_2(\mu)H^2 \quad (8)$$

Thus, the empirical formulae are obtained for various values of μ, H in the ranges: $\mu=0.1 \sim 0.3$, $H=0.000 \sim 0.125$. The formulae are shown as follows:

$$h_{opt} = \{2(2+\mu)(1+\mu)\}^{-1/2} - (0.719 - 2.45\mu + 5.3\mu^2) \quad (9)$$

$$H + (1.47 - 17.6\mu + 42\mu^2)H^2 \quad (9)$$

$$\eta_{max} = [\mu/(2+\mu) + (2.008 - 0.52\mu)H - (0.19 + 0.8\mu - \mu^2) \quad (10)$$

$$H^2]^{-1} \quad (10)$$

Tolerance or permissible error ranges attached to the above equations are found to be less than 1% for $\mu=0.03 \sim 0.40$, and $H=0.00 \sim 0.15$ (with sufficient accuracy); these are the ranges for the practical application of μ and H . In case of $H=0$, the above formulae are found to agree with the results obtained by Dr. J.E. Brock⁽¹⁾. Fig.2 shows as an example, ($\mu=0.1$, $H=0.0625$) the response curve (broken line) of amplitude ratio of a main system in which the optimum value of h obtained by Newton's iteration method is used. Comparing, in Fig.2, the broken line with the solid line, it is evident that the magnitude of the maximum amplitude ratio of the former is lower than that of the latter.

2.2 Case II (Magnitude of exciting force is proportional to the square of exciting force.)

Generally speaking, vibration arising from an unbalance of rotational parts of machinery corresponds to the case in which the magnitude of exciting force is proportional to the square of exciting frequency; such phenomena are widely seen in mechanical fields.

The vibration model of this case is

Fig.2 Magnitude of amplitude ratio of the main vibration system with the Houde damper

shown as a system with two degrees of freedom in which an alternate exciting force which is proportional to the square of exciting frequency is exerted on the main vibration system. To begin with, the following two items will be searched for: 1) h_{opt} , non-dimensional optimum damping ratio of Houde damper without damping in the main vibration system and 2) η_{max} , maximum magnitude of amplitude ratio of the main vibration system without damping. In order to obtain these values, an analysis is performed with a vibration model which is similar to the case in which a constant steady alternate exciting force is exerted in regard to frequency⁽⁵⁾.

$$\begin{cases} m_1 \ddot{x}_1 + c_2 \dot{x}_1 + k_1 x_1 - c_2 \dot{x}_2 = P_0 g^2 e^{j\omega t} \\ m_2 \ddot{x}_2 + c_2 \dot{x}_2 - c_2 \dot{x}_1 = 0 \end{cases} \quad (11)$$

Assuming that the solutions are of the forms $x_1 = X_1 e^{j\omega t}$, $x_2 = X_2 e^{j\omega t}$, the magnitude of amplitude ratio of the main system is as follows:

$$\eta = |X_1|/X_{st} = (a_2/b_2)^{1/2} \quad (12)$$

wherein $a_2 = a_1 g^4$, $b_2 = (b_1)_{H=0}$. h_{opt} , optimum damping of the Houde damper is found as follows: First, find fixed points⁽⁵⁾ lying on the response curve by use of Eq.(12). Second, the value of h_{opt} is found as a solution of equation: $\partial \eta / \partial g = 0$, wherein $\partial \eta / \partial g$ means a slope of this curve. Finally h_{opt} is obtained.

$$h_{opt} = [\{(5+6\mu+2\mu^2)^{1/2} - (1+\mu)\} / \{2(2+\mu)^2\}]^{1/2} \quad (13)$$

In this case, the maximum response magnitude of the main vibration system is as follows.

