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Asymmetric Shrink Fit of a Cylindrical Shell*

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In this paper the point-matching method is used to calculate the asymmetric contact pressure and stress distributions of a cylindrical shell induced by shrinking an asymmetric disk (which has a circular disk, sectors and ribs).

The classical theory of thin shells developed by Flügge is used to calculate the asymmetric displacement of the shell and two-dimensional elasticity theory is used to calculate that of the disk. In calculating the radial displacements of the disk, sectors and ribs are in contact with the circular disk and the stress functions for the sectors and circular disk are used.

The calculated distribution of the radial displacements of the cylindrical shell is in good agreement with the experimental data.

The axisymmetric stress resultant N_ϕ , moment resultant M_ϕ and asymmetric moment resultants $M_\theta, M_{\theta\phi}$ are dominant stresses in the shell.

1. Introduction

The problem of finding the contact stresses of a shell using the point matching method is discussed. Partitioning the contact length into several small parts the contact pressures and shearing forces are considered as constants in this part. Using this method, the problems of a cylindrical shell or a cylinder shrunk onto a shaft were solved by Conway⁽¹⁾ and Oda⁽²⁾ et al. And the contact problems of sector-to-cylinder were solved by Matsumoto et al⁽³⁾⁽⁴⁾.

In this paper, we deal with the contact problem of a cylindrical shell induced by shrinking an asymmetric disk (which has a circular disk, sectors and ribs) at the end of the shell.

This problem is characterized by the following two points.

(1) The cylindrical shell is shrunk onto the asymmetric disk where sectors and ribs are pressed onto the circular disk.

(2) The asymmetric deformations of the cylindrical shell occur.

and the approximation method mentioned above has not yet been applied for this problem.

The influence factors of the disk are calculated for the following two cases. In the first case, the pressure is applied at the sector parts, and the contact pressure between sectors and the circular disk is evaluated. In the second case, the pressure is applied at the rib parts and the radial deformations of the rib and circular disk are calculated.

For the calculation of the influence factors of the cylindrical shell, the out-of-plane shearing forces by contact pressures work at its end and Flügge's equations⁽⁵⁾ are used.

Using these influence factors and shrinkage allowance, the radial deformations and stresses in the cylindrical shell are calculated. Comparing these radial deformations with the experimental data, the analysis model for this problem is verified.

2. Theoretical analysis

2.1 Nomenclature

- m, q : numbers of partitions and ribs in $\frac{1}{4}$ circle
- $\theta_j, 2\epsilon_j$: partitioning point, width of partitioning ($j=1, 2, \dots, m+q$)
- $a_k^{(j)}$: radial deformation at θ_k point of the outer surface of disk loaded at θ_j point ($j, k=1, 2, \dots, m+q$)
- $b_k^{(j)}$: radial deformation at θ_k point of the cylindrical shell loaded at θ_j point ($j, k=1, 2, \dots, m+q$)
- p_j : contact pressure between circular disk and sector ($j=1, 2, \dots, m$) contact pressure between disk and cylindrical shell ($j=1, 2, \dots, m+q$)
- $P_i, P_i^{(j)}$: unit pressure working at θ_i point of sectors ($i=1, 2, \dots, m$), $\sum_{j=1}^m P_i^{(j)} = P_i = 1$
- 2α : angle of sectors
- r_i, r_o : outer and inner radii of sectors
- R_o, R_i : outer and inner radii of circular disk
- k_1, k_2 : $k_1 = R_o/R_i$, $k_2 = r_1/r_o$
- $\mu = \pi/\alpha$
- u_j^s : radial deformation of sector at the inner surface loaded by p_j at $r=r_o$ and $P_i^{(j)}$ at $r=r_i$ ($j=1, 2, \dots, m$)
- u_j^R : radial deformation of circular disk at the outer surface by p_j at $r=R_o$ ($j=1, 2, \dots, m$)
- Φ, Ψ : stress functions for the sector and for the circular disk
- E, ν : Young's modulus and Poisson's ratio of the disk and the cylindrical shell
- u : radial deformation of the disk and the cylindrical disk or axial displacement of the cylindrical shell
- v, w : tangential and radial displacements of the cylindrical shell
- a, h, l : radius, thickness and length of the cylindrical shell
- $D = Eh^3/(1-\nu^2)$
- $K = Eh^3/(12(1-\nu^2))$
- $u', u'', \dots : \partial u / \partial \phi, \partial^2 u / \partial \phi^2, \dots$
- $u', u'', \dots : a \partial u / \partial x, a^2 \partial^2 u / \partial x^2, \dots$
- $u', u'', \dots : a \partial^2 u / (\partial x \partial \phi), a \partial^2 u / (\partial \phi^2 \partial x), \dots$
- $N_\phi, N_\theta, N_{\theta\phi}, N_{\phi\theta}$: stress resultants of the cylindrical shell

