Float-area-type Flow Meters with Constant Discharge Coefficients

(General Flow Rate Characteristics When the Gradient of a Tapered Tube is Large)*

By Hiroshi MAKI** and Yasuo MORI***

Discharge coefficients, including general flow rate characteristics when the gradient of a tapered tube is large, of float-area-type flow meters are generally discussed.

It is experimentally pointed out that the discharge coefficients became constant when a float, having the small length of the maximum diameter part of the float and having 180° vertex angle of the cone portions upstream of the maximum diameter part of the float, and a particular tapered tube having the gradient $k = 0.545$ are employed.

When employing such a combination of the float and the tapered tube, a semi-empirical formula to predict the flow rate characteristics is obtained and the limits of applicability of the formula are experimentally explained.

Finally, a float-area-type flow meter having linear relations between the flow rate and the position of the float is manufactured.

1. Introduction

It has been reported⁽²⁾ that the discharge coefficient of float-area-type flow meters is generally not constant for varying working conditions and that it remains constant at a value of 0.788 within an accuracy of $\pm 1\%$ over a wide range of characteristic Reynolds numbers Re when $Re > 3 \times 10^3$ only in the case where a float with small $l^{(1)}$ and having a particular vertex angle $\theta_1 = 96^\circ$ is employed.

The constant discharge coefficient in float-area-type flow meters means that the flow rate characteristics, i.e. the relations between the flow rate and the position of the float, of them are unaffected by a change in the viscosity of the working fluid and are influenced only by a change in the density of it.

It has been also reported in Reference (2) that conversion equations of the flow rate given in the JIS-Z8761-1965, for example, are applicable, according to the conversion accuracy, only when employing float-area-type flow meters with a constant discharge coefficient. The results reported in Reference (2), however, were obtained when the gradient of tapered tubes employed was smaller than roughly 10^{-2} .

Recently, there is a tendency to employ tapered tubes having a large gradient for the purpose of reducing the manufacturing cost even sacrificing the sensitivity⁽³⁾ of float-area-type flow meters. When employing tapered tubes having a large gradient, the length of the tapered tube becomes naturally shorter. Moreover, there are important merits that a transmitter-mechanism becomes simpler when the flow rate, i.e. the position of the float, of the float-area-type flow meters is converted to a pneumatic or an electric signal.

The purpose of this report is to study experimentally the general flow rate characteristics of the float-area-type flow meters when the gradient of the tapered tube is large. Moreover, a value of the gradient of the tapered tube where the discharge coefficient of this kind of flow meter has a constant value will be found out when a vertex angle of the cone portion upstream of the maximum diameter part of the float is 180° . This makes a contrast with the result report-

ed in Reference (2) where a vertex angle of the cone portion upstream of the maximum diameter part of the float was found when the gradient of the tapered tube was considerably small.

2. Experimental apparatus and procedure

The working fluid employed in the present study is water. Water is introduced into the float-area-type flow meter through a flow control valve from a head tank placed at height of 5 m from the ground. The head of water in the head tank is stabilized by overflow. The flow rate is measured by means of a balance and a stop watch after flowing out from the float-area-type flow meter.

Figure 1 shows a typical assembly drawing of the float-area-type flow meter employed in this study. Three kinds of the float-area-type flow meters, according to the maximum diameter d_f of the float

Fig. 1 Assembly drawing of the experimental apparatus

* Received 14th June, 1977.
 ** Assistant Professor, Science University of Tokyo, Noda.
 *** Professor, Tokyo Institute of Technology.

described below, were employed. The constructions of them, however, are roughly the same as those shown in Fig. 1 and the respective tapered tubes are exchangeable.

