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Study on Bending Fatigue Strength of Helical Gears*

(1st Report, Effect of Helix Angle on
Bending Fatigue Strength)

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This paper presents an investigation into the effect of helix angle on the root fillet stresses and the bending fatigue strength of helical gear teeth with a comparatively small face width. The root fillet stresses were investigated based on the principle of superposition and the moment-image method and compared with the results of strain-gage investigation. The maximum root stress occurs under the condition that the meshing point in the central transverse plane coincides with the outer point of single tooth pair contact for any degree of helix angle ($\beta_0 \leq 30^\circ$).

Furthermore the effect of helix angle on bending fatigue strength was also investigated using the bending fatigue testing machine for cylindrical gears, which had been developed by the authors. It was found that the bending fatigue limit loads (circumferential loads) for helical gears are almost the same regardless of degree of helix angle.

1. Introduction

Helical gears are used increasingly often as power transmitting gears since not only they can carry larger loads because of their lower dynamic load but also their noise in operation is smaller compared with spur gears. Most of investigations^{(1)~(7)} on helical gears in the past have dealt with concentrated stresses on the root fillet, load distributions on the contact line, dynamic behaviors and so on; and the bending fatigue test results for helical gears are very few. The bending fatigue breakage, which is a basic problem on the bending strength of gear teeth, and the change of bending fatigue strength under various loading conditions were hardly investigated. The bending strength design is still carried out by modifying approximately the strength formulas for spur gears on the basis of the data obtained for spur gears.

The aim of this study is to investigate the bending fatigue breakage of helical gear teeth by carrying out the bending fatigue test for helical gears of various materials and under various heat treatment conditions and to obtain more precise data on the bending strength design for helical gears by making clear the change of bending fatigue strength under various conditions.

In the present paper, the authors

investigated the effects of helix angle on the bending fatigue strength of helical gears with a small overlap ratio ($\epsilon_{sp} < 1.1$), in which the bending strength poses a specific problem. A theoretical calculation of root stresses in helical gear teeth is carried out, and the effects of helix angle on the root stresses in helical gear teeth, the worst meshing condition under which the maximum root stress occurs and the total length of line of contact are clarified by carrying out a static loading test. Furthermore the bending fatigue strength of helical gear teeth is obtained by carrying out bending fatigue tests for helical gears with various degrees of helix angle ($\beta_0 = 0^\circ, 10^\circ, 20^\circ, 30^\circ$) using the bending fatigue testing machine, which was developed by the authors, and discussion is made on the basis of the static loading test results. Consequently the effect of helix angle on bending fatigue strength has been clarified considerably.

2. Calculation of root stresses

2.1 Calculating formulas of root stresses⁽¹⁾

The stresses in helical gear teeth are calculated, considering the helical gear teeth as a cantilever plate with a finite length and the distributed load along the contact line of the tooth as a group of concentrated loads per unit length of the line of contact. The load distribution along the line of contact in helical gear tooth is calculated from the empirical equation developed by G. Niemann and W. Richter⁽²⁾. The factor β for bending moment at the clamped edge changes

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with x , the distance between the loading position and the position for calculating the stress, η , the distance between the loading position and the free edge, and ξ , the distance between the loading position and the middle of tooth trace, taking the coordinate axes for gear tooth as shown in Fig.1. As for $\beta = \beta_{\xi=0}$ due to the load acting at the middle of the tooth trace, the value of β calculated for a cantilever plate with infinite length by K. Fujita⁽³⁾ is used, and for $\beta = \beta_{\xi=\xi_i}$ due to the load acting at a position separated by distance ξ_i from the middle of the tooth trace, the values of β obtained by applying the moment-image method proposed by E. J. Wellauer and A. Seireg⁽⁴⁾ to the values calculated by K. Fujita are used. Coordinate axes for helical gear tooth and the notations for calculation of stress at root fillet are taken as shown in Fig.1. The intersecting point of the free edge AB

