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Visco-elastic Half-space Subjected to
Impact Torsion by a Rigid Cylinder*By Hiroyuki MATSUMOTO**, Ichiro NAKAHARA***
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The three dimensional analysis of stress and strain in linear visco-elastic bodies is considered when the loading is impulsive.

It is shown that removal of the time variable by applying the Laplace transformation enables a solution to be obtained in terms of an associated linear elasto-dynamic problem. Thus the analytical methods based on the stress functions can be utilized in visco-elastic analysis.

As an example, the mixed boundary value problem of a semi-infinite body subjected to impulsive torsion on its surface by a rigid cylinder is analyzed assuming the material to be a 3-parameter standard solid model. Stress, displacement and stress intensity factor are shown for the cases of a given rotational angle and a given torque of the cylinder.

1. Introduction

It is well known that the time history of the measured impact stress shows a wave decaying phenomenon. The main sources of it are the following:

- (1) Visco-elastic properties of the materials.^{(1),(2)}
- (2) Effects of the experimental conditions.
 - (a) Energy release from the support of the specimen.
 - (b) Energy emission to the air from the vibrating surface of the specimen.⁽³⁾
 - (c) Energy absorption in the impacted portion where the plastic deformation occurs.
- (3) Effects of the dispersion phenomenon of the wave propagation as coupled with the effects of (1) and (2).
- (4) Distortion of the wave due to the experimental arrangement.
 - (a) Effects of the strain gauge and the glue.
 - (b) Effect of the capacitance of the electric circuit.

It is not easy to separate these effects from the measured wave shape.

The main purpose of this paper is to get some theoretical information about the effect of the viscosity on the impact stress.^{(4),(5)}

The analytical method for the dynamic theory of elasticity can be applied to the dynamic theory of linear visco-elasticity based on the correspondence principle.^{(1),(2),(6)} Namely, the static and dynamic solutions of the theory of elasticity correspond to the

quasi-static and dynamic solutions of the linear visco-elastic problem, respectively, in the transformed domain of the Laplace transformation.

Recently, the fundamental and applied researches of the dynamic mixed boundary value problem have been receiving significant attention of many people in the fields of dynamic fracture mechanics,⁽⁷⁾⁻⁽⁹⁾ the dynamic contact problem^{(10),(11)} and the impact problem.⁽¹²⁾ The authors have analyzed the problem of an elastic half-space subjected to an impulsive torsion by a rigid cylinder attached on it, based on the three dimensional dynamic theory of elasticity.⁽¹⁰⁾

In this paper, the method applied to the previous work has been evolved to analyze the dynamic visco-elastic problem.

2. Basic equations

The solutions of the linear dynamic theory of elasticity are expressed using several kinds of stress functions. The Laplace transformation method is one of the powerful techniques to analyze the impulsive stress problems. It will be shown below that they can be applied to the linear dynamic theory of visco-elasticity.

The stress-strain relations for homogeneous isotropic linear visco-elastic materials are assumed as follows:^{(1),(2)}

$$\left. \begin{aligned} P_1(D)S_{ij} &= Q_1(D)E_{ij} \\ P_2(D)s &= Q_2(D)e \end{aligned} \right\} \dots\dots\dots (1)$$

where

$$P_k(D) = \sum_{n=0}^{N_k} p_n D^n, \quad Q_k(D) = \sum_{n=0}^{N_k} q_n D^n$$

$D = \partial/\partial t$: p_n and q_n are material constants.

In Eq.(1), S_{ij} , E_{ij} , s and e are deviator stresses, deviator strains, the sum of principal stresses and the dilatation, respectively. They are defined by using the stress components σ , τ and the strain components ϵ , γ as follows:

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$$\left. \begin{aligned} S_{ij} &= \begin{cases} \sigma_i - s/3, & (i=j) \\ \tau_{ij}, & (i \neq j) \end{cases} \\ E_{ij} &= \begin{cases} \varepsilon_i - e/3, & (i=j) \\ \gamma_{ij}/2, & (i \neq j) \end{cases} \\ s &= \sigma_1 + \sigma_2 + \sigma_3, \quad e = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \end{aligned} \right\} \dots\dots\dots (2)$$

The engineering material constants of a visco-elastic body which is initially at rest are defined in the image domain of the Laplace transformation as follows:

- (i) Uniaxial stress field
 $p\bar{E}(p) \equiv \bar{\sigma}_i/\bar{\varepsilon}_i, \quad \bar{\nu}(p) \equiv -\bar{\varepsilon}_j/\bar{\varepsilon}_i$
- (ii) Simple shear stress field
 $p\bar{G}(p) \equiv \bar{\tau}_{ij}/\bar{\gamma}_{ij}$
- (iii) Hydrostatic pressure field
 $3p\bar{K}(p) \equiv \bar{s}/\bar{e}$

where $\bar{f}(p) = \int_0^\infty f(t)e^{-pt}dt$

and p is the parameter of the Laplace transformation for t .

Then, stress-strain relations in the image domain become similar to that of the case of an elastic body and are given by

$$\left. \begin{aligned} \bar{\sigma}_i &= 2p\bar{G}(p) \left(\bar{\varepsilon}_i + \frac{\bar{\nu}(p)}{1-2\bar{\nu}(p)} \bar{e} \right) \\ \bar{\tau}_{ij} &= p\bar{G}(p) \bar{\gamma}_{ij} \end{aligned} \right\} \dots\dots\dots (3)$$

Thus, the equation of the displacement vector $U(u,v,w)$ in Cartesian coordinate system (x,y,z) has the following form in the image domain.

$$\nabla^2 \bar{U} + \frac{1}{1-2\bar{\nu}(p)} \text{grad } \bar{e} = \frac{\gamma p}{g\bar{G}(p)} \bar{U} \dots\dots\dots (4)$$

where $\bar{U} = (\bar{u}, \bar{v}, \bar{w})$; γ and g are weight per unit volume and the acceleration of gravity. The solution of Eq. (4) can be given by using stress functions (13), (14)

$$\left. \begin{aligned} \bar{u} &= \frac{\partial \bar{\psi}_0}{\partial x} + \frac{\partial \bar{\lambda}_1}{\partial y} - \frac{\partial \bar{\lambda}_2}{\partial z} + z \frac{\partial \bar{\chi}}{\partial y} - y \frac{\partial \bar{\chi}}{\partial z} \\ &\quad + \frac{\partial^2 \bar{\psi}_1}{\partial x^2} - p^2 \bar{\psi}_1 + \frac{\partial^2 \bar{\psi}_2}{\partial x \partial y} + \frac{\partial^2 \bar{\psi}_3}{\partial x \partial z} \\ \bar{v} &= \frac{\partial \bar{\psi}_0}{\partial y} + \frac{\partial \bar{\lambda}_1}{\partial z} - \frac{\partial \bar{\lambda}_2}{\partial x} + x \frac{\partial \bar{\chi}}{\partial z} - z \frac{\partial \bar{\chi}}{\partial x} \\ &\quad + \frac{\partial^2 \bar{\psi}_1}{\partial x \partial y} + \frac{\partial^2 \bar{\psi}_2}{\partial y^2} - p^2 \bar{\psi}_2 + \frac{\partial^2 \bar{\psi}_3}{\partial y \partial z} \\ \bar{w} &= \frac{\partial \bar{\psi}_0}{\partial z} + \frac{\partial \bar{\lambda}_2}{\partial x} - \frac{\partial \bar{\lambda}_1}{\partial y} + y \frac{\partial \bar{\chi}}{\partial x} - x \frac{\partial \bar{\chi}}{\partial y} \\ &\quad + \frac{\partial^2 \bar{\psi}_1}{\partial x \partial z} + \frac{\partial^2 \bar{\psi}_2}{\partial y \partial z} + \frac{\partial^2 \bar{\psi}_3}{\partial z^2} - p^2 \bar{\psi}_3 \end{aligned} \right\} \dots\dots\dots (5)$$

where

$$\bar{p}_1^2 \bar{\psi}_0 = \bar{p}_2^2 \bar{\lambda}_1 = \bar{p}_2^2 \bar{\psi}_1 = \bar{p}_2^2 \bar{\chi} = 0 \dots\dots\dots (6)$$

and

$$\begin{aligned} \bar{p}_1^2 &\equiv p^2 - \frac{1-2\bar{\nu}(p)}{2(1-\bar{\nu}(p))} \frac{\gamma p}{g\bar{G}(p)} \\ \bar{p}_2^2 &\equiv p^2 - \frac{\gamma p}{g\bar{G}(p)} \\ p^2 &\equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \end{aligned}$$

The results given above show the existence of a correspondence rule between the dynamic elastic and visco-elastic problems. This confirms that the change of the longitudinal modulus of elasticity E , Poisson's ratio ν and the transverse modulus of elasticity G in the transformed solution of a dynamic problem of elasticity to $p\bar{E}(p)$, $\bar{\nu}(p)$ and $p\bar{G}(p)$, respectively, gives a corresponding transformed solution of the linear dynamic problem of visco-elasticity.