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$M_s, M_\phi, M_{s\phi}, M_{\phi s}$: moment resultants of the cylindrical shell
 Q_s, Q_ϕ : out-of-plane shearing forces of the cylindrical shell
 T_s, S_s : Kirchhoff's equivalent in-plane and out-of-plane shearing forces
 $N_{zn}^{(i)}, M_{zn}^{(i)}, T_{zn}^{(i)}, S_{zn}^{(i)}$: complex values for N_s, M_s, T_s, S_s ($i=1, 2, \dots, 8; n=1, 2, \dots$)
 $R_c(\lambda_i)$: real part of complex λ_i ($i=1, 2, \dots, 8$)
 $\eta_i = 1 [R_c(\lambda_i) > 0]$ ($i=1, 2, \dots, 8$)
 $\eta_i = 0 [R_c(\lambda_i) < 0]$ ($i=1, 2, \dots, 8$)
 $\delta_{jk} = 1$ ($j=k$), $\delta_{jk} = 0$ ($j \neq k$)
 $(j, k = m+1, m+2, \dots, m+q)$
 $\sigma_r, \sigma_\theta, \tau_{r\theta}$: radial, tangential and shearing stresses of the disk

2.2 Analysis method and assumptions

Supposing only the normal pressures are working at the contact parts, the following equation is obtained.

$$\Phi = a_0 \ln r + b_0 r^2 + c_0 r^2 \ln r + \sum_{n=1}^{\infty} (a_n r^{\mu n} + b_n r^{\mu n+2} + a_n' r^{-\mu n} + b_n' r^{-\mu n+2}) \cos \mu n \theta \quad (2)$$

$$\left. \begin{aligned} \theta = \pm \alpha \quad \tau_{r\theta} = 0, \quad \int_{r_0}^{r_1} \sigma_\theta dr = 0, \quad \int_{r_0}^{r_1} \sigma_r dr = 0 \\ r = r_1 \quad \sigma_r = \sum_{n=0}^{\infty} A_n' \cos \mu n \theta, \quad \tau_{r\theta} = 0, \quad r = r_0 \quad \sigma_r = \sum_{n=0}^{\infty} A_n \cos \mu n \theta, \quad \tau_{r\theta} = 0 \end{aligned} \right\} \quad (3)$$

Constants a_n, b_n, a_n', b_n' are calculated by the well known theory of the two-dimensional elasticity.