2.1 Floats

The floats employed in the present experiment are made of brass, stainless steel or vinyl chloride for varying Re . The maximum diameters of them are 25, 42 and 57 mm, respectively, and the ratio $l/d_f = \beta_2$, where l denotes the length of the part of the maximum diameter of the float, is roughly 0.01 ~ 1. The dimensions of them were accurately measured by means of micro-meters. Since these floats were naturally unstable, three kinds of shafts, having a diameter matching the maximum diameter of the float, were employed as a stabilizer and the floats were fixed to them with a small screw. The shafts were guided by journal bearings as shown in Fig. 1. In order to vary Re closely, the weight of the float system was adjusted by means of a weight installed at the bottom of the shaft as shown in Fig. 1. The diameter of a cylindrical part upstream of the maximum diameter part of the float was set at $0.6 d_f$ to prevent an influence on the flow rate characteristics. Accordingly, the diameter of the weight was also set at $0.6 d_f$.

2.2 Tapered tubes

The tapered tubes employed in the present experiment are made of duralumin or stainless steel and have values of the gradients as listed in Table 1. The inner diameter distributions of them were accurately measured by means of inner micrometers and their gradients were calculated from those distributions.

$d_f(\text{mm})$	k
25	0.0846
	0.126
	0.177
	0.260
	0.385
	0.545
	0.577
	1.014
42	0.545
	0.577
	1.011
57	0.546

Table 1 Gradients of the tapered tubes employed

The equilibrium position of the float has been defined⁽⁴⁾ as the location at the minimum annular clearance formed between the maximum diameter part of the float and the tapered tube. In the case of employing tapered tubes having a large gradient, the location of the minimum clearance is not at a widely used distance aa' when employing tapered tube having a very small gradient, where a' denotes a point extended horizontally from a point a upstream edge of the maximum diameter part of the float as shown in Fig. 2, but at a distance aa'' . Where a'' is a point of intersection of a line on the inner wall of the tapered tube and a line normal to it as shown in Fig. 2. However, it is practical to adopt the equilibrium position of the float as a' instead of a'' . Consequently, the equilibrium position of the float was

Fig. 2 Explanatory drawing of the minimum annular clearance

measured at a' and after that the position a' was transformed into a'' when arranging the experimental data. Denoting the inner diameter of the tapered tube at a' by D_t and putting $\angle a'a'' = \theta$, the hydraulic length at the minimum annular clearance is $(D_t - d_f) \cos \theta / 4$. Therefore, the dimensionless equilibrium position β_1 of the float in the present study is expressed as $\beta_1 = (D_t - d_f) \cos \theta / 4 d_f$.

2.3 Measurement of the equilibrium position of the float

Since the tapered tubes employed in the present experiment are made of metals, we can not read directly the position of the float. Moreover, since the displacement of the float is small on account of the tapered tubes having a large gradient, accurate measurement can not be expected from direct measuring method. In order to measure accurately the position of the float, consequently, a differential transformer having an effective measuring range of 50 mm was installed at the top of the tapered tube and an iron core was also installed at the top of the shaft as shown in Fig. 1. The displacement of the float was converted by the differential transformer and the iron core to an electric voltage and read by a voltmeter. A storage battery of 6 volts was employed as the input to the differential transformer. The output characteristics of the differential transformer when the iron core was immersed in water and the relative relation between the location of the differential transformer and that of the equilibrium position of the float were preliminarily and accurately calibrated.

2.4 Experimental procedure and arranging methods of the experimental data

The float with small l defined in Reference (1) had $\beta_2 = l/d_f \leq 0.01$ when employing a tapered tube having its gradient k smaller than roughly 10^{-2} . It was expected that the limit of the value of β_2 for the float with small l was changed because of employing the tapered tubes having a large gradient. Therefore, in the beginning of the experiments, all floats employed were set with $\beta_2 = 0.01$ and a gradient of the tapered tube to give a constant discharge coefficient of the float-area-type flow meters was sought. The experiments were carried out using the tapered tubes listed in Table 1 and the floats having $\beta_2 = 0.01$ and the relation between the position of the float and the flow rate was obtained by changing the characteristic Reynolds number according to the varied weight of the float system.