Fig.1 Coordinate system for distributed load on helical gear tooth

with a perpendicular to AB through the point G_i is taken as the origin, where the point G_i is the intersecting point of the line of action of the distributed load p_i with the middle plane ABCD. And the free edge is taken as x-axis and the direction at right angles to x-axis through the origin as y-axis. And the distance from the middle of free edge O_0 to the origin O_i corresponding to each loading position is taken as ξ_i , and the distance from each origin to the loading position G_i as η_i . Thereby, the bending root stress σ_{ξ_i} which occurs at the point Q_i separated by ξ_i from the middle of tooth trace can be calculated from the following Eq.(1) proposed by H. Kugimiya⁽¹⁾.

$$\sigma_{\xi_i} = \sum_i \frac{\beta_{\xi=\xi_i, \eta=\eta_i, x=-(\xi_i-\xi_j)}}{\beta_{\xi=\xi_i, \eta=\eta_i, x=0}} \sigma'_{\xi=\xi_i, \eta=\eta_i, x=0} \quad (1)$$

where $\sigma'_{\xi=\xi_i, \eta=\eta_i, x=0}$ and $\beta_{\xi=\xi_i, \eta=\eta_i, x=0}$ are the bending stress and the factor for bending moment respectively, which occur

Fig.2 Meshing positions for calculation and measurement of root stresses (G.B)

Table 1 Dimensions of test gears for calculation and measurement of root stresses

Gear sign		G.A	G.B	G.C
Tooth profile		Standard	Standard	Standard
Normal module	m_n	6	6	6
Normal pressure angle	α_n	20°	20°	20°
Helix angle	β_0	10° (R · L)	20° (R · L)	30° (R · L)
Number of teeth	z	36	36	36
Pitch circle diameter	d_0	219.33 mm	229.86 mm	249.42 mm
Face width	b	40 mm	40 mm	40 mm
Transverse contact ratio	ϵ_s	1.66	1.55	1.38
Overlap ratio	ϵ_{sp}	0.37	0.73	1.06
Whole depth	H	13.98 mm	13.95 mm	13.91 mm
Chordal thickness at root fillet	s_n	12.75 mm	12.93 mm	13.60 mm

at the corresponding root point Q_i due to the distributed load p_i acting at a point separated by ξ_i in x-axis direction from the middle of tooth trace and by η_i in y-axis direction from the tip of the tooth, and $\beta_{\xi, \eta, \gamma, \alpha} = -(\xi_i - \xi_j)$ is the factor for bending moment for a root point separated by $-(\xi_i - \xi_j)$ in x-axis direction from the loading position.

2.2 Calculating method

Dimensions of gears used for calculation of root stresses are shown in Table 1. Figure 2 shows the meshing positions selected for calculation and measurement of root stresses for G.B. Root stresses of G.B are calculated under a circumferential load of 1000 kg with regard to the case when the line of contact passes through each meshing point dividing the length of action into 13 (26, 17 for G.A, G.C respectively) equal parts where the beginning and the end of engagement in the central transverse plane are denoted by the position numbers ① and ⑭ respectively. The position of critical section is determined by applying Hofer's method on the normal section. The length H from critical section to the tip of tooth and the chordal thickness s_n at the critical section are measured to be used for stress calculation. The values of H and s_n for each gear are given in Table 1.

3. Measurement of root stresses

3.1 Test gears

Dimensions of test gears are the same as those of gears used for calculation and given in Table 1. Test gears are of SNC21 steel and teeth surfaces are ground after casehardening. Accuracy of gears is of 1st class in JIS. Figure 3 shows the tooth profile and lead error diagrams for G.B. Tooth profile errors of L.-face and R.-face are 7μ and 5μ respectively, and lead errors are 5μ and 4μ respectively.

The stresses in helical gear teeth were measured by electric resistance strain gages with 1 mm gage length using

static strain meter and switch box under a circumferential load of 1000 kg.

Strain gages were pasted at seven points of the critical section determined by applying Hofer's method on the normal section along tensile fillets for gears with any degree of helix angle as shown in Fig.4. For G.B strain gages were also pasted at the root fillet of compressive side, and both tensile and compressive stresses were measured at the same time. Stress values are obtained by multiplying strain readings by modulus of elasticity $E = 21000 \text{ kg/mm}^2$.