$$\left. \begin{aligned} a_n &= -\frac{A_n}{2(\mu n-1)} \frac{r_0^{\mu n}}{\Delta} (r_0^2 M + \mu n r_1^{2\mu n} N) + \frac{A_n'}{2(\mu n-1)} \frac{r_1^{\mu n}}{\Delta} (r_1^2 M + \mu n r_0^{2\mu n} N) \\ b_n &= \frac{A_n}{2(\mu n+1)} \frac{r_0^{\mu n}}{\Delta} (M + \mu n r_1^{2\mu n-2} N) - \frac{A_n'}{2(\mu n+1)} \frac{r_1^{\mu n}}{\Delta} (M + \mu n r_0^{2\mu n-2} N) \\ a_n' &= -\frac{A_n}{2(\mu n+1)} \frac{r_1^{2\mu n} r_0^{\mu n+2}}{\Delta} (M + \mu n r_0^{2\mu n-2} N) + \frac{A_n'}{2(\mu n+1)} \frac{r_0^{2\mu n} r_1^{\mu n+2}}{\Delta} (M + \mu n r_1^{2\mu n-2} N) \\ b_n' &= \frac{A_n}{2(\mu n-1)} \frac{r_1^{2\mu n-2} r_0^{\mu n}}{\Delta} (r_1^2 M + \mu n r_0^{2\mu n} N) - \frac{A_n'}{2(\mu n-1)} \frac{r_0^{2\mu n-2} r_1^{\mu n}}{\Delta} (r_0^2 M + \mu n r_1^{2\mu n} N) \end{aligned} \right\} \quad (4)$$

where

$$\Delta = M^2 - (\mu n)^2 r_0^{2\mu n-2} r_1^{2\mu n-2} N, \quad M = r_0^{2\mu n} - r_1^{2\mu n}, \quad N = r_0^2 - r_1^2$$

For a_0, b_0, c_0 the following linear equations are obtained from Eq. (3).

$$\begin{pmatrix} \ln \frac{r_0}{r_1} & r_1^2 - r_0^2 & r_1^2 \ln r_1 - r_0^2 \ln r_0 \\ \frac{1}{r_0^2} & 2 & 1 + 2 \ln r_0 \\ \frac{1}{r_1^2} & 2 & 1 + 2 \ln r_1 \end{pmatrix} \begin{pmatrix} a_0 \\ b_0 \\ c_0 \end{pmatrix} = \begin{pmatrix} 0 \\ A_0 \\ A_0' \end{pmatrix} \quad (5)$$

Solving above equations, we have

Fig.1 Asymmetric disk

Fig.2 Coordinate system of the cylindrical shell

Fig.3 Sector

$$\left. \begin{aligned} a_0 &= \frac{r_0^2}{L} [(2 \ln k_2 + (k_2^2 - 1)) A_0 - (2 k_2^2 \ln k_2 + (k_2^2 - 1)) A_0'] \\ b_0 &= \frac{A_0}{L} [-(1 + \ln r_1) + k_2^{-1} (1 + \ln r_0) - \ln k_2 (1 + 2 \ln r_1)] \\ &\quad + \frac{A_0'}{L} [k_2^2 (1 + \ln r_1) - (1 + \ln r_0) + \ln k_2 (1 + 2 \ln r_0)] \\ c_0 &= \frac{1}{L} [(1 - k_2^{-1} + 2 \ln k_2) A_0 + (1 - k_2^2 - 2 \ln k_2) A_0'] \end{aligned} \right\} \quad (6)$$

$$[(\alpha_k^{(j)} + (b_k^{(j)}))] \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_{m+q} \end{pmatrix} = \begin{pmatrix} s \\ s \\ \vdots \\ s \end{pmatrix} \quad (1)$$

where $\alpha_k^{(j)}$ and $b_k^{(j)}$ are the influence factors of the disk and the cylindrical shell and s is the shrinkage allowance.

Calculating the influence factors of the asymmetric disk as shown in Fig. (1), the sectors and ribs are pressed onto the circular disk. We divide the sector parts into m sections, each rib corresponding to each section.

In calculating the influence factors of the cylindrical shell as shown in Fig. (2), the out-of-plane shearing forces by the contact pressures are supposed to act at the end part of it, and Flügge's equations are used.

