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Influence of the Gas-film Inertia Forces on the Dynamic Characteristics of Externally Pressurized, Gas Lubricated Journal Bearings*

--- Part I: Proposal of Governing Equations ---

By Atsunobu MORI**, Kuniaki AOYAMA*** and Haruo MORI****

The conventional analyses based on the classical Reynolds equation are incomplete to get fine predictions of the dynamic characteristics of externally pressurized gas bearings. This seems to be attributed to the fact that the Reynolds equation ignores entirely the inertia forces of the gas-film.

This paper proposes modified Reynolds equations, which are derived by averaging out the inertia forces in Navier-Stokes equations across the film thickness, and their adequate boundary conditions, in order to determine the influence of the inertia forces on the dynamic characteristics such as the whirl instability and the squeeze damping.

1. Introduction

Inertia forces of the gas-film are generally ignored in the analysis of the dynamic characteristics of externally pressurized gas bearings as well as in the analysis of oil-film lubrication.

In the externally pressurized gas bearings, the inertia forces have little effect upon the static performances except the pressure depression just downstream of the feed holes, but they seem to act with considerable magnitude on the dynamic characteristics such as the whirl instability and the squeeze damping.

As for the whirl instability, the conventional theory based on the classical Reynolds equation, which is, for example, found in the paper by Lund[1], is not able to analyze the following important facts which have been experienced experimentally in inherently compensated journal bearings; i.e., the fact that, in such bearings, the rotational speed of the journal at threshold of instability is generally greater than twice the whirl speed[2], [3], and that the ratio of the threshold speed to the whirl speed is affected little by the eccentricity[4] but considerably by the axial mass rate of flow supplied through the feed holes[5]. This seems to be attributed to neglect of the inertia forces of the gas-film.

At the JSME annual meeting on April 2nd, 1977, H. Izumi firstly pointed out that the above mentioned experimental facts would be explained by considering the growth of the tangential velocity profile for axial pressure flow. The authors have the same opinion.

As for the squeeze damping, the experimental damping coefficient tends to be larger than the conventional analytical result as found in the paper by Mullan & Richardson [6] for example. The tendency also seems to grow with the mass rate of flow by the external pressurization. This may be caused by the inertia forces of the squeezed gas-film which are convectively magnified by a high speed pressure flow, unlike in a self-acting squeeze bearing. The tendency must also be attributed to the inertia forces, though it has, up to this time, been attributed to the difficulty of isolating the experimental system from external sources of damping.

Thus, the analyses of the dynamic characteristics of the externally pressurized gas bearings stand in need of some evaluation of the effect of the inertia forces of the gas-film.

In the previous paper[7], the authors have discussed the influence of the inertia forces on the squeeze damping of a inherently compensated, parallel thrust collar bearing with multiple holes admission by using the modified Reynolds equations which were derived by averaging out the inertia forces across the film thickness, and, from the comparison with the experiment, they have confirmed that such a method would be able to give good evaluation of the inertia effect.

In this paper, based on the same approach, modified Reynolds equations and their boundary conditions are introduced to determine the influence of the inertia forces of the gas-film on the dynamic characteristics of the externally pressurized gas journal bearings such as the whirl instability and the squeeze damping. This approach yields well-formed governing equations considering the boundary conditions.

2. Nomenclature

C_D : discharge coefficient of the inherent orifice.
 C_r : radial clearance.

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** Assistant, Faculty of Technology, Kyoto University, Kyoto.

