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Analysis of Free Forging by Rigid-Plastic Finite Element Method

Based on the Plasticity Equation for Porous Metals*

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The rigid-plastic finite element method based on the plasticity equation for porous metals has been extended and applied for the analysis of plastic deformation of ordinary pore-free metals. The relative density is assumed to be very close to 100%, e.g. 99.5% or 99.95% in the calculation. The stress and strain states can be calculated with this method up to large deformation. Upsetting of a cylindrical billet with grooved dies and compression of square and rectangular thin plates with flat dies without lubrication are analysed. The analytical results of stress, load and deformed shape show good agreements with experimental results.

1. Introduction

For the purpose of predicting the occurrence of defects and the shape of the product in metal forming, it is necessary to develop a method by which stress and strain states in a plastically deforming material can be calculated. Among various methods which have been developed so far for analysing metal forming, the elastic-plastic finite element methods for the analysis of small deformation problem^{(1),(2)} and large deformation problem⁽³⁾ are considered to simulate the forming processes very well, since work-hardening characteristics of the material or complicated boundary conditions can be taken into consideration. However, these methods require a large number of iterations in calculation to follow a large deformation; otherwise the error of calculated stress increases markedly.

The rigid-plastic finite element method is suitable for analysing problems of large plastic strain in relatively short computing time. In this method, elastic deformation in the material is neglected and a velocity field is so determined that the power required for plastic deformation and friction is minimized. Several authors have developed the finite element methods for rigid-plastic material by which not only deformation state but also stress can be calculated: Kobayashi et al.⁽⁴⁾ minimized the power by the

Lagrange multiplier method in which the multiplier is equal to the mean stress; Alexander et al.⁽⁵⁾ integrated the stress equilibrium equations from the stress boundary; Kitahara et al.⁽⁶⁾ optimized the upper and lower bounds of power separately.

The authors⁽⁷⁾ applied the rigid-plastic finite element method to the analysis of forming processes of porous (compressible) metals. Since, in this analysis, volume constancy is not necessarily satisfied, the formulation for minimization of power is simplified. It is presumed that this method can be applied for approximate analysis of plastic deformation of pore-free metals, if the relative density is assumed to be very high, say 99.5%. In this study, upsetting of a cylindrical billet and compression of thin plates are treated to demonstrate the possibility of the method.

2. Basic concept

For plastic deformation of porous metals, which are compressible, a kinematically admissible velocity field must satisfy only the velocity boundary conditions; a proof was given of the upper bound theorem based on the plasticity theory for porous metals⁽⁸⁾ in a similar manner to the conventional upper bound theorem. The power, $\dot{W}$, derived from a kinematically admissible velocity field is:

$$\dot{W} = \int_V \rho \sigma_{eq} \dot{\epsilon}_{eq} dV + \int_{S_T} \tau_t \dot{v}_t dS - \int_{S_A} F_i \dot{v}_i dS \dots \dots \dots (1)$$

where ρ is the relative density (apparent density of the porous material divided by the density of pore-free matrix), $\dot{\epsilon}_{eq}^*$ is the equivalent strain rate derived from any kinematically admissible velocity field $\dot{v}_i^*$, Δv^* is a velocity discontinuity at a plane, S_T , where it occurs, such as die-workpiece interfaces and S_A is the surface where the traction F_i is prescribed. It is considered that the exact velocity field may be approximated by a

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velocity field which minimizes $\dot{W}$.

From the plasticity theory for porous metals⁽⁹⁾, the equivalent strain rate $\dot{\epsilon}_{eq}$ is:

$$\dot{\epsilon}_{eq} = \rho^{n-1} \sqrt{\frac{2}{9} \left\{ (\dot{\epsilon}_x - \dot{\epsilon}_y)^2 + (\dot{\epsilon}_y - \dot{\epsilon}_z)^2 + (\dot{\epsilon}_z - \dot{\epsilon}_x)^2 + \frac{3}{2} (\dot{\gamma}_{xy}^2 + \dot{\gamma}_{yz}^2 + \dot{\gamma}_{zx}^2) \right\} + (f \dot{\epsilon}_v)^2} \quad (2)$$

$$n=2.5, \quad f = \frac{1}{2.49(1-\rho)^{0.514}}, \quad \dot{\epsilon}_v = \dot{\epsilon}_x + \dot{\epsilon}_y + \dot{\epsilon}_z$$