The general flow rate characteristics of the float-area-type flow meters when employing a float

with small l were expressed using Eq. (2) in Reference (2) in the region of the characteristic Reynolds numbers larger than the critical characteristic Reynolds number, as

$$\left. \begin{aligned} Q/vd_f &= C_1 4\sqrt{2\pi} f(\beta_1) Re^{C_2} \\ f(\beta_1) &= \frac{\beta_1(1+2\beta_1)}{\sqrt{1 - \{(1+2\beta_1)^2(8\beta_1)^2/(1+4\beta_1)^4\}}} \end{aligned} \right\} \quad (1)$$

where $Re = [(W - F)/\nu^2 \rho]^{1/2}$, W denotes the weight of the float system, F the buoyant force acting on the float system, ν and ρ the kinematic viscosity and the density, respectively, Q the volume flow rate and C_1 and C_2 the experimental constants varying with the vertex angle of the cone portion upstream of the maximum diameter part of the float and the gradient of the tapered tube. As shown in Fig. 2, since the floats employed in the present experiments have no cone portion, i.e. the vertex angle is 180° , C_1 and C_2 become functions of only the gradient of the tapered tube. In the present study, assuming the relation expressed by Eq. (1) even when employing tapered tubes having a large gradient, experimental data obtained from respective tapered tube are plotted on the logarithmic scales with Q/vd_f and Re as rectangular coordinates and with β_1 as parameter. Since the experimental data for a fixed value of β_1 were ascertained to lie along a straight line, the value of C_2 was obtained from the gradient of the latter. Substituting the value of C_2 into Eq. (1), C_1 was obtained as an arithmetic mean of a number of experimental values. In the process of arranging the experimental values described above, the gradient of the tapered tube was found when the value of C_2 became unity. Subsequently, employing the tapered tubes having the gradient discovered, i.e. giving $C_2 = 1$, a limiting value of Re where the experimental values were off from the line with $C_2 = 1$, i.e. the critical characteristic Reynolds number Re_c , and a limiting value of β_2 of the float with small l were explored experimentally.

3. Experimental results and discussions

3.1 Relations between C_1 and C_2 in Eq. (1) and the gradients of the tapered tubes

Figure 3 shows the experimental results obtained by the above mentioned procedures. The relations shown in Fig. 3 are obtained from the experimental values for the regions of $Re > Re_c$ and $1.5 \times 10^{-2} < \beta_1 < 7 \times 10^{-2}$. However, since the value of C_1 was a little varied with the values of β_1 as shown later, the experimental results plotted in Fig. 3 were obtained as arithmetic means of a number of the experimental values.

Fig. 3 Relations between k and C_1 and C_2

As seen from Fig. 3, C_2 becomes unity when employing a tapered tube having $k = 0.545$. As reported in Reference (2), $C_2 = 1$ means that the flow rate characteristics of the float-area-type flow meters are unaffected by a change in the kinematic viscosity of the working fluid. It is seen from Fig. 3 that the region of k is roughly $0.40 < k < 0.61$, allowing for

the spread in the values of C_2 which is $\pm 1\%$. Namely, we express 2θ instead of k , the extent of 2θ becoming $43.6^\circ \sim 62.8^\circ$. As seen from these experimental facts, the value of C_2 is considered to be considerably insensible to the gradient of the tapered tube as compared with the vertex angle of the cone portion upstream of the maximum diameter part of the float reported in Reference (2).

Figure 4 shows the relations between C_1 and β_1 for typical values of Re when employing tapered tubes having $k = 0.545$ and when putting $C_2 = 1$ in Eq. (1). As seen from Fig. 4, the value of C_1 becomes 0.720 with an accuracy of $\pm 3\%$ in the range of $1.5 \times 10^{-2} < \beta_1 < 7 \times 10^{-2}$. Thus, it is ascertained experimentally that Eq. (1) may be rewritten as

$$\left. \begin{aligned} Q/vd_f &= 0.720 \times 4\sqrt{2\pi} f(\beta_1) Re \\ f(\beta_1) &= \frac{\beta_1(1+2\beta_1)}{\sqrt{1 - \{(1+2\beta_1)^2(8\beta_1)^2/(1+4\beta_1)^4\}}} \end{aligned} \right\} \quad (2)$$

Fig. 4 Relation between C_1 and β_1

3.2 Critical characteristic Reynolds number

It was reported in Reference (2) that the critical characteristic Reynolds number Re_c , where the discharge coefficient of the float-area-type flow meter remained constant, was roughly 3×10^3 , at $\beta_1 = 10^{-2}$ when employing a tapered tube having a gradient smaller than roughly 10^{-2} .