From the results shown in this chapter, the dynamic problem of visco-elasticity can be solved directly using stress functions, in the theoretical sense, and the solution for the elastic body can be derived as a limiting process for the parameters of the material.

3. Method of solution

In this chapter, an analytical method is proposed for the mixed boundary value problem of a visco-elastic half-space in axisymmetric torsion. A rigid cylinder perfectly attached on the surface is forced to rotate impulsively in two different ways.

[1] Time history f_2 of its rotational angle is given.

[2] Time history f of the external torque applied to it is known.

Here, the material of the half-space is assumed to be a linear standard solid (Fig.1). Then, the stress-strain relation has the following form

$$(G_1 + G_2 + c \frac{\partial}{\partial t}) \tau = G_1 (G_2 + c \frac{\partial}{\partial t}) \gamma \dots\dots\dots (7)$$

For this mathematical model, the shear wave speed is given by $c_2 = \sqrt{G_1 g / \gamma}$.

In the axisymmetric impulsive torsion problem about z axis in cylindrical coordinate system (r, θ, z) , there are one non-zero displacement v_θ which is independent of θ , and two non-zero stress components $\tau_{r\theta}$ and $\tau_{\theta z}$.

Hereafter, nondimensional quantities shown below are used to simplify the analytical treatment.

Standard units for these quantities are taken to be radius a of the rigid cylinder, time a/c_2 and instantaneous transverse modulus G_1 of elasticity.

$$\begin{aligned} \rho &= r/a, \quad \xi = z/a, \quad \eta = c_2 t/a \\ v(\rho, \xi, \eta) &= v_\theta(r, z, t)/a, \quad \tau_{\theta z} = \tau_{\theta z}(r, z, t)/G_1 \\ \tau_{r\theta} &= \tau_{r\theta}(r, z, t)/G_1, \quad \alpha = ap/c_2 \end{aligned}$$

In the first problem [1], the boundary conditions are given by

$$\left. \begin{aligned} \bar{v}(\rho, 0, \alpha) &= \rho \bar{f}_2(\alpha), \quad (0 \leq \rho \leq 1) \\ \bar{\tau}_{\theta z}(\rho, 0, \alpha) &= 0, \quad (1 < \rho < \infty) \end{aligned} \right\} \dots\dots\dots (8)$$

where

$$\bar{F}(\alpha) = \int_0^\infty F(\eta) e^{-\alpha \eta} d\eta$$

Fig.1 Visco-elastic half-space subjected to impulsive torsion
(a) The coordinate system
(b) The standard linear solid

The parameter of the material for the standard linear solid takes the form

$$p\bar{G}(p) = \alpha\bar{G}(\alpha) = \frac{G_1(\Gamma_1 + \Gamma_2\alpha)}{1 + \Gamma_1 + \Gamma_2\alpha} \quad (9)$$

where $\Gamma_1 = G_2/G_1$, $\Gamma_2 = c_2c/aG_1$.

This problem can be solved by using the stress function⁽¹⁰⁾ and its solution is given by:

$$\left. \begin{aligned} \bar{u} &= \int_0^\infty \frac{\beta}{\sqrt{\beta^2 + \alpha^2}} A(\beta, \alpha) J_1(\beta\rho) e^{-\sqrt{\beta^2 + \alpha^2} \xi} d\beta \\ \bar{\tau}_{\theta\xi} &= -\frac{\alpha\bar{G}(\alpha)}{G_1} \int_0^\infty \beta A(\beta, \alpha) J_1(\beta\rho) e^{-\sqrt{\beta^2 + \alpha^2} \xi} d\beta \\ \bar{\tau}_{\rho\theta} &= -\frac{\alpha\bar{G}(\alpha)}{G_1} \int_0^\infty \frac{\beta^2}{\sqrt{\beta^2 + \alpha^2}} A(\beta, \alpha) J_2(\beta\rho) e^{-\sqrt{\beta^2 + \alpha^2} \xi} d\beta \end{aligned} \right\} \quad (10)$$

where

$$A(\beta, \alpha) = \frac{1}{\pi} \bar{f}_2(\alpha) \int_0^1 \Psi(x, \alpha) \sin \beta x dx \quad (11)$$

and $\Psi(x, \alpha)$ satisfies the following Fredholm integral equation of the second kind.