If the relative density is very close to 100%, f in Eq.(2) is infinitely large; therefore $\dot{\epsilon}_{eq}$ becomes very large if the volumetric strain rate $\dot{\epsilon}_v$ is not very small. Thus, a velocity field which minimizes $\dot{W}$ makes $\dot{\epsilon}_v$ very close to zero. The degree of volume change will be examined in the later section. This method is similar to the penalty function approach, in which a large positive number is multiplied to the volume integral of the square of the volumetric strain rate being added to the power. Unlike the penalty function approach, however, stress can be calculated directly from the constitutive equations.

3. Upsetting of a cylindrical billet

3.1 Analytical method

The problem treated here is upsetting of a cylindrical billet with completely rough platens, in which a radial flow of the material at the die-workpiece interfaces is not allowed.

The power required for plastic deformation, $\dot{W}_D$, is expressed by

$$\dot{W}_D = \sum_{i=1}^m (a_{i,j} \dot{\epsilon}_{i,j} V)_i = \dot{W}_D(u_1, v_1, \dots, u_n, v_n) \quad (3)$$

where m is the number of elements and n the number of nodes. Since the strain rate components in each element are linear functions of the nodal velocities, $\dot{W}_D$ is a function of the nodal velocities.

In upsetting with completely rough platens, the friction loss at the die-workpiece interface is zero, because there is no relative slip. Therefore, the total power $\dot{W}$ is equal to $\dot{W}_D$.

$$\dot{W} = \dot{W}_D = \dot{W}(u_1, v_1, \dots, u_n, v_n) \quad (4)$$

A minimum value of $\dot{W}$ can be sought by varying the nodal velocities under the velocity boundary conditions,

$$u_k = \text{const.}, \quad v_k = \text{const.} \quad (5)$$

where k refers to the nodes at which the velocity is prescribed. The minimization of $\dot{W}$ is carried out by the perturbation method using an initial velocity field at each step.

The initial velocity field at the first step is employed as follows:

$$u = c_1 r(c_2 - z), \quad v = c_1(c_2 - z)^2 + c_3 \quad (6)$$

where c_1 , c_2 and c_3 are constants which are determined from the boundary conditions. This velocity field gives no relative slip at the die-workpiece interface and satisfies the incompressibility condition everywhere in the workpiece. However, since this velocity field is not an

optimum one, a few iterations (5 ~ 10 iterations) may suffice.

In the subsequent step, the velocity field for the previous step is modified and used as the initial one. In this case, only three to five iterations are necessary up to convergence.

From the obtained velocity field, the stress can be calculated using the stress-strain rate relation⁽⁷⁾.

3.2 Results of analysis

3.2.1 Computed model

The material of the model was assumed to be a 0.25% carbon steel. The stress-strain curve for this material was obtained from a simple compression test and was approximated by the following equation:

$$\sigma_{eq} = 76.3(\epsilon_{eq} + 0.0397)^{0.21} \quad \text{kg/mm}^2 \quad (7)$$

Fig.1 shows the meridional plane of a cylindrical billet divided into 49 quadrilateral elements. The initial height and diameter of the billet were equally 20mm and the relative density was assumed to be 99.5% at each step of deformation. The reduction in height at each deformation step was 1.0%~1.2% and the calculation was continued up to a reduction in height of 45%. When folding of a node at the free surface (the side surface) occurred, the node was restricted to the die surface and not allowed to move radially any further.

3.2.2 Friction-free upsetting

Friction-free upsetting was also calculated to check the validity of the method. In this case, homogeneous stress and strain were obtained; the magnitudes of both σ_r and σ_θ were less than 10^{-4} kg/mm². The load derived from the power was compared with that obtained from the stress-strain curve. The difference between the two loads was within 2%.

3.2.3 Upsetting with completely rough platens

As a standard test for workability of materials in cold forging, upsetting of a cylindrical billet with grooved dies is often employed⁽¹⁰⁾. When the workpiece is compressed with these dies, a radial flow of the workpiece at the end surface hardly occurs. In the present experiment, the platens with concentric grooves as shown in Fig.2 were used.