In the present study of $k = 0.545$, on the other hand, in order to determine the value of Re_c , i.e. the range of applicabilities of Eq. (2), we show the general flow rate characteristics in Fig. 5. In Fig.

Fig. 5 General flow rate characteristics when $k = 0.545$

5, the solid lines show the semi-empirical values calculated from Eq. (2) and the symbols $\circ$ denote the experimental values. As seen from Fig. 5, Re_c at $\beta_1 = 1.5 \times 10^{-2}$ may be considered to be roughly 3.5×10^3 .

3.3 Limiting value of the aspect ratio

$$\beta_2 = l / df \text{ for the float with small } l$$

In order to verify the limiting value of β_2 experimentally, Fig. 6 is given the relation between Q/vdf and β_2 when $Re = 3.16 \times 10^4$ as a typical example. In Fig. 6, the solid lines express Eq. (2) and the symbols $\circ$ denote the experimental values. As seen from Fig. 6, they agree well when $\beta_2 < 1$.

Fig. 6 Limits for the float with small l

Therefore, it is evident that the floats having even $\beta_2 = 1$ are considered to be ones with small l when employing tapered tubes having $k = 0.545$. Further, the same results are obtained for the other characteristic Reynolds numbers.

The reason for cutting out the experiment at $\beta_2 = 1$ is as follows. Judging from the purpose of manufacturing a float-area-type flow meter shortened as described in Introduction, it was considered that a suitable limit of β_2 was nearly unity. Moreover, from the practical point of view, it was judged that a type in which the float-system was held with springs by setting β_2 smaller than unity would be superior to one designed with a large β_2 to increase the weight of the float-system. If the spring-held-type is adopted, it will yield a merit that the float-area-type flow meter can be installed in not only a vertical pipe-line but a horizontal one.

Since the flow rate characteristics of a spring-held-type float-area-type flow meter are also those of a force-balanced-type flow meter, the analytical results for the float-area-type flow meter known so far can not be applied directly. The general flow rate characteristics in this case, however, were reported in detail by Mori and Ochiai⁽⁶⁾.

3.4 Influence of the gradient of the tapered tube on the flow rate characteristics when employing a float with small l

The elemental relation for the flow rate characteristics of a float-area-type flow meter has been reported in the previous report⁽¹⁾ as

$$Q/vdf = f(\beta_1, \beta_2, k, Re) \quad (3)$$

When employing specially designed tapered tubes or floats having peculiar shapes, the factors depending on them come into Eq. (3). For example, tri-flat tubes⁽⁶⁾ or spheres⁽⁷⁾ in place of usually employed

floats are just the case in point. Here, it should be emphasized the need for a comprehensive investigation on the flow rate characteristics of the float-area-type flow meters.

It has been reported⁽¹⁾ that k and β_2 in Eq. (3) can be eliminated when employing a tapered tube having small k and a float with small β_2 , and thereby, Eq. (3) becomes

$$Q/vdf = f(\beta_1, Re) \quad (4)$$

The influence of β_2 , eliminated in process of rewriting Eq. (3) into Eq. (4), on the flow rate characteristics can be determined from the theoretical results reported in Reference (4) in the laminar region of the flow in the annular clearance between the maximum diameter part of the float and the tapered tube. On the other hand, the influence of k on the flow rate characteristics has not been reported so far because of difficulty of the theoretical treatment depending on the separation of flow in the annular clearance when employing tapered tubes having large value of k . Judging from the experimental fact that a float having β_2 up to unity is considered to be one with small l described in Section 3.3, it can be considered that there is no necessity for taking account of the float with large l . Therefore, Eq. (3) becomes

$$Q/vdf = f(\beta_1, k, Re) \quad (5)$$

In order to show Eq. (5) in a diagram, it is seen, the values of β_1 and Re must be fixed.