2.3 Deformation analysis of sectors

We use the following stress function and boundary conditions for the sector as shown in Fig. (3).

where $L = -4(\ln k_2)^2 + (k_2 - k_2^{-1})^2$

From Eq. (4) and Eq. (6), the radial displacements for the sector at $r=r_0$ and $r=r_1$ are,

$$Eu|_{r=r_0} = \left[\frac{2[4 \ln k_2 + (k_2^2 - k_2^{-2})]}{4(\ln k_2)^2 - (k_2 - k_2^{-1})^2} + (1-\nu) \right] r_0 A_0 - \frac{4[(k_2 - k_2^{-1}) + (k_2 + k_2^{-1}) \ln k_2]}{4(\ln k_2)^2 - (k_2 - k_2^{-1})^2} r_1 A_0' \\ + \sum_{n=1}^{\infty} \frac{2}{\Delta_n} [(2\mu n + 1 - \nu) k_2^{2\mu n - 2} + (\mu n)^2 (1 + \nu) + 2(1 - \nu)((\mu n)^2 - 1) k_2^{-2}] r_0 A_n \cos \mu n \theta \\ - (3 - \nu)(\mu n)^2 k_2^{-4} - (2\mu n - 1 + \nu) k_2^{-2\mu n - 2}] r_1 A_n \cos \mu n \theta \\ - \sum_{n=1}^{\infty} \frac{4\mu n}{\Delta_n} [(\mu n + 1)(k_2^{\mu n - 1} - k_2^{-\mu n - 1}) - (\mu n - 1)(k_2^{\mu n - 1} - k_2^{-\mu n - 1})] r_1 A_n' \cos \mu n \theta - A \cos \theta \quad (7)$$

$$Eu|_{r=r_1} = - \left[\frac{2[4 \ln k_2 + (k_2^2 - k_2^{-2})]}{4(\ln k_2)^2 - (k_2 - k_2^{-1})^2} - (1-\nu) \right] r_1 A_0' + \frac{4[(k_2 - k_2^{-1}) + (k_2 + k_2^{-1}) \ln k_2]}{4(\ln k_2)^2 - (k_2 - k_2^{-1})^2} r_0 A_0 \\ - \sum_{n=1}^{\infty} \frac{2}{\Delta_n} [(2\mu n - 1 + \nu) k_2^{2\mu n - 2} + (\mu n)^2 (3 - \nu) - 2((\mu n)^2 - 1)(1 - \nu) k_2^{-2}] r_1 A_n' \cos \mu n \theta \\ - (\mu n)^2 (1 + \nu) k_2^{-4} - (2\mu n + 1 - \nu) k_2^{-2\mu n - 2}] r_0 A_n \cos \mu n \theta + A \cos \theta \quad (8)$$

where

$$\Delta_n = 2[(\mu n)^2 - 1][(1 - k_2^{-2})^2 - k_2^{-2}(k_2^{-\mu n} - k_2^{\mu n})^2]$$

Then the radial displacement of the sector at $r=r_0$ loaded by p_j at $r=r_0$ and $P_i^{(U)}$ at $r=r_1$ as shown in Fig. (3) is given by Eq. (9).

$$Eu_j^R|_{r=r_0} = \alpha(k_2, \varepsilon_j) p_j + \beta(k_2, \varepsilon_i) P_i^{(U)} \\ + \sum_{n=1}^{\infty} \gamma_n(k_2, \theta_j, \varepsilon_j) p_j \cos \mu n \theta + \sum_{n=1}^{\infty} \delta_n(k_2, \theta_i, \varepsilon_i) P_i^{(U)} \cos \mu n \theta + A_j \cos \theta \quad (9)$$

$$\alpha(k_2, \varepsilon_j) = - \left[\frac{2[4 \ln k_2 + (k_2^2 - k_2^{-2})]}{4(\ln k_2)^2 - (k_2 - k_2^{-1})^2} + (1 + \nu) \right] \frac{2r_0 \varepsilon_j}{\alpha}$$

$$\beta(k_2, \varepsilon_i) = \frac{8[(k_2 - k_2^{-1}) + (k_2 + k_2^{-1}) \ln k_2]}{4(\ln k_2)^2 - (k_2 - k_2^{-1})^2} \frac{r_1 \varepsilon_i}{\alpha}$$