Fig.3 shows the calculated distortion of the grid pattern after 26.2% and 45% reductions in height. The profiles of the

Fig.1 Finite element mesh for cylindrical billet in upsetting

Fig.2 Platen with concentric grooves

Fig.3 Calculated distribution of grid pattern in upsetting with completely rough platens

Fig.4 Comparison between calculated and experimentally derived upsetting loads with completely rough platens

side surface obtained by the calculation are not smooth, probably because the element at the corner can hardly follow a complicated deformation and the deformation of neighbouring elements is affected. The profiles would be made smoother by dividing the material near the corner into smaller elements.

Fig.4 shows a comparison between the loads obtained by the calculation and the experiment, where the calculated load was derived from the power. Point A (26.2%) and point B (42%) correspond to nodes folded at the free surface. The calculated load is smaller than the experimental one. This difference would be due to the area of the end surface; the area in the experiment expands gradually as the reduction in height increases, whereas the area in the calculation does not increase up to the occurrence of folding.

Fig.5 Distributions of calculated stress components and equivalent strain at a reduction in height of 45%

Fig.5 shows the calculated distributions of stress components and the equivalent strain at 45% reduction. The equivalent strain ϵ_{eq} is small near the centre of the end surface. Examination of the stress is shown below. Fig.6 shows a comparison between the stress components at the equatorial free surface obtained by the calculation and the experiment. The latter was obtained as follows: the axial strain increment was obtained from the change in the distance between two hardness indentations which were so located that the equator lay in between, while the hoop strain increment was obtained from the change in the diameter of the equator; the axial and hoop stress components were calculated by using these axial and hoop strain increments and the Lévy-Mises equations⁽¹⁰⁾. At point A and B, where the folding occurred, the calculated curves change sharply, but the calculated axial and hoop stress components are respectively in good agreement with the experimental ones. The calculated radial stress is within $\pm 5 \text{ kg/mm}^2$; the stress boundary condition at the free surface is satisfied quite well.

To examine the accuracy of the stress, the load derived from the die pressure was

Fig.6 Stress components at the equatorial free surface obtained by calculation and experiment

Fig.7 Comparison between upsetting load derived from power and the one from die pressure (completely rough platens)

compared with that derived from the power as shown in Fig.7. The former agrees well with the latter. It is considered that the stress in the material near the die-workpiece interface can be calculated fairly accurately.

As this method is based on the plasticity theory for porous metals, the volume of each element changes by plastic deformation. The total volume change of the billet was as small as 0.61% after 45% reduction in height, showing that this method was successfully applicable to the pore-free metals.

It is presumed that the occurrence of ductile fracture can be predicted by combining these results and a ductile fracture criterion⁽¹¹⁾.

4. Compression of thin plates

4.1 Analytical method

4.1.1 Relation between strain rate and nodal velocity

Compression of thin plates with parallel platens is treated here. It is assumed that the plate is deformed uniformly in the thickness direction (z-direction). The plate is divided into quadrilateral elements. The strain rate components are given by:

$$\begin{Bmatrix} \dot{\epsilon}_x \\ \dot{\epsilon}_y \\ \dot{\gamma}_{xy} \end{Bmatrix} = \frac{2}{J'} \begin{bmatrix} B_i' & 0 & B_j' & 0 & B_m' & 0 & B_k' & 0 \\ 0 & C_i' & 0 & C_j' & 0 & C_m' & 0 & C_k' \\ C_i' & B_i' & C_j' & B_j' & C_m' & B_m' & C_k' & B_k' \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_m \\ v_m \\ u_k \\ v_k \end{Bmatrix}$$

$$\dot{\epsilon}_z = \text{const.}, \quad \dot{\gamma}_{xz} = \dot{\gamma}_{yz} = 0$$