Figure 7 shows, as a typical example, the general flow rate characteristics when $Re = 3.16 \times 10^4$. It is seen from Fig. 7 that the dimensionless flow rate Q/vdf increases linearly with an increasing k on the rectangular coordinate with logarithmic scales.

Moreover, it is also seen from Fig. 7 that straight lines linking the experimental values of respective β_1 are parallel to each other. This means that a measuring range of flow rates is unaffected by a change in k within the limits of this experiment.

Fig. 7 General flow rate characteristics when $Re = 3.16 \times 10^4$

3.5 Discharge coefficient

The discharge coefficient C of the float-area-type flow meters has been defined as

$$C = Q / A \sqrt{2gV_f(\gamma_f - \gamma_o)} / A_f \gamma_o \quad (6)$$

where $A = \pi(D_t^2 - d_f^2)/4$, $V_f(\gamma_f - \gamma_o) = W - F$, $A_f = \pi d_f^2/4$ and $g/\gamma_o = 1/\rho$. Since the viscosity of the working fluid contains implicitly the discharge

coefficient C in Eq. (6), it is evident that the discharge coefficient of the float-area-type flow meter is generally not constant for a varying viscosity.

One of the authors reported in Reference (1) that the discharge coefficient of this kind of flow meter became constant when employing a tapered tube having a small value of k as one limit of the gradient of the tapered tube and a float with small l and with the vertex angle $\theta_1 = 96^\circ$ as described above and when $Re > Re_{ec}$ and $10^{-2} < \beta_1 < 6 \times 10^{-2}$.

On the other hand, in the present investigation, the gradient of the tapered tube where the discharge coefficient becomes constant is ascertained experimentally when $\theta_1 = 180^\circ$ as the other limit of the vertex angle. Figure 8 is a qualitative illustration of

Fig. 8 Quantitative relation between k and θ_1

these relations. It may be possible to manufacture innumerable float-area-type flow meters with constant discharge coefficients, if various combinations of θ_1 and k obtained from a point on a two-dot chain line drawn provisionally in Fig. 8 are adopted. Since the value of Re_{ec} and the range of β_1 are considered to be more or less different according to a combination of θ_1 and k , it is expected to obtain nearly the same values of them by referring to the results obtained in Reference (1) and in the present studies.

In order to obtain the discharge coefficient of this kind of flow meter in the case of $k = 0.545$, values of C calculated from Eqs. (2) and (6) are plotted in Fig. 9. In calculating C in Fig. 9, the value

Fig. 9 Discharge coefficient

of C_1 was not fixed at 0.720 but was obtained from Fig. 4 for each value of β_1 . Therefore, it becomes evident from Fig. 9 that the discharge coefficient of the float-area-type flow meter, when employing a float with small l and a tapered tube with $k = 0.545$, is estimated to be 0.755 within an accuracy of $\pm 5\%$ irrespective of Re and β_1 for $3.5 \times 10^3 < Re$ and $1.5 \times 10^{-2} < \beta_1 < 7 \times 10^{-2}$.

However, since $Q/\nu df$ in Eq. (1) is very sensitive to C_2 , conversion of the flow rate over a figure of Re by using Eq. (2) should be avoided to maintain the accuracy.