$$\eta_{max} = 2/\mu \quad (14)$$

Equations of motion of the vibration system in which the main system has damping are as follows:

$$\begin{cases} m_1 \ddot{x}_1 + (c_1 + c_2) \dot{x}_1 + k_1 x_1 - c_2 \dot{x}_2 = P_0 g^2 e^{j\omega t} \\ m_2 \ddot{x}_2 + c_2 \dot{x}_2 - c_2 \dot{x}_1 = 0 \end{cases} \quad (15)$$

As the result, η , magnitude of amplitude ratio of the main system can be analysed similarly to that of 2.1, which will be shown as

$$\eta = |X_1|/X_{st} = (a_3/b_3)^{1/2} \quad (16)$$

wherein, $a_3 = a_1 g^4$, $b_3 = b_1$. In this case, η , a function of g , can be acknowledged as a curve with a hump: h and η_{max} which corresponds to the vibration system (in which the main system has damping) can be found by Newton's method used in 2.1. As to the initial values which are used in the above iteration, Eq.(13) and Eq.(14) can be used. In this case, empirical formulae to find h_{opt} and η_{max} , as function of μ , H are given as follows:

$$h_{opt} = [\{(5+6\mu+2\mu^2)^{1/2} - (1+\mu)\} / \{2(2+\mu)^2\}]^{1/2} + (4.086 - 11.82\mu + 10.2\mu^2)H - (20.51 - 87.0\mu - 81\mu^2) \cdot H^2 \quad (17)$$

$$\eta_{max} = [\mu / \{2 + (2.010 + 0.48\mu)H - (0.20 - 0.2\mu)H^2\}]^{-1} \quad (18)$$

Errors arising from these equations amount to merely less than 1% in the practical range of μ and H ($\mu = 0.03 \sim 0.40$, $H = 0.00 \sim 0.15$).

2.3 Case III (alternate displacement is given to the foundation of the main system.)

Vibration structure equipped with a mechanical apparatus is widely seen in mechanical fields, especially in the machinery such as aircraft, vehicles and so on. Fig.3 shows a vibration model of two degrees of freedom in which an alternate displacement is given to the foundation of the main system. It is an important problem, in this case, to analyse a deflection of the main system,

as well as fatigue strength against the deflection; of course the former should be lowered. First of all, consider h_{opt} and ξ_{max} of the main vibration system without damping ($c_1 = 0$), wherein h_{opt} : optimum damping factor of the Houde damper; ξ_{max} : magnitude of relative amplitude between the mass of the main system and the foundation. In order to obtain ξ_{max} , equations of motion are derived by putting $y_1 = x_1 - y_0$, $y_2 = x_2 - y_0$.

$$\begin{cases} m_1 \ddot{y}_1 + c_2 \dot{y}_1 + k_1 y_1 - c_2 \dot{y}_2 = m_1 a_0 \omega^2 e^{j\omega t} \\ m_2 \ddot{y}_2 + c_2 \dot{y}_2 - c_2 \dot{y}_1 = m_2 a_0 \omega^2 e^{j\omega t} \end{cases} \quad (19)$$

The response ratio of relative amplitude between the main system and the foundation is gained.

$$\xi = |X_1 - a_0|/a_0 = (a_4/b_4)^{1/2} \quad (20)$$

wherein, $a_4 = g^4 + 4(1+\mu)^2 g^4 h^2$, $b_4 = g^2(g^2 - 1)^2 + 4h^2 \cdot \{1 - (1+\mu)g^2\}^2$. Using Eq.(20), h_{opt} and ξ_{max} are obtained in a similar manner to 2.2. as follows:

$$h_{opt} = [(2+\mu)/\{8(1+\mu)^2\}]^{1/2} \quad (21)$$

$$\xi_{max} = (2+\mu)/\mu \quad (22)$$

Equations of motion of a vibration model in which the main system has damping are as follows:

$$\begin{cases} m_1 \ddot{y}_1 + (c_1 + c_2) \dot{y}_1 + k_1 y_1 - c_2 \dot{y}_2 = m_1 a_0 \omega^2 e^{j\omega t} \\ m_2 \ddot{y}_2 + c_2 \dot{y}_2 - c_2 \dot{y}_1 = m_2 a_0 \omega^2 e^{j\omega t} \end{cases} \quad (23)$$