$$\gamma_n(k_2, \theta_j, \varepsilon_j) = - \frac{8r_0}{n\pi \Delta_n} [(2\mu n + 1 - \nu) k_2^{2\mu n - 2} + (\mu n)^2 (1 + \nu) \\ + 2(1 - \nu)((\mu n)^2 - 1) k_2^{-2} - (3 - \nu)(\mu n)^2 k_2^{-4} - (2\mu n - 1 + \nu) k_2^{-2\mu n - 2}] \cos \mu n \theta_j \sin \mu n \varepsilon_j$$

$$\delta_n(k_2, \theta_i, \varepsilon_i) = \frac{16\mu r_1}{\pi \Delta_n} [(\mu n + 1)(k_2^{\mu n - 1} - k_2^{-\mu n - 1}) - (\mu n - 1)(k_2^{\mu n - 1} - k_2^{-\mu n - 1})] \cos \mu n \theta_i \sin \mu n \varepsilon_i$$

2.4 Deformation analysis of the circular disk

The stress function and boundary conditions for the circular disk loaded by p_j at the outer surface as shown in Fig. (4) are given as follows.

$$\psi = a_0 \ln r + b_0 r^2 + \sum_{n=1}^{\infty} (a_n r^{2n} + b_n r^{2n+2} + a_n' r^{-2n} + b_n' r^{-2n+2}) \cos 2n\theta \quad (10)$$

$$r=R_i \quad \sigma_r=0, \quad \tau_{r\theta}=0, \quad r=R_o \quad \sigma_r = \sum_{n=0}^{\infty} A_n \cos 2n\theta, \quad \tau_{r\theta}=0 \quad (11)$$

Fig.4 Circular disk

Using the Fourier series expansion of p_j and Eqs. (10)(11) the radial displacement of the circular disk at $r=R_o$ is,

$$Eu_j^R|_{r=R_o} = -K(k_1, \theta_j, \varepsilon_j) R_o p_j \cos 2n\theta \quad (12)$$

$$K(k_1, \theta_j, \varepsilon_j) = \frac{4}{\pi} \left[\frac{1 + \nu + (1 - \nu) k_1^2}{k_1^2 (1 - k_1^{-2})} \varepsilon_j - \sum_{n=1}^{\infty} \frac{\sin 2n\varepsilon_j \cos 2n\theta_j}{n \Delta^*} [(4n - 1 + \nu) k_1^{4n - 2} \right. \\ \left. + (3 - \nu)(2n)^2 - 2(1 - \nu)((2n)^2 - 1) k_1^{-2} - (1 + \nu)(2n)^2 k_1^{-4} - (4n + 1 - \nu) k_1^{-4n - 2}] \right] \\ \Delta^* = (4n^2 - 1)(4n^2(1 - k_1^{-2})^2 - k_1^{-2}(k_1^{2n} - k_1^{-2n})^2)$$

2.5 Influence factors of the disk

Calculating the radial displacement of the disk loaded at θ_i section of the sector, we discuss at first the contact pressures between sectors and circular disk. We consider the radial displacements of the disk loaded by P_i at θ_i section of the outer surface as the superposition of the displacements of the sector loaded by $P_i^{(U)}$ and p_j at the outer and inner surfaces and of the circular disk loaded by p_j at the outer surface.

From Eqs. (9)(12) discussed before, we equalize the radial displacements at $\theta_j (j=1, 2, \dots, m)$,

$$\sum_{j=1}^m Eu_j S|_{r=r_0} = \sum_{j=1}^m Eu_j R|_{r=R_0} \dots\dots\dots (13)$$

Rewriting the above equations, we obtain the following equations.