..... (8)

$$J' = \{(-x_i + x_j + x_m - x_k) + (x_i - x_j + x_m - x_k)\eta\} \cdot$$

$$\{(-y_i - y_j + y_m + y_k) + (y_i - y_j + y_m - y_k)\xi\}$$

$$- \{(-x_i - x_j + x_m + x_k) + (x_i - x_j + x_m - x_k)\xi\} \cdot$$

$$\{(-y_i + y_j + y_m - y_k) + (y_i - y_j + y_m - y_k)\eta\}$$

$$\begin{Bmatrix} B_i' \\ B_j' \\ B_m' \\ B_k' \end{Bmatrix} = \begin{Bmatrix} y_{jk} - y_{mk}\xi - y_{jm}\eta \\ -y_{im} + y_{mk}\xi + y_{ik}\eta \\ -y_{jk} + y_{ij}\xi - y_{ik}\eta \\ y_{im} - y_{ij}\xi + y_{jm}\eta \end{Bmatrix}, \quad \begin{Bmatrix} C_i' \\ C_j' \\ C_m' \\ C_k' \end{Bmatrix} = \begin{Bmatrix} -x_{jk} + x_{mk}\xi + x_{jm}\eta \\ x_{im} - x_{mk}\xi - x_{ik}\eta \\ x_{jk} - x_{ij}\xi + x_{ik}\eta \\ -x_{im} + x_{ij}\xi - x_{jm}\eta \end{Bmatrix}$$

$$x_{ij} = x_i - x_j, \quad y_{ij} = y_i - y_j$$

4.1.2 Power

The friction loss at the die-workpiece interfaces, $\dot{W}_F$, is

$$\dot{W}_F = \sum_{i=1}^m (2\tau_f \Delta v S)_i \quad \text{..... (9)}$$

where τ_f is the frictional shear stress, Δv is the relative velocity at the interfaces, S is the contact area and m is the number of elements. Using the contact pressure p_0 at the previous step, the frictional shear stress τ_f at the present step is approximated by

$$\left. \begin{aligned} \tau_f &= \mu p_0 \left(\mu p_0 < \frac{1}{\sqrt{3}} \sigma_{eq} \right) \\ \tau_f &= \frac{1}{\sqrt{3}} \sigma_{eq} \left(\mu p_0 \geq \frac{1}{\sqrt{3}} \sigma_{eq} \right) (\text{sticking}) \end{aligned} \right\} \quad \text{..... (10)}$$

where μ is the coefficient of friction. The relative velocity, Δv , at the interface is defined by the mean value which is

calculated from the velocities at the centroidal points of the nine parts into which an element is divided,

$$\dot{v} = \frac{1}{9} \sum_{i=1}^9 \sqrt{u_i^2 + v_i^2} \quad (11)$$

Using the power necessary for plastic deformation, $\dot{W}_D$, which is of a similar form to Eq.(3), the total power $\dot{W}$ is expressed by a function of the nodal velocities as follows:

$$\dot{W} = \dot{W}_D + \dot{W}_F = \dot{W}(u_1, v_1, \dots, u_n, v_n) \quad (12)$$

4.1.3 Initial velocity field

The initial velocity field employed at the first step is (12)

$$u = -C_4 \varepsilon_x, \quad v = -(1-C_4) \varepsilon_y \quad (0 < C_4 < 1) \quad (13)$$

Constant C_4 was so chosen that the power was minimized. In the subsequent step, the velocity field for the previous step was used as the initial one, but it was so modified that the incompressibility condition was nearly satisfied for the present shape.

4.2 Results of analysis

4.2.1 Computed model

The material of the plates used in the experiment was an annealed commercially pure aluminum of 3mm thick. The stress-strain curve for the material was obtained from a simple compression test and was approximated by the following equation:

$$\sigma_{eq} = 13.01(\varepsilon_{eq} + 0.00008)^{0.25} \text{ kg/mm}^2 \quad (14)$$

Fig.8 shows the finite element meshes of a quarter of the plates: (a) a right square plate and (b) a rectangular plate. The initial length of the right square plate was 20mm and the initial length and the width of the rectangular plate were 30mm and 20mm, respectively. The initial thickness was 3mm in both plates. The relative density was assumed to be 99.95% at each step of deformation. The thickness strain in each step was 1.0% and the

Fig.8 Finite element meshes for plates in compression

calculation was continued up to 42% reduction in thickness.

4.2.2 Compression with dry friction ($\mu=0.25$)

The calculated results for $\mu=0.25$ are compared with the experimental ones with dry friction, since the coefficient of friction in upsetting with dry friction is supposed to be $0.2 \sim 0.25$ (13). The experiment was carried out with parallel flat platens degreased with acetone. The surface roughness of the die was about $0.1 \mu\text{m}$.