*** Graduate student, Faculty of Technology, Kyoto University, Kyoto.

**** Professor, Faculty of Technology, Kyoto University, Kyoto.

D : bearing diameter.
 d_s : feed hole diameter.
 $F = \frac{\mu R f}{p a}$: dimensionless form of f .
 f : effective pressure gradient in x -direction including inertia effect, defined by Eq. (5).
 $G = \frac{\mu R g}{p a}$: dimensionless form of g .
 g : effective pressure gradient in z -direction including inertia effect, defined by Eq. (6).
 $H = h/C_r$: dimensionless local clearance.
 h : local clearance, see Fig. 1.
 $\dot{m}_{in}, \dot{m}_{out}$: mass rates of flow through the inherent orifices and through the clearance space at the feeding section.
 $P = p/p_a$: dimensionless pressure.
 p : film pressure.
 p_s, p_a : supply pressure and ambient pressure.
 ΔQ : assumed value of volumetric rate of tangential flow at the feeding section.
 $R = D/2$: bearing radius.
 IR : gas constant, defined as $p/\rho = IR T$.
 T_s, T : temperatures of the gas in the supply port and in the clearance space.
 t : time.
 U : sliding speed.
 u, v, w : velocity components in x -, y - and z -directions.
 x, y, z : Cartesian coordinates.
 $\alpha = \frac{C_D d_s n}{\Psi 4R}$: supply parameter.
 $\beta = \frac{p_a C_r^2 \omega}{24 IR T \mu}$
 $\gamma = \frac{p_a^2 C_r^4}{120 IR T \mu^2 R^2}$
 $\delta = \frac{(R \omega)^2}{6 IR T}$: dimensionless inertia parameters
 $\Gamma = \frac{12 \mu \alpha \sqrt{IR T} R}{p_a C_r^2}$: designing parameter.
 $\zeta = z/R$: dimensionless axial coordinate, see Fig. 1.
 θ : angular coordinate, see Fig. 1.
 κ : ratio of specific heats of the gas.
 $\Lambda = \frac{6 \mu \omega}{p_a} \left(\frac{R}{C_r}\right)^2$: compressibility number.
 μ : viscosity.
 $\xi = 1 - \frac{\Delta Q}{(hU/2)}$: correction factor for the boundary condition of F .
 ρ : density.
 $\tau = \omega t$: dimensionless time.
 ϕ : dimensionless speed of gas through the inherent orifice, defined by Eqs. (21-a,b).
 Ψ : correction factor for dispersion effect of discrete feed.
 ω : angular speed of rotation.

3. Modified Reynolds Equations

Based on the usual assumption of thin film lubrication theory, the equation of mo-

tion for a fluid element are;

$$\frac{\rho}{\mu} \left\{ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right\} + \frac{1}{\mu} \frac{\partial p}{\partial x} = \frac{\partial^2 u}{\partial y^2} \quad (1)$$

$$\frac{\partial p}{\partial y} = 0 \quad (2)$$

$$\frac{\rho}{\mu} \left\{ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right\} + \frac{1}{\mu} \frac{\partial p}{\partial z} = \frac{\partial^2 w}{\partial y^2} \quad (3)$$

where u, v and w are the velocity components in the x -, y - and z -directions respectively; y is the coordinate across the film thickness; t is time; p the pressure, ρ the density and μ the viscosity.

The continuity equation for a compressible fluid is given by;

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \quad (4)$$

Now assume that the inertia forces are relatively small and that unsteadiness of the flow is weak. Then, average out the inertia forces in Eqs. (1) and (3) across the film thickness, and define f and g as;

$$f = \frac{1}{\mu} \frac{\partial p}{\partial x} + \frac{\rho}{\mu h} \int_0^h \left\{ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right\} dy \quad (5)$$

$$g = \frac{1}{\mu} \frac{\partial p}{\partial z} + \frac{\rho}{\mu h} \int_0^h \left\{ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right\} dy \quad (6)$$

where h denotes the film thickness. Thence, Eqs. (1) and (3) become;

$$\frac{\partial^2 u}{\partial y^2} = f \quad (7)$$

$$\frac{\partial^2 w}{\partial y^2} = g \quad (8)$$

The velocity boundary conditions are;

$$y = 0 : u = v = w = 0 \quad (9)$$

$$y = h : u = U \quad (10)$$

$$v = U \frac{\partial h}{\partial x} + \frac{\partial h}{\partial t} \quad (11)$$

$$w = 0 \quad (12)$$

where U gives the sliding speed. Since both f and g are independent of y , the velocity components, u and w , can be integrated as;

$$u = \frac{f}{2} y(y - h) + \frac{U}{h} y \quad (13)$$

$$w = \frac{g}{2} y(y - h) \quad (14)$$

under those boundary conditions.

According to the thin film assumption, we assume that the change of state of the gas-film is isothermal. The continuity equation, Eq. (4), can then be written in the form of;

$$\frac{\partial p}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \quad (15)$$

From substitution of Eqs. (13) and (14) into this, the velocity component, v , can be ob-

tained as;

$$\begin{aligned} v = & -\frac{1}{\mu} \frac{\partial p}{\partial t} y - \frac{1}{12} \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial z} \right) y^2 (2y - 3h) \\ & - \frac{U}{2\mu} \frac{\partial}{\partial x} \left(\frac{p}{h} \right) y^2 - \frac{f}{4} \frac{\partial h}{\partial x} y^2 \\ & - \frac{f}{12\mu} \frac{\partial p}{\partial x} y^2 (2y - 3h) \\ & - \frac{g}{12\mu} \frac{\partial p}{\partial z} y^2 (2y - 3h) \end{aligned} \quad (16)$$