$$\begin{aligned} \sum_{j=1}^m \{K(k_1, \theta_j, \varepsilon_j) R_0 \cos 2n\theta_k + \alpha(k_2, \varepsilon_j) + \sum_{n=1}^{\infty} \gamma_n(k_2, \theta_j, \varepsilon_j) \cos \mu n\theta_k\} p_j \\ + \beta(k_2, \varepsilon_k) + \sum_{n=1}^{\infty} \delta_n(k_2, \theta_k, \varepsilon_k) \cos \mu n\theta_k + A \cos \theta_k = 0 \dots\dots\dots (14) \end{aligned}$$

Substituting $p_j = p_j^{(1)} + A p_j^{(2)}$ into Eq. (14),

$$\begin{aligned} \sum_{j=1}^m \{K(k_1, \theta_j, \varepsilon_j) R_0 \cos 2n\theta_k + \alpha(k_2, \varepsilon_j) + \sum_{n=1}^{\infty} \gamma_n(k_2, \theta_j, \varepsilon_j) \cos \mu n\theta_k\} p_j^{(1)} \\ = -\beta(k_2, \varepsilon_k) - \sum_{n=1}^{\infty} \delta_n(k_2, \theta_k, \varepsilon_k) \cos \mu n\theta_k \dots\dots\dots (15) \end{aligned}$$

$$\sum_{j=1}^m \{K(k_1, \theta_j, \varepsilon_j) R_0 \cos 2n\theta_k + \alpha(k_2, \varepsilon_j) + \sum_{n=1}^{\infty} \gamma_n(k_2, \theta_j, \varepsilon_j) \cos \mu n\theta_k\} p_j^{(2)} = -\cos \theta_k \dots\dots\dots (16)$$

where A is given in Eq. (17) by the equilibrium condition for the sector.

$$A = \frac{k_2 \sin \varepsilon_k \cos \theta_k - \sum_{j=1}^m p_j^{(1)} \sin \varepsilon_j \cos \theta_j}{\sum_{j=1}^m p_j^{(2)} \sin \varepsilon_j \cos \theta_j} \dots\dots\dots (17)$$

Calculating the contact pressures p_j from Eqs. (15) - (17), the radial displacements of the sector at $r=r_1$ are given as follows.

$$\begin{aligned} Eu|_{r=r_1} = \left\{ \frac{4 \ln k_2 + (k_2^2 - k_2^{-2})}{4 (\ln k_2)^2 - (k_2 - k_2^{-1})^2} - \frac{1-\nu}{2} \right\} \frac{4 \varepsilon_k r_1}{\alpha} - \sum_{j=1}^m \frac{(k_2 - k_2^{-1}) + (k_2 + k_2^{-1}) \ln k_2}{4 (\ln k_2)^2 - (k_2 - k_2^{-1})^2} \frac{8 r_0 \varepsilon_j p_j}{\alpha} \\ + \sum_{j=1}^m \sum_{n=1}^{\infty} \frac{16 \mu r_0 p_j}{\pi \Delta_n} \{(\mu n + 1)(k_2^{-\mu n - 1} - k_2^{\mu n - 1}) - (\mu n - 1)(k_2^{-\mu n - 1} - k_2^{\mu n - 1})\} \cos \mu n\theta_j \sin \mu n\theta_k \cos \mu n\theta \\ + \sum_{n=1}^{\infty} \frac{8 r_1}{n \pi \Delta_n} \{[2(\mu n + 1 + \nu) k_2^{2\mu n - 2} + (\mu n)^2 (3 - \nu) - 2\{(\mu n)^2 - 1\}(1 - \nu) k_2^{-2} - (\mu n)^2 (1 + \nu) k_2^{-4} \\ - (2\mu n + 1 - \nu) k_2^{-2\mu n - 2}] \cos \mu n\theta_j \sin \mu n\theta_k \cos \mu n\theta + A \cos \theta \} \dots\dots\dots (18) \end{aligned}$$

The radial deformations of the ribs end loaded at θ_j section of the sector are equal to those of the circular disk at the same points and given by,

$$Eu|_{r=r_1} = - \sum_{j=1}^m K(k_1, \theta_j, \varepsilon_j) R_0 p_j \cos 2n\theta \dots\dots\dots (19)$$