The solution of Eq.(23) is as follows:

$$\xi = |X_1 - a_0|/a_0 = (a_5/b_5)^{1/2} \quad (24)$$

wherein, $a_5 = a_4 g^2$, $b_5 = \{g^4 - (1+4Hh)g^2\}^2 + \{-2(h+H+\mu h)g^2 + 2h\}^2 g^2$. The values of h_{opt} and ξ_{max} (main system has damping) are obtained just by the procedure shown above in 2.2.

$$h_{opt} = [(2+\mu)/\{8(1+\mu)^2\}]^{1/2} - (0.346 + 1.54\mu - 5.3 \cdot \mu^2)H - (0.49 - 17.6\mu + 49\mu^2)H^2 \quad (25)$$

$$\xi_{max} = [\mu / \{2 + (2.010 + 0.50\mu + 0.2\mu^2)H - (0.19 - 0.1\mu)H^2\}]^{-1} \quad (26)$$

Errors which are found in the solutions of these equations are found to be less than 1% in the practical range of μ and H (like in the cases I and II).

3. Investigation of a vibration system in which the main system has negative damping.

This chapter treats the vibration prevention effects by a Houde damper which is attached to the main vibration system with negative damping (self-excited vibration system). It has been pointed out that the self-excited vibration system can be regarded as a system with negative damping.^{(4),(5),(6)}

Fig.3 Vibration model

The object of this chapter is to obtain the optimum damping coefficient, h_{opt} , of the Houde damper, which gives the optimum critical value satisfying the stability condition. Putting $p_0=0$ in Fig.1, equations of motion are given as follows, wherein $c_1 < 0$.

$$\begin{cases} m_1 \ddot{x}_1 + (c_1 + c_2)x_1 + k_1 x_1 - c_2 \ddot{x}_2 = 0 \\ m_2 \ddot{x}_2 + c_2 \ddot{x}_2 - c_2 \ddot{x}_1 = 0 \end{cases} \quad (27)$$

Assuming that the solutions are of exponential forms such that $x_1 = X_1 e^{st}$, $x_2 = X_2 e^{st}$, the characteristic equation is obtained as follows:

$$s^3 + \{c_2/m_2 + (c_1 + c_2)/m_1\}s^2 + \{c_1 c_2/(m_1 m_2) + k_1/m_1\}s + k_1 c_2/(m_1 m_2) = 0 \quad (28)$$

Putting Eq.(28) $s^3 + A_2 s^2 + A_1 s + A_0 = 0$, the necessary and sufficient condition of Eq.(28) is given by $A_1 A_2 - A_0 > 0$ ⁽⁵⁾. It is found that the following relations are necessary for the stability of the vibration system.

$$4(1+\mu)Hh^2 + (4H^2 + \mu)h + H \geq 0 \quad (29)$$

Fig.4 shows, as an example, a stability region in which $\mu=0.10$ is used. From this curve, negative damping factor, $(-H)$, of the main vibration system takes the maximum value at one point (point C) on this curve, and the ordinate of the point shows the optimum viscous damping factor, $(h)_{opt}$, of a sub-system with a Houde damper. The value of $(-H)_{cr}$ depends on the area of stability and the larger the value of $(-H)_{cr}$, the larger the area of the stability region becomes. As a matter of fact, it is difficult to foresee the value of $(-H)$, therefore it is preferable to use the value $(h)_{opt}$ as a damping factor of the Houde damper. Self-excited vibration system shows an unstable state in the outer area of the critical value of the negative damping which is shown as the abscissa of stability. (see Figure). The critical value of $(-H)$ and the optimum value of h can be obtained from the following two equations: $\partial(-H)/\partial h = 0$ and the formula for stability limit. Thus, we have $8(1+\mu)Hh + 4H^2 + \mu = 0$ ---(30) and using the equations (29) and (30), the values of $(h)_{opt}$ and $(-H)_{cr}$ are obtained as follows:

Fig.4 An example of the stability region of the self-excited vibration system. ($\mu=0.1$)

$$(h)_{opt} = 1/\{2\sqrt{1+\mu}\} \quad (31)$$

$$(-H)_{cr} = \mu/4 \quad (32)$$

The critical curve for stability (see Fig.4) is a curve having a certain curvature which differs from the critical curve for stability⁽³⁾ given to the value of f_{opt} ($=\kappa_{opt}$) of the normal type absorber. A comparison of the critical value of negative damping to that of the normal type absorber shows that the value of the former becomes less than that of the latter, while the value of the optimum damping ratio becomes larger. When the equivalent negative damping value can be estimated in the original self-excited vibration system, it is possible to stabilize the system by use of Houde damper. In this case it is preferable to increase the value of μ at least greater than $\mu_{min}(=4H)$,---(33) and that the value given by Eq.(31) is to be used for h_{opt} .

4. Investigation of a vibration system in which the main system with negative damping is subjected to an alternate disturbing external force.

Consider the main vibration system with negative damping (self-excited vibration system); even if it is situated in the stability region of the self-excited vibration, it is preferable that the response to the outer exciting force be minimum. Therefore, it is important to investigate how to prevent the forced self-excited vibration as well as the outer exciting force from the vibration system.

4.1 Constant steady alternate exciting force is acting on the mass of the main system.

The analysis can be done in a similar way to the case in which the value of H is positive and the magnitude of amplitude ratio of the main system is given by Eq.(2). As an example, Fig.5 shows the curve (solid line) of the magnitude of amplitude ratio of the main system for $\mu=0.1$, $H=-0.01$ and $h=\{2(2+\mu)\cdot(1+\mu)\}^{-1/2}$. The response curve of the main vibration system forms one peak for the exciting frequency just like in the case where $H>0$. However, the available region of damping of the main system should not be outside of the critical curve of negative damping previously treated. In case the main

Fig.5 Magnitude of amplitude ratio of the main vibration system with negative damping when the Houde damper is used.

system has negative damping, it is preferable that the response to the exciting force be minimum in the stability region of self-excited vibration. The optimum value of h as well as the value of the corresponding η_{max} is obtained by iterative procedure similar to the one given in chapter 2. The values of μ , H are $\mu=0.1\sim0.3$, $H=0.00\sim0.02$.

$$h_{opt} = \{2(2+\mu)(1+\mu)\}^{-1/2} + (1.057 - 66.65\mu + 5.2\mu^2) \cdot H + (48.51 - 7.0\mu - 516\mu^2)H^2 \quad (34)$$

$$\eta_{max} = [\mu/(2+\mu) + (2.001 - 0.51\mu)H + (0.40 - 3.5\mu + 6\mu^2)H^2]^{-1} \quad (35)$$

These formulae have errors of merely less than 1% in the practical range which is shown by the values of $\mu=0.03\sim0.40$ in case the value of $(-H)$ is given in the stability region. Fig.5 shows, as an example, a response curve (broken curve) of the magnitude of amplitude ratio of the main system in which the optimum value of h is used, wherein $\mu=0.1$ and $H=-0.01$. Comparing this curve with the solid curve, the magnitude of amplitude ratio of the main system (broken line) is found lower. Fig.6 shows, as an example, computational results on alternate exciting force in the stability region for the self-excited vibration by use of h_{opt} , the optimum damping value of the sub-system for $\mu=0.1$. It is evident that in the stability region, the value of h for which the response becomes minimum for the alternate constant steady exciting force increases slowly and linearly with an increase of $(-H)$ and passes through a point adjacent to the one which corresponds to the critical value of negative damping of self-exciting system. The most favourable condition for the alternate exciting force with negative damping consists in that h , damping of the Houde damper, be situated on the above curve. However, it seems to be difficult to find the value of equivalent negative damping; therefore the optimum value of h can be