Substitutions of Eqs. (13), (14) and (16) into Eqs. (5) and (6), and the velocity boundary condition, Eq. (11), yield the following three simultaneous equations among the three unknown quantities, p , f and g ;

$$\begin{aligned} \frac{\partial}{\partial x}(ph^3f) + \frac{\partial}{\partial z}(ph^3g) \\ = 6U \frac{\partial}{\partial x}(ph) + 12 \frac{\partial}{\partial t}(ph) \end{aligned} \quad (17-a)$$

$$\begin{aligned} f = & \frac{1}{\mu} \frac{\partial p}{\partial x} + \frac{1}{\mu R T h} \left\{ \frac{1}{120} \frac{\partial}{\partial x}(ph^5f^2) \right. \\ & - \frac{1}{120} \frac{\partial}{\partial z}(ph^5fg) - \frac{U^2}{6} \frac{\partial}{\partial x}(ph) \\ & + \frac{U}{24} \frac{\partial}{\partial z}(ph^3g) - \frac{1}{12} \frac{\partial}{\partial t}(ph^3f) \\ & \left. - \frac{U}{2} \frac{\partial}{\partial t}(ph) \right\} \end{aligned} \quad (17-b)$$

$$\begin{aligned} g = & \frac{1}{\mu} \frac{\partial p}{\partial z} + \frac{1}{\mu R T h} \left\{ \frac{1}{120} \frac{\partial}{\partial x}(ph^5fg) \right. \\ & + \frac{1}{120} \frac{\partial}{\partial z}(ph^5g^2) - \frac{U}{24} \frac{\partial}{\partial x}(ph^3g) \\ & \left. - \frac{1}{12} \frac{\partial}{\partial t}(ph^3g) \right\} \end{aligned} \quad (17-c)$$

where R is the gas constant and T is the temperature of the gas-film; f and g represent the effective pressure gradients in x - and z -directions including inertia effect; and the second terms of the right-hand sides in Eqs. (17-b) and (17-c) denote the contributions from the inertia forces. Omission of those terms yields the classical Reynolds equation for a compressible fluid.

Now we imagine applications to the journal bearings, and normalize the equations by means of the following dimensionless quantities;

$$\begin{aligned} \theta = \frac{x}{R}, \quad \zeta = \frac{z}{R}, \quad H = \frac{h}{C_r}, \quad \omega = \frac{U}{R}, \quad \tau = \omega t, \\ \Lambda = \frac{6\mu\omega}{p_a} \left(\frac{R}{C_r} \right)^2, \quad \beta = \frac{p_a C_r^2 \omega}{24 R T \mu}, \quad \gamma = \frac{p_a^2 C_r^4}{120 R T \mu^2 R^2}, \\ \delta = \frac{(R\omega)^2}{6 R T}, \quad P = \frac{p}{p_a}, \quad F = \frac{\mu R f}{p_a}, \quad G = \frac{\mu R g}{p_a} \end{aligned}$$

where R is the bearing radius, C_r the radial clearance, ω the rotational angular speed and p_a the ambient pressure. Normalized forms of Eqs. (17-a,b,c) are as follows;

$$\begin{aligned} \frac{\partial}{\partial \theta}(PH^3F) + \frac{\partial}{\partial \zeta}(PH^3G) \\ = \Lambda \frac{\partial}{\partial \theta}(PH) + 2\Lambda \frac{\partial}{\partial \tau}(PH) \end{aligned} \quad (18-a)$$

$$\begin{aligned} HF = & H \frac{\partial P}{\partial \theta} + \gamma \left\{ \frac{\partial}{\partial \theta}(PH^5F^2) + \frac{\partial}{\partial \zeta}(PH^5FG) \right\} \\ & + \beta \left\{ \frac{\partial}{\partial \zeta}(PH^3G) - 2 \frac{\partial}{\partial \tau}(PH^3F) \right\} \\ & - \delta \left\{ \frac{\partial}{\partial \theta}(PH) + 3 \frac{\partial}{\partial \tau}(PH) \right\} \end{aligned} \quad (18-b)$$

$$\begin{aligned} HG = & H \frac{\partial P}{\partial \zeta} + \gamma \left\{ \frac{\partial}{\partial \theta}(PH^5FG) + \frac{\partial}{\partial \zeta}(PH^5G^2) \right\} \\ & - \beta \left\{ \frac{\partial}{\partial \theta}(PH^3G) + 2 \frac{\partial}{\partial \tau}(PH^3G) \right\} \end{aligned} \quad (18-c)$$

For analyses of the dynamic characteristics of externally pressurized gas bearings the authors would like to propose these simultaneous equations which include the effect of the gas-film inertia forces, in place of the classical Reynolds equation. Those should be named "modified Reynolds equations".