Fig.9 shows comparison between the shapes obtained by the calculation and the experiment. The calculated shapes agree well with the experimental ones. Fig.10 shows a comparison between the loads ob-

Fig.9 Comparison between calculated and experimentally derived shapes

Fig.10 Comparison between calculated and experimentally derived loads

tained by the calculation and the experiment. There is a good agreement between them. However, at a large reduction in thickness, the calculated load is higher than the experimentally obtained one. This may be due to the fact that the frictional shear stress in the calculation at a large reduction is overestimated⁽¹³⁾ because a constant coefficient of friction is assumed in the calculation.

Fig.11 shows the calculated distributions of stress at a reduction in thickness of 40%. The stress boundary conditions at the free surface are satisfied well; σ_x at the side surface roughly normal to the x-direction is almost zero and so is σ_y at the side surface roughly normal to the y-direction. Fig.12 shows a comparison between the loads computed from the power and from the die pressure. The former agrees well with the latter. It is considered that the stress field in the material is calculated fairly accurately. The total volume change after 42% reduction in thickness was as small as 0.42% in the right square plate and 0.50% in the rectangular plate. It can thus be said that the incompressibility condition was satisfied quite well.

(a) Square (b) Rectangle
Fig.11 Distributions of calculated stress components at a reduction of 40%

5. Discussion

It is presumed that the calculated results may vary when the assumed density is changed. In order to check this, the relative density of a cylindrical billet in upsetting was assumed to be 99.95% and the results were compared with those for $\rho=99.5\%$; the difference in the stress components between the two cases was within 7% and the difference in loads was 1%. These differences would become smaller, if the minimization of the power was carried out more rigorously. On the other hand, the volume change after 12% reduction in height is 0.18% for $\rho=99.5\%$ and 0.04% for $\rho=99.95\%$. It is thus considered that the volume change decreases as the relative density increases. However, when the relative density is extremely high, the magnitude of the calculated stress becomes unreasonably large in some elements. Further, the computing time for the lower relative density is shorter than that for the higher one. Therefore, unless the condition of volume constancy is required to a very good accuracy, the assumed relative density of 99.5% furnishes sufficient results.

In the analysis of upsetting of a cylindrical billet, the initial velocity field which was quite different from the present one resulted in an erroneous solution, i.e. the stress boundary condition was not satisfied. Such a velocity field is obviously deemed incorrect. In the analyses treated in the present paper, the initial velocity fields which yield good final results are easily found. In practical forming processes, appropriate velocity fields are not easily found. Therefore, an attempt should be made to develop a "fool-proof" technique which provides a reasonable initial velocity field for any forming process.

The present method is an application of the upper bound technique and the stress field is obtained from a kinematically admissible velocity field. Since the obtained stress field does not necessarily satisfy the stress equilibrium condition and boundary conditions, the accuracy of the calculated stress field must be examined. In this study, only the stress

Fig.12 Comparison between compression load derived from power and the one from die pressure

boundary conditions were checked.

It was assumed that the entire body is deforming plastically in the present analysis, whereas a rigid zone may arise near the end surface in upsetting of a cylindrical billet⁽¹⁴⁾. In the analysis shown here, the equivalent strain rate near this region was very small. A calculation was also carried out by assuming that these regions were rigid, but the stress and the strain were not affected significantly.

Since there is no condition of volume constancy, the formulation for minimization of power is simplified and the number of variables is smaller compared with that by Kobayashi et al.⁽⁴⁾.

In the analysis of metal forming, the formulation for the frictional condition is an important problem. In the present analysis of compression of plates, it was assumed that the coefficient of friction is constant. However, the coefficient of friction differs in reality with the position at the die-workpiece interfaces and with reduction in thickness. The difference between the calculated and experimental results would be mainly due to the difference in the frictional conditions.

Since the approximate frictional shear stress is given by using the contact pressure at the previous deformation step, the error by the approximation would increase with increasing deformation in one step. The error would be decreased by using a small deformation step or by employing an iterative technique in which the contact pressure is treated as a variable.

6. Conclusions

By the assumption that the relative density is 99.5% ~ 99.95%, the rigid-plastic finite element method based on the plasticity equation for porous metals is applied to the analysis of the forming processes of pore-free metal. In this method, the formulation for minimization of power is simplified and the stress can be calculated easily. Upsetting of a cylindrical billet and compression of plates are treated as examples of the analysis. The calculated results agree well with the experimental ones.

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