$$\Psi(x, \alpha) + \int_0^1 \Psi(y, \alpha) K(x, y, \alpha) dy = x \quad (0 \leq x \leq 1, 0 \leq y \leq 1) \quad (12)$$

In Eq.(12), kernel $K(x, y, \alpha)$ is symmetric for x and y and is given by a finite integral below.

$$K(x, y, \alpha) = \frac{\alpha^2}{\pi} \int_0^\pi \cos \varphi (e^{-\alpha_2 |x-y| \cos \varphi} - e^{-\alpha_2 |x+y| \cos \varphi}) d\varphi \quad (13)$$

where

$$\alpha_2 = \frac{G_1}{\bar{G}(\alpha)} \alpha, \quad J_n(x): \text{ Bessel function of the 1st kind.}$$

When the time history of the torque is given by $a^3 G_1 T_0 f(\eta)$ on the rigid cylinder, the following non-dimensional relation holds between the applied torque and the shear stress on the surface of the half space.

$$T_0 \bar{f}(\alpha) = \int_0^1 2\pi \rho^2 (\bar{\tau}_{\theta\xi})_{\xi=0} d\rho \quad (14)$$

In Eq.(14), the standard unit for the torque is taken to be $a^3 G_1$. Substituting Eqs.(10) and (11) into Eq.(14), we have the relation between $\bar{f}_2(\alpha)$ and $\bar{f}(\alpha)$ as follows:

$$\frac{1}{\pi} \bar{f}_2(\alpha) \int_0^1 \Psi(x, \alpha) x dx = -\frac{T_0}{4\pi} \frac{1}{\alpha\bar{G}(\alpha)} \bar{f}(\alpha) \quad (15)$$

The above results are derived directly by using stress function and they coincide with the results of applying the correspondence rule to the dynamic problem of elasticity.

4. Special case of stress and displacement

4.1. Static solution

In the problem [1], if the function $\bar{f}_2$ tends to θ_0 as η goes to infinity, the final values of stress and displacement are obtained from the final value theorem of

the Laplace transformation. These results correspond to the static solutions for the elastic half-space with the transverse modulus of elasticity $G = G_1 \Gamma_1 / (1 + \Gamma_1)$.

In the problem [2], if the function $\bar{f}$ tends to unity as η goes to infinity, the solution is obtained from that for the problem [1] by changing θ_2 to $-(3/16)(1 + \Gamma_1)T_0/\Gamma_1$.

4.2. Stress and displacement on the surface

In the problem [1], transformed solutions of stress and displacement on the surface $\xi = 0$ are given by:

$$\left. \begin{aligned} (\bar{u})_{\xi=0} &= \frac{1}{\pi} \bar{f}_2(\alpha) \int_0^1 \Psi(x, \alpha) \left\{ \frac{x}{\rho \sqrt{\rho^2 - x^2}} \right. \\ &\quad \left. + \frac{\pi}{2} \int_0^1 \sin \frac{\pi y}{2} K\left(x, \rho \sin \frac{\pi y}{2}, \alpha\right) dy \right\} dx \\ &\quad (1 < \rho < \infty) \\ (\bar{\tau}_{\theta\xi})_{\xi=0} &= -\frac{4}{\pi} \bar{f}_2(\alpha) \alpha \bar{G}(\alpha) \left[\frac{\rho}{\sqrt{1-\rho^2}} \Psi(1, \alpha) \right. \\ &\quad \left. + \frac{\sqrt{1-\rho^2}}{\rho} \{\Psi(1, \alpha) - \Psi'(1, \alpha)\} \right. \\ &\quad \left. + \frac{1}{\rho} \int_0^1 \Psi''(x, \alpha) \sqrt{x^2 - \rho^2} dx \right] \\ &\quad (0 \leq \rho < 1) \\ (\bar{\tau}_{\rho\theta})_{\xi=0} &= -\frac{4}{\pi} \bar{f}_2(\alpha) \alpha \bar{G}(\alpha) \left[\int_0^1 \Psi'(x, \alpha) \right. \\ &\quad \left. \times \left\{ I(x) + \frac{\rho^2 - 2x^2}{\rho^2 \sqrt{\rho^2 - x^2}} \right\} dx - \Psi(1, \alpha) \right. \\ &\quad \left. \times \left\{ I(1) + \frac{\rho^2 - 2}{\rho^2 \sqrt{\rho^2 - 1}} \right\} \right] \quad (1 < \rho < \infty) \end{aligned} \right\} \quad (16)$$