4. Boundary Conditions

In this chapter the boundary conditions required for the unknown quantities, P , F and G , in the modified Reynolds equations are introduced. The objective is an inherently compensated journal bearing with single plane feeding, which is familiar as a high speed bearing. It is schematically shown in Fig. 1. The feed holes are located in the midplane, with respect to which the bearing is symmetric. The number of feed holes is assumed large enough to be replaced by an equivalent line feed.

Fig. 1 Inherently compensated bearing with single plane feeding.

Condition for Film Pressure at Bearing Ends

At the bearing ends, the film pressure becomes equal to the ambient pressure. Thence;

$$\begin{aligned} P|_{\zeta=\pm L/D} &= 1 \end{aligned} \quad (19)$$

Continuity of Mass Rate of Flow at Feeding Section

At this section, the mass rate of flow through the inherent orifices must be equal to the mass rate of the axial flow through the bearing clearance. The former per circumferential unit length can be given by;

$$\dot{m}_{in} = \frac{C_D \pi d_s n}{2 \pi R} \frac{p_s}{\sqrt{RT_s}} h \phi \quad (20)$$

where

$$\phi = \sqrt{\frac{2\kappa}{\kappa-1} \left\{ \left(\frac{p|_{z=0}}{p_s} \right)^{\frac{2}{\kappa}} - \left(\frac{p|_{z=0}}{p_s} \right)^{\frac{\kappa+1}{\kappa}} \right\}} \quad (21-a)$$

$$\text{for } \frac{p|_{z=0}}{p_s} > \left(\frac{2}{\kappa+1} \right)^{\frac{\kappa}{\kappa-1}}$$

or

$$\phi = \sqrt{\frac{2\kappa}{\kappa+1} \left(\frac{2}{\kappa+1} \right)^{\frac{2}{\kappa-1}}} \quad (21-b)$$

$$\text{for } \frac{p|_{z=0}}{p_s} \leq \left(\frac{2}{\kappa+1} \right)^{\frac{\kappa}{\kappa-1}}$$

where p_s is the supply pressure, C_D the discharge coefficient of the inherent orifice, d_s the feed hole diameter, n the number of feed holes and κ the ratio of specific heats of the gas. On the other hand, based on the line source concept, the latter per circumferential unit length can be given by;

$$\begin{aligned} \dot{m}_{out} &= 2 \frac{p|_{z=0}}{RT} \Psi \int_0^h w|_{z=0} dy \\ &= - \frac{\Psi}{6RT} h^3 p|_{z=0} g|_{z=0} \end{aligned} \quad (22)$$

where Ψ is the correction factor for the dispersion effect of the discrete source. For an incompressible isothermal flow in which viscous forces are predominant, that is given by the following equation by means of the complex potential theory;

$$\begin{aligned} \frac{1}{\Psi} &= \frac{D}{Ln} \sum_k (-1)^{k+1} \times \\ &\ln \left[\frac{\cosh\{nd_s(1 - 2kL/d_s)/D\} - 1}{\cosh\{nL(1 - 2k)/D\} - 1} \right] \end{aligned} \quad (23)$$

where $k = 0, \pm 1, \pm 2, \dots$

In a usual case T_s is nearly equal to the temperature of the bearing bushing. The temperature of the gas-film, T , is also equal to this. T_s should be replaced by T .

Now introducing the designing parameter defined as;

$$\Gamma = \frac{12\mu \alpha \sqrt{RT} R}{p \alpha C_r^2} \quad (24)$$

where

$$\alpha = \frac{C_D d_s n}{\Psi 4R} \quad (25)$$

we can write the continuity equation, $\dot{m}_{in} = \dot{m}_{out}$, in dimensionless form as;

$$\Gamma P_s \phi = - H^2 \frac{P|}{\zeta=0} \frac{G|}{\zeta=0} \quad (26)$$

Condition for Tangential Flow at Feeding Section

Fig. 2 shows schematically a velocity profile downstream of the feeding section. The actual discrete feed is conceptually replaced by a continuous line feed. Figure (a) is for an ideal, complete line feed and assumes no tangential flow at the entrance,

Fig. 2 Models of velocity profile downstream of the feeding section.