(a) Tooth profile error (b) Lead error

Fig.3 Tooth profile and lead error diagrams of test gears

Fig.4 Positions of strain gages (G.B)

Table 2 Dimensions of test gears for bending fatigue test

Gear sign		G.0		G.1		G.2		G.3	
Tooth profile		Standard		Standard		Standard		Standard	
Normal module	m_n	3		3		3		3	
Normal pressure angle	α_n	20°		20°		20°		20°	
Number of teeth	z	36		36		36		36	
Helix angle	β_0	0°		10°		20°		30°	
Helix direction		—		Left	Right	Left	Right	Left	Right
Pitch circle diameter	d_0	108.00 mm		109.67 mm		114.93 mm		124.71 mm	
Face width	b	20 mm	30 mm	20 mm	30 mm	20 mm	30 mm	20 mm	30 mm
Transverse contact ratio	ϵ_s	1.69		1.66		1.55		1.38	
Overlap ratio	ϵ_{sp}	0		0.37		0.73		1.06	

($b = 30 \text{ mm}$: for supporting gear)

3.2 Static loading apparatus

In the static loading apparatus used in this experiment, a test gear can be loaded at arbitrary meshing positions with a manual pump and a loading bar by fixing a supporting gear. The applied load value is detected from the reading of the strain gages pasted at the shaft.

The meshing positions are determined by measuring the distance between two specific gear teeth in one pair of gears with a reading microscope. The backlash can be set by changing a center distance, and a circumferential backlash $c_0 = 110 \mu$, which is equal to a half of the minimum value specified in JIS, is selected in this experiment.

4. Bending fatigue test

4.1 Test gears

Dimensions of gears for bending fatigue test are shown in Table 2. Test gears are made of S45C normalized steel and cut with a hob, and accuracy of gears is of 4th or 5th class in JIS. Supporting gears meshing with test gears are made of SCM21 steel and teeth surfaces are ground after casehardening, and the accuracy of gears is of 1st class in JIS. Dimensions of each gear for bending fatigue test are the same as those of gears used in the measurement of root stresses except normal module and face width.

4.2 Bending fatigue testing machine for cylindrical gear

Figure 5 shows the bending fatigue testing machine for cylindrical gear, which was developed by the authors. This testing machine is of hydraulic type and consists of a fuel injection pump and its driving apparatus, a pressure controller, a pulsating torque generator and a gear supporting apparatus. The torque generator consists of a rotary wing with four blades and its casing. The rotary wing and the test gear are attached to the same shaft and the test gear is meshed with the supporting gear to be fixed at arbitrary meshing positions. Thus gear teeth are loaded by a torque generated by driving the rotary wing with hydraulic pressure. The applied torque is detected with strain

Fig.5 Bending fatigue testing machine

Fig.6 Comparison of strain wave forms at root fillet and at shaft

gages pasted at the shaft. The center distance is variable from 105 mm to 130 mm. The maximum applied torque T is 200 kgm, and the frequency of load repetitions is about 500 c/min. And the meshing positions can be determined by measuring the distance between two specific gear teeth with a reading microscope.

Bending fatigue test was carried out with gears meshing at the worst loading position on the basis of the static loading test results. The backlash was set by changing a center distance so that it might be of maximum value specified in JIS.

Figure 6 shows the strain wave forms measured at the root fillet of gear tooth and at the shaft. It is seen from this figure that both wave forms are similarly good and of perfect one side loading condition. Compared with the running test using a power circulating type gear testing machine, a bending fatigue test using this testing machine has advantages in that no effect of dynamic load is involved in the test results and that the S-N curve can be obtained economically with only one pair of gears.