Fig.6 Characteristics of the optimum damping factor of the Houde damper in case the vibration system is subjected to an alternate exciting force in the stability region of the self-excited vibration.

taken approximately equal to $h_{opt} = 1/(2\sqrt{1+\mu})$, optimum value of h for the self-excited vibration. In this case the value of damping of the Houde damper, as a vibration prevention equipment against alternate exciting force (constant periodic force), can be given as the above h_{opt} .

4.2 Case in which the magnitude of exciting force is proportional to the square of exciting force.

Vibration system in which the main system has negative damping and is subjected to the alternate exciting force (proportional to the square of exciting frequency) can be analysed similarly to the system developed in 2.2. The magnitude of amplitude ratio of the main system is shown by Eq.(16). However, $H < 0$. The optimum values of h as well as the value of the corresponding η_{max} are obtained by iterative procedure (Newton's method) similar to the one given in 4.1. The values of μ , H are $\mu=0.1\sim0.3$, $H=0.00\sim0.02$. Empirical formulae are here shown by the following:

$$h_{opt} = \{[(5+6\mu+2\mu^2)^{1/2} - (1+\mu)]/[2(2+\mu)^2]\}^{1/2} - (23.230 - 84.64\mu + 154.5\mu^2)H - (965.09 - 4715.1\mu + 8632\mu^2)H^2 \quad (36)$$

$$\eta_{max} = [\mu/2 + (2.003 + 0.44\mu + 0.5\mu^2)H + (0.51 - 5.6\mu + 35\mu^2)H^2]^{-1} \quad (37)$$

These formulae, in the practical range, have errors of merely less than 1%. It is well understood as in the statement of 4.1 that the value of h for which the response to the external exciting force (alternate force proportional to the square of frequency) becomes minimum can be found as a value nearly equal to the critical value of negative damping of the self-excited vibration system and as the optimum value of damping of the Houde damper, the value of h_{opt} can be taken

Fig.7 Maximum magnitude of amplitude ratio of the main vibration system with the change of the mass ratio.

in practice for prevention of self-excited vibration.

4.3 Alternate displacement is given to the foundation of the main system.

Vibration system in which the main system has negative damping and an alternate displacement is given to the foundation of the main system can be analysed similarly to the system developed in 2.3.

The magnitude of relative amplitude of main vibration system and foundation is given by Eq.(24). However, $H < 0$. The vibration system for which the main system has negative damping is analysed similarly to the previous cases and the empirical formulae for $\xi_{\max}$ and h_{opt} ($\mu = 0.1 \sim 0.3$, $H = 0.00 \sim -0.02$) are given as follows:

$$h_{\text{opt}} = [(2+\mu)/\{8(1+\mu)^2\}]^{1/2} - (0.923 + 0.54\mu - 8.4 \cdot \mu^2)H + (8.81 - 441.7\mu + 1379\mu^2)H^2 \quad (38)$$

$$\xi_{\max} = [\mu/(2+\mu) + (1.998 - 0.48\mu + 0.1\mu^2)H + (0.29 - 2.1\mu + 4\mu^2)H^2]^{-1} \quad (39)$$

The errors arising from the above formulae are found to be less than 1% in the practical range ($\mu = 0.03 \sim 0.40$) for the value of $(-H)$ given in the stability range. The value of h_{opt} is nearly equal to the value of $(-H)_{\text{cr}}$ in the stability region of the self-excited vibration. The results are quite similar to those of the previous two cases.

5. Comparison of vibration prevention effects of normal type absorber and Houde damper.

The optimum design of the normal type absorber was reported in our previous paper.^{(2),(3)} Now a comparison of the vibration prevention effects of these two types will be made.