Fig. 3 Actual and analyzed tangential velocity profiles.

while figure (b) is modified for an actual feed and assumes some amount of the tangential flow at the entrance.

In case of an ideal line feed, at the entrance to the bearing clearance space, the tangential drag flow generated by the journal rotation may be restricted to within a very thin layer near the journal surface, and the major part of the supplied gas may be oriented in the axial direction. Downstream of the feeding section the profile of the tangential velocity grows and approaches gradually that of a fully developed Couette flow.

Now we aim at the growth of the displacement thickness of the tangential flow under concentric condition. We can find that the modified Reynolds equations give an approximate solution, if we set at zero the boundary condition at the feeding section for the net flow rate in the tangential direction, whereas the velocity profile itself is different from the actual one as shown in Fig. 3 because of inherent assumption of fully developed axial and tangential flows. This fact indicates that the modified Reynolds equations with such a boundary condition can appropriately evaluate the effect of the inertia forces on the growth of the tangential drag flow which dominates the hydrodynamic wedge action.

Thus, in case of an ideal, complete line feed, we set the following condition

for the tangential flow at the feeding section;

$$\int_0^h u \Big|_{z=0} dy = 0 \quad (27)$$

Substitution of Eq. (13) into this equation yields;

$$f \Big|_{z=0} = \frac{6U}{h^2} \quad (28)$$

In dimensionless form this can be written as;

$$F \Big|_{\zeta=0} = \frac{\Lambda}{H^2} \quad (29)$$

In case of a discrete feed in an actual application, such inertia effect of the supplied gas may be weakened in the intervening space between the feed holes, and it may be natural to assume some amount of the tangential flow even at $z=0$ as shown in Fig. 1(b). Thence, the following condition can be considered instead of Eq. (27);

$$\int_0^h u \Big|_{z=0} dy = \Delta Q \quad (30)$$

This yields;

$$f \Big|_{z=0} = \xi \frac{6U}{h^2}, \quad \xi = 1 - \frac{\Delta Q}{hU/2} \quad (31)$$

In dimensionless form;

$$F \Big|_{\zeta=0} = \xi \frac{\Lambda}{H^2} \quad (32)$$

Theoretical determination of the value of ΔQ or ξ may be impossible, and this will be done experimentally. It may, at least, be true that the value of ΔQ is strongly influenced by the value of the supply parameter, α , and that ΔQ approaches zero or $hU/2$ respectively when α approaches unity which is corresponding to an ideal line feed or zero which is corresponding to a self-acting bearing without feed.

Periodicity of Unknown Quantities

Remaining condition is periodicity of the unknown quantities in the θ -direction, i.e.;

$$P(\theta, \zeta) = P(\theta+2\pi, \zeta) \quad (33-a)$$

$$F(\theta, \zeta) = F(\theta+2\pi, \zeta) \quad (33-b)$$

$$G(\theta, \zeta) = G(\theta+2\pi, \zeta) \quad (33-c)$$

Equations (19), (26), (29) or (32) and (33-a,b,c) are the requisite boundary conditions for the unknown quantities, P , F and G . The modified Reynolds equations with those boundary conditions will be able to evaluate the effects of the inertia forces on the wedge action as well as on the squeeze action.

Note that the dimensionless inertia parameters, β , γ and δ , in the modified Reynolds equations are given by;

$$\beta = \left(\frac{\alpha}{\Gamma}\right)^2 \Lambda \quad (34)$$

$$\gamma = 1.2 \left(\frac{\alpha}{\Gamma}\right)^2 \quad (35)$$

$$\delta = \frac{2}{3} \left(\frac{\alpha}{\Gamma}\right)^2 \Lambda^2 \quad (36)$$

where α is a new parameter which has never appeared in the conventional analyses. The bearing forces are characterized by this new parameter, α , besides the conventional parameters such as L/D , P_s , Γ and Λ . As found from the definition by Eq.(25), α represents not only the feeding magnitude but also the approach to the ideal line feed, and it shall be named "supply parameter".

5. Conclusions

Modified Reynolds equations were derived by averaging out the inertia forces across the film thickness. They have three unknown quantities, the film pressure and the effective pressure gradients in x - and z -directions which include the inertia forces of the gas film. Reasonable boundary conditions for those unknown quantities were also derived.

They would be able to evaluate the effects of inertia forces both on the wedge action and on the squeeze action, and would, consequently, be able to analyze the incomprehensible experimental facts about the whirl instability and the squeeze damping. This will be done in Part II.

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