4. Float-area-type flow meters having a linear relation between the equilibrium position of the float and the flow rate

The relation between the equilibrium position of the float and the flow rate of the float-area-type flow meter is expressed generally as a concave line on orthogonal coordinate system in which the ordinate is the flow rate and the abscissa is the equilibrium position of the float. For example, Figs. 6 and 7 in Reference (1) are typical examples. Therefore, when the flow rate is converted to a pneumatic or an electric signal, the transmitter-mechanism becomes naturally complicated on account of the non-linearity of the flow rate characteristics. One of the useful methods of eliminating this fault is to manufacture a tapered tube with not uniform gradient distribution. As seen from Fig. 3, fortunately, a range of k maintaining $C_2 = 1 \pm 1\%$ is as wide as $0.40 < k < 0.61$. Therefore, we tried to manufacture a tapered tube with not uniform gradient distribution by varying D_t -distribution within the range of above mentioned k .

Such gradient distribution, i.e. inner diameter distribution, of the tapered tube can be obtained by selecting a combination of C_1 and β_1 from Figs. 3 and 4 independently of Re in the range of $Re > Re_{ec}$. The combination of them, however, must be taken such that $C_1 f(\beta_1)$ in Eq. (1) becomes an equation of the first degree. The solid line in Fig. 10 shows the inner diameter distribution of the tapered tube when $df = 25$ mm as a typical example. In Fig. 10, the

Fig. 10 Typical example of inner diameter distribution of the tapered tube

two-dot chain line denotes the uniform inner diameter distribution for comparison and the abscissa x denotes the distance toward the downstream of the tapered tube from the starting point of the taper.

As a means of producing the tapered tube as shown in Fig. 10, a numerical control lathe was considered suitable. However, the widely used lathe was employed in the present study, because it was a trial manufacture.

Figure 11 shows the relation between the equilibrium position x of the float and the flow rate Q obtained by employing the above-mentioned tapered tube. Figure 11 shows a typical example only when $Re = 4.16 \times 10^4$, however, it is evident that the relation between Q and x is linear enough for practical use. That is to say, it becomes evident from the present study that a float-area-type flow meter having the merits of linear relation between the equilibrium position of the float and the flow rate and independent of the change in the kinematic viscosity of the working fluid can be manufactured.

Fig. 11 Relation between x and Q

5. Conclusions

General considerations were made on the geometrical dimensions of the tapered tubes and the float of the float-area-type flow meters such that a constant discharge coefficient of the latter could be obtained the results obtained are as follows:

(1) It was found experimentally that the discharge coefficient of the float-area-type flow meter remained constant at 0.755 within an accuracy of $\pm 5\%$ when a float with small l and a particular vertex angle $\theta_1 = 180^\circ$ was employed for the regions of $3.5 \times 10^3 < R$ and $1.5 \times 10^{-2} < \beta_1 < 7 \times 10^{-2}$.

(2) It was found experimentally that the ratio of the measuring ranges of the flow rate of the float-area-type flow meter was unchanged when a float with small l was employed for the range of $0.0846 < k < 1.014$.

(3) By referring to the result obtained in the present study, a float-area-type flow meter having a linear relation between the equilibrium position of the float and the flow rate and having a constant discharge coefficient irrespective of R_e and β_1 over a limiting range of this study was trially manufactured, and this was verified by experiments.

These experiments were partly performed by Mr. Kenichi Komatsu, Mr. Kaoru Kitajima, Mr. Masaharu Okanishi and Mr. Ryûzo Ôshima as part of their undergraduate thesis at Science University of Tokyo and the authors are grateful to them for their cooperation.

References

- (1) Maki, H., Mori, Y., Bull. JSME, 15-83 (1972-5), 585.
- (2) Maki, H., Bull. JSME, 17-103 (1974-1), 83.
- (3) Mori, Y., Jour. Japan Soc. Preci. Engng. (in Japanese), 23-3(1957-3), 96.
- (4) Maki, H., Mori, Y., Bull. JSME, 13-65 (1970-11), 1316.
- (5) Mori, Y., Ochiai, H., Trans. JSME (in Japanese), 40-339 (1974-11), 3129.
- (6) Aikawa, N., Kanazumi, J., Preprint of JSME (in Japanese), No. 764-4 (1976-3), 1.
- (7) Maki, H., Hydraulic Engng. (in Japanese), 13-8 (1977-8), 460.