As an example, the vibration prevention effects of a system which is subjected to a constant alternate exciting frequency are as follows: Assuming that the value of H is constant and takes a small value for a small value of μ , the maximum magnitude of amplitude of the main system should have a value nearly equal to $\sqrt{2/\mu^2}$ for normal type absorber, but $2/\mu$ for Houde damper. Fig.7 shows a graph of $\eta_{\max}$ as a function of μ , in case the main

system is subjected to a constant alternate exciting frequency. First of all, in the case of $H=0$ (see Fig.7), the maximum magnitude of amplitude of the main system with normal type absorber is lower in comparison with the one having Houde damper. Therefore, the former is better for the purpose of vibration prevention. The value of $\eta_{\max}$ should take a value of about $1/(2H)$ at $\mu=0$ (see Fig.7), but the errors of the empirical formula increase in the range of $\mu < 0.05$ (not in practical range). The value of $\lim_{\mu \rightarrow 0} \eta_{\max}$ differs slightly from that of $1/(2H)$. In each case, as the damper mass ratio μ increases, the vibration prevention effect increases. Normal type absorber has a better prevention effect compared with the one having Houde damper in the following cases:

- 1) The main system which is subjected to the exciting frequency.
 - 2) Alternate displacement is given to the foundation of the main vibration system.
- In the stability region of the self-excited vibration system, the critical value of negative damping is equal to the value of $\sqrt{\mu}/2$ ⁽³⁾ for the normal type absorber, but to the value of $\mu/4$ for the Houde damper. Therefore the former is more advantageous than the latter. Fig.8 shows a graph of $(-H)_{\text{cr}}$ as a function of μ . The critical value of the negative damping of the normal type absorber takes a larger value than that of the Houde damper. Therefore the former is better than the latter. Fig.8 shows the curve of $(-H)_{\text{cr}}$ as a function of μ ; $(-H)_{\text{cr}} = \sqrt{\mu}/2$ for the normal type absorber and $(-H)_{\text{cr}} = \mu/4$ for the Houde damper. Conversely, if the value of $(-H)_{\text{cr}}$ is given, the value of μ is shown as follows: $\mu = 4(-H)_{\text{cr}}^2$ for the normal type absorber and $\mu = 4(-H)_{\text{cr}}$ for the Houde damper. The critical value of the negative damping increases as the value of μ increases in the practical range of dampers. Fig.9 shows a graph of $(h)_{\text{opt}}$ as a function of μ . The value of $(h)_{\text{opt}}$ of the normal type absorber is lower than that of the Houde damper. The empirical formula of $(h)_{\text{opt}}$ differs slightly in case μ is relatively small.

Fig.8 The critical value of negative damping factor of the main vibration system with the change of the mass ratio.

Fig.9 The optimum damping factor of the auxiliary system with the change of the mass ratio.

6. Conclusions

The conclusions drawn from the above analyses of the Houde damper attached to the main system with damping are as follows:

- (1) Empirical formulae about the optimum damping of the Houde damper and the corresponding value of maximum amplitude of the main system obtained here are valid in practice.
- (2) It is evident that the magnitude of the maximum amplitude ratio of the main vibration system with damping is lower than that without damping.
- (3) Stability region for self-excited vibration system can exist only in the region for relatively small values of the negative damping. It is shown that this region can be enlarged up to the critical value, if the optimum damping is given to the Houde damper.
- (4) It is found that the optimum value of damping of the Houde damper, for which the main system has negative damping and is subjected to various alternate exciting forces, comes close to the optimum value of damping of the Houde damper attached to the self-excited vibration system.
- (5) Concerning the vibration prevention effects of Houde damper, the maximum magnitude of amplitude ratio of the main system is relatively large comparing to that of the normal type absorber and the stability region of the self-excited vibration system is relatively small. Therefore the normal type absorber has the advantage of the vibration prevention effects. However, the Houde damper is sometimes convenient for practical use because of the simplicity of its structure.

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