5. Calculated and experimental results and discussions

5.1 Calculated and measured results of root stresses

Figure 7(a), (b), (c) show the calculated root stress distributions in longitudinal direction for G.A, G.B, G.C respectively, and Fig.8(a), (b), (c) show the measured ones. Each number in these figures indicates the meshing position number. It is found from Figs.7 and 8 that maximum root stresses occur at the meshing positions ⑮ and ⑨ for G.A and G.B respectively both in calculated and measured results. These meshing positions coincide almost with the outer point of single tooth pair contact in the central

transverse plane. As for G.C, meshing position for maximum root stress is ⑮ in measured results, and ⑰ in calculated results. But the difference between maximum root stresses at the positions ⑮ and ⑰ is small in calculated results as seen from Fig.7(c). It may be considered, therefore, that the maximum root stress occurs near the meshing position ⑮.

(a) G.A ($\beta_0 = 10^\circ$)

(b) G.B ($\beta_0 = 20^\circ$)

(c) G.C ($\beta_0 = 30^\circ$)

Fig.7 Calculated root stress distribution in longitudinal direction ($P = 1000$ kg)

The meshing position ⑮ lies close to the outer point of single tooth pair contact in the central transverse plane. Figure 9 shows the measured root stress distributions in longitudinal direction at the compressive side for G.B. As seen from Fig.8(b) and Fig.9, the absolute value of the ratio of the maximum root stress at the compressive side to that at the tensile side is about 1.15. This may be attributed to the effect of the radial component of the tooth load.

Figure 10 compares the calculated root stresses with the measured ones at

(a) G.A ($\beta_0 = 10^\circ$)

(b) G.B ($\beta_0 = 20^\circ$)

(c) G.C ($\beta_0 = 30^\circ$)

Fig.8 Measured root stress distribution in longitudinal direction ($P = 1000$ kg, $c_0 = 110 \mu$)

the middle of face width. In Fig.10 the maximum stresses are given as 1.0 and the stresses occurring at the other meshing positions are given as the ratios to the maximum stress. It is found from Fig.10 that the meshing position of maximum root stress calculated on the basis of Niemann-Richter distribution is in good agreement with the experimental result, but that calculated on the basis of uniform distribution differs slightly from the experimental result.

Figure 11 shows changes in the total length of line of contact with the progress of meshing for test gears. Main dimensions of spur gear ($\beta_0 = 0^\circ$) are the same as those of helical gears. The positions of the beginning and the end of engagement are taken on the central transverse plane. Arrows in this figure indicate the meshing positions of maximum root stresses. The meshing position of maximum root stress coincides almost with the outer point of single tooth pair contact (E) in the central transverse plane for any degree of helix angle ($\beta_0 \leq 30^\circ$) in calculated and measured results. The position of maximum root stress tends to shift a little toward a position at which the total length of line of contact becomes minimized from the outer point of single tooth pair contact.

Figure 12 shows the relation between

Fig.9 Measured root stress distribution in longitudinal direction at compressive side (G.B)

Fig.10 Comparison of calculated root stresses with measured ones at middle of face width (G.B)

the maximum root stress and the helix angle for G.A, G.B, G.C used in calculation and measurement of root stresses. In this figure the curve (a) shows the relation between the helix angle and the calculated maximum root stress, and it is seen therefrom that the maximum root stress tends to decrease with an increasing degree of helix angle. The curves (b), (c), (d) show the relations between the helix angles and the maximum root stresses measured by the static loading tests for various backlashes. The measured maximum root stresses tend to increase slightly but conversely the calculated ones do to decrease with an increasing degree of helix angle. This might be attributed to the fact that the calculated value is a so-called nominal

Fig.11 Change of total length of line of contact with helix angle

Fig.12 Relation between maximum root stress and helix angle

stress, and that the compressive stress due to the radial component, shearing stress due to the circumferential component of the tooth load, effects of stress concentration, gear errors and incomplete parts of tooth ends⁽⁸⁾ are not considered in the calculation of stresses. The backlashes $c_0 = 110, 220, 640 \mu$ correspond to a half of minimum backlash, minimum backlash and maximum backlash specified in JIS for test gear pairs respectively. As is seen from the curves (b), (c), (d), the effect of backlash on maximum root stress is very small even if the backlash changes from maximum to minimum specified in JIS. Hence it may be considered that the maximum root stresses are almost the same regardless of degree of helix angle under a constant circumferential load.