where $\Psi'(x, \alpha) = \frac{\partial}{\partial x} \Psi(x, \alpha)$

$$\Psi'(1, \alpha) = [\Psi'(x, \alpha)]_{x=1}$$

$$I(x) = \int_0^\infty \left(\frac{\beta}{\sqrt{\beta^2 + \alpha^2}} - 1 \right) \cos \beta x J_2(\beta\rho) d\beta$$

If we define the stress intensity factor k_3 for $\tau_{\theta\xi}$ as

$$\lim_{\rho \rightarrow 1} \sqrt{2(1-\rho)} (\bar{\tau}_{\theta\xi})_{\xi=0}$$

Then, from Eq.(16), $\bar{k}_3$ is given by

$$\bar{k}_3 = -\frac{4}{\pi} \alpha \bar{G}(\alpha) \bar{f}_2(\alpha) \Psi(1, \alpha) \quad (17)$$

and the stress intensity factor for $\tau_{\rho\theta}$ coincides with k_3 .

And also, nondimensional torque T can be derived from Eq.(14) as follows

$$\bar{T} = -16\alpha \bar{G}(\alpha) \bar{f}_2(\alpha) \int_0^1 x \Psi(x, \alpha) dx \quad (18)$$

In the case of the problem [2], the corresponding solutions are obtained from the above results by changing $\bar{f}_2$ to $\bar{f}$ using Eq.(15).

5. Numerical calculation

Because of the difficulty to get an analytical solution of the integral equation (12), it is not easy to invert analytically the Laplace transforms obtained above. Combining the numerical method for the solution of the integral equation and the numerical inversion method of the Laplace transformation,⁽¹⁵⁾ we can get the solution numerically.

From Eq. (12), the following set of linear equations for $\Psi(x_j, \alpha)$ is derived by applying the weighted quadrature formula.

$$\sum_{j=1}^M (\delta_{ij} + K_{ij} W_j) \Psi_j = x_i, \quad (i=1 \sim M) \quad (19)$$

where δ_{ij} is Kronecker's delta, $K_{ij} = K(x_i, x_j, \alpha)$, $\Psi_j = \Psi(x_j, \alpha)$, $W_j = W(x_j)$ and x_j denotes the points at which Ψ_j are to be evaluated and $W(x_j)$ are denotes the weights to be attached to these values.

For a given values of α , the solution of Eq. (19) gives the values of $\Psi(x_j, \alpha)$ at M distinct points of x .

The approximate value $\Psi(x, \alpha)$ at arbitrary point of x can be calculated by an interpolation formula.

From Eq. (15), the relation between f_2 and f is given as follows.

$$\frac{4}{\pi} f_2(\alpha) \sum_{i=1}^M \Psi_i x_i W_i = -\frac{T_0}{4\pi} \frac{f(\alpha)}{\alpha G(\alpha)} \quad (20)$$

Eliminating Ψ_i from Eqs. (19) and (20), we have

$$f_2(\alpha) = -\frac{T_0 H(\alpha)}{16 \alpha G(\alpha)} f(\alpha) \quad (21)$$

$$\text{where } \frac{1}{H(\alpha)} = A_1 A_2^{-1} A_3$$

and $A_1 = [x_j W_j]$, $(j=1 \sim M)$; $A_2 = [\delta_{ij} + W_j K_{ij}]$, $(i, j=1 \sim M)$ and $A_3 = [x_i]$, $(i=1 \sim M)$ are row vector, square matrix and column vector of M th order, respectively. A_2^{-1} is an inverse matrix of A_2 .