5.2 Results of bending fatigue test

Figure 13 shows the bending fatigue test results for test gears G.0, G.1, G.2 and G.3 carried out with gears meshing at the worst loading positions on the basis of the above-mentioned static loading test results. It is found from this figure that the bending fatigue limit loads (circumferential loads) for helical gears are almost the same regardless of degree of helix angle. This might be explained

Fig.13 Bending fatigue test results

Fig.14 Relation between transmitted torque and helix angle

from Fig.12 as follows. The maximum root stresses (measured values) are almost the same regardless of degree of helix angle under a constant circumferential load. On the other hand, the time strength tends to increase largely with an increasing degree of helix angle. The reasons may be as follows. The maximum root stresses σ_{tmax} for helical gears are almost the same regardless of degree of helix angle under a constant circumferential load as shown in Fig.12, and the stress concentration factor increases with an increasing degree of helix angle, as the radius of curvature at root fillet decreases with an increasing degree of helix angle. Therefore the larger the stress concentration factor at the root fillet (the larger the degree of helix angle) is, the smaller the nominal stress is ($\alpha_n = \sigma_{tmax} / \alpha$). And since the radius of curvature at root fillet has no effect on the stresses at the tip of a crack after the crack appears, the crack propagates more slowly in the helical gear with a larger degree of helix angle. Consequently the number of load cycles from initiation of a crack to tooth breakage may become larger with an increasing degree of helix angle.

Figure 14 shows the relation between the transmitted torque corresponding to the bending fatigue limit load and the helix angle for helical gears with various degrees of helix angle. In Fig.13 the bending fatigue limit loads (circumferential loads) are found almost the same regardless of degree of helix angle, so it may be considered that the transmitted torque increases with an increasing degree of helix angle, since the pitch circle diameter increases with an increasing degree of helix angle for standard helical gears with the same dimensions except for helix angle. Figure 15 shows the positions of crack initiation and the paths of crack propagation in each normal section. The positions of crack initiation lie in the neighborhood of critical section determined by Hofer's method, but the paths of crack propagation differ in each

Fig.15 Position of crack initiation and path of crack propagation(G.B)

Fig. 16 Effect of meshing position on bending fatigue strength

section. This might be because the stress conditions of root fillet differ in each section since the loading positions in each section differ as indicated by arrows because of inclination of contact line in helical gear tooth. In order to examine the effect of the meshing positions on the bending fatigue strength of helical gear teeth, bending fatigue tests were carried out at the meshing position ⑨ of maximum root stress (see Fig. 2) and at the positions ⑧, ⑩ distant slightly from the position ⑨. The bending fatigue test results are summarized in Fig. 16. It is found from this figure that both the time strength and the bending fatigue strength vary little even if the meshing position in the fatigue test shifts slightly from the position ⑨.

6. Conclusions

The effect of helix angle on bending fatigue strength was investigated by static loading and bending fatigue tests for helical gears with different degrees of helix angle and comparatively small face width. The main results obtained from this investigation are summarized as follows.

- (1) The meshing position of maximum root stress (worst loading position) for helical gears lies in the close vicinity of the outer point of single tooth pair contact in the central transverse plane regardless of degree of helix angle ($\beta_0 \leq 30^\circ$).
- (2) The maximum root stress occurs at a position distant slightly from the end of tooth trace toward the middle.
- (3) The maximum root stress at the compressive side is larger than that at the tensile side, and the ratio is about 1.15.

(4) The maximum root stresses are almost the same regardless of the degree of helix angle and the amount of backlash under a constant circumferential load.

(5) The bending fatigue limit loads (circumferential loads) for helical gears are almost the same regardless of degree of helix angle. Accordingly the transmitted torque increases with an increasing degree of helix angle. And the time strength for normalized steel helical gears increases with an increasing degree of helix angle.

(6) The positions of crack initiation in each normal section lie in the neighborhood of critical section determined by Hofer's method, but the paths of crack propagation differ in each section.

(7) Both the time strength and the bending fatigue strength vary little even if the bending fatigue tests are carried out at the meshing position distant slightly from the worst loading position.

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