The solutions $\Psi(x, \alpha)$ of Eq. (12) for the elastic solid ($\alpha_2 = \alpha$) are shown in Fig. 2.

To get the values of K from Eq. (13), the interval of integration in Eq. (13) is divided into 12 parts and Legendre-Gauss's quadrature formula is used. To get the values of $\Psi(x, \alpha)$, we put $M=12, 24$ and 48 in Eq. (19).

From the numerical evaluation, it is seen that we have to use larger values of M for larger values of α .

Fig. 2 The solutions of integral equation (12). (For the case of elastic body)

The following results are obtained by using Bellman's numerical inversion method and putting $\alpha=1, 2, 3, \dots, 7$ and time scales 2.0, 2.5 and 3.0.

The values of $\Psi(x, \alpha)$ at arbitrary points of x are calculated by Aitken's interpolation method.

6. Numerical results

The numerical results are obtained for four cases by varying the parameters Γ_1, Γ_2 of the visco-elastic material.

- [I] $\Gamma_1 = \infty$ (Elastic Body)
- [II] $\Gamma_1 = 5, \Gamma_2 = 50$
- [III] $\Gamma_1 = 1, \Gamma_2 = 10$
- [IV] $\Gamma_1 = 0.1, \Gamma_2 = 1$

In Figs. (4)-(9), solid, dotted, chain and double dotted chain lines are used to indicate the results for [I], [II], [III] and [IV], respectively.

Loading time functions $f_2(\eta)$ and $f(\eta)$ are assumed as follows (Fig. 3).

$$f_2(\eta)/\theta_0 = \begin{cases} \frac{1}{2} \{1 - \cos(\pi \eta / \eta_0)\}, & (0 \leq \eta \leq \eta_0) \\ 1 & (\eta_0 \leq \eta) \end{cases}$$

and the value of η_0 is fixed to be 2.0.

Figures (4)-(7) show the numerical results for a given rotatory angle.

Fig. 3 Time history of the torque and the angle of rotation

Fig. 4 Time history of the stress on the contact surface (For the case of given rotational angle)

The stresses $(\tau_{\theta z})_{z=0}$ on $\rho=0.7$ and 0.9 are shown in Fig.4. The values shown on the right hand side of this figure are the final values (the static solutions). The solution for an elastic body increases monotonously to the final value. For the initial response, the visco-elastic body behaves like an elastic body with the elastic constant G_1 , and for the final response, it reacts as an elastic body with the elastic constant $G_1\Gamma_1/(1+\Gamma_1)$.

Fig.5 Time history of the stress on the free surface (For the case of given rotational angle)

Fig.6 Time history of the stress intensity factor (For the case of given rotational angle)

Fig.7 Time history of the torque (For the case of given rotational angle)

Fig.8 Time history of the rotational angle (For the case of the given torque)

Fig.9 Time history of the stress intensity factor (For the case of the given torque)

The stresses $(\tau_{r\theta})_{z=0}$ on the free surface at $\rho = 1.1$ and 1.3 are shown in Fig.5. They reach static values after some oscillation.

Figures (4) and (5) show the relaxation phenomena for stresses and Fig.(7) shows that for the torque.

The results for a given torque are shown in Figs.8 and 9. Figure 8 shows the displacement on the surface at $\rho = 1$ which is proportional to the rotatory angle. From this figure, we can see the effect of the dynamic creep.

Figure 9 shows the time history of the stress intensity factor. The response of a visco-elastic material is similar to that of an elastic body with the elastic constant $G_1\Gamma_1/(1+\Gamma_1)$ in the case of a given torque.

7. Conclusion

In this paper, an analytical attempt has been made to study the impulsive response of a visco-elastic half-space the surface of which is subjected to a suddenly applied torque on displacement by a rigid cylinder attached on it. The numerical results are given for the standard linear solid by using the numerical inversion method of the Laplace transformation.

In conclusion, it is shown that the stress function method and the numerical method proposed for the dynamic theory of elasticity are applicable for the solution of a three-dimensional dynamic problem of linear visco-elasticity.

The analytical treatment of a dynamic stress relaxation problem has given relation between the displacement and the force in the transformed domain. It should be pointed out that the dynamic creep problem can also be solved by using this relation.

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