The radial deformations of the disk loaded at θ_j section of ribs are equal to those of the circular disk at point loaded at θ_k section. Adding the radial deformation of itself for $k=j$, we have

$$Eu|_{r=r_1} = -K(k_1, \theta_j, \varepsilon_j) R_0 k_2 \cos 2n\theta_k + k_2 (r_1 - r_0) \ln k_2 \delta_{jk} / (k_2 - 1) \dots\dots\dots (20)$$

2.6 Deformation analysis of cylindrical shell

Now, we discuss the asymmetric deformation of the cylindrical shell using Flügge's equations. Defining u , v , w as the deformations in x , ϕ , and z directions, the stress and moment resultants are given as follows.

$$\left. \begin{aligned} N_\phi &= \frac{D}{a} (v' + w + \nu u') + \frac{K}{a^2} (w + w''), & N_z &= \frac{D}{a} (u' + \nu v' + \nu w) - \frac{K}{a^2} w'' \\ N_{\phi z} &= \frac{D}{a} \frac{1-\nu}{2} (u' + v') + \frac{K}{a^2} \frac{1-\nu}{2} (u' + w''), & N_{z\phi} &= \frac{D}{a} \frac{1-\nu}{2} (u' + v') + \frac{K}{a^2} \frac{1-\nu}{2} (v' - w'') \\ M_\phi &= \frac{K}{a^2} (w + w'' + \nu w''), & M_z &= \frac{K}{a^2} (w'' + \nu w' - u' - \nu v') \\ M_{\phi z} &= \frac{K}{a^2} (1-\nu) \left(w'' + \frac{1}{2} u' - \frac{1}{2} v' \right), & M_{z\phi} &= \frac{K}{a^2} (1-\nu) (w'' - v') \end{aligned} \right\} \dots\dots\dots (21)$$

From above equations and equilibrium equations for stress and moment resultants, the well known linear differential equations for u , v , w are obtained. Solving these equations we express u , v , w as follows.

$$\left. \begin{aligned} u &= \sum_{n=0}^{\infty} \sum_{i=1}^8 A_i \exp [(\lambda_i x - \eta_i R_0(\lambda_i) l) / a] \cos n\phi, & v &= \sum_{n=0}^{\infty} \sum_{i=1}^8 B_i \exp [(\lambda_i x - \eta_i R_0(\lambda_i) l) / a] \sin n\phi \\ w &= \sum_{n=0}^{\infty} \sum_{i=1}^8 C_i \exp [(\lambda_i x - \eta_i R_0(\lambda_i) l) / a] \cos n\phi \end{aligned} \right\} \dots\dots\dots (22)$$

Where λ_i is given by Eq. (23) and A_i , B_i , C_i are given by Eq. (24).

$$\lambda^3 - 2(2n^2 - \nu)\lambda^2 + \left\{ -\frac{12(1-\nu^2)a^2}{h^2} + 6n^2(n^2 - 1) \right\} \lambda^4 - 2n^2 \{ 2n^4 - (4-\nu)n^2 + (2-\nu) \} \lambda^2 + n^4(n^2 - 1)^2 = 0 \quad (23)$$

$$\left[\begin{array}{cc} \lambda_i^2 - \frac{1-\nu}{2} n^2 \left(1 + \frac{h^2}{12a^2} \right), & \frac{1+\nu}{2} \lambda_i n \\ \frac{1+\nu}{2} \lambda_i n, & -\frac{1-\nu}{2} \lambda_i^2 + n^2 - \frac{h^2}{8a^2} (1-\nu) \lambda_i^2 \end{array} \right] \begin{pmatrix} \alpha_i \\ \beta_i \end{pmatrix} = \begin{pmatrix} -\nu \lambda_i + \frac{h^2}{12a^2} \left(\lambda_i^3 + \frac{1-\nu}{2} \lambda_i n^2 \right) \\ -n + \frac{(3-\nu)h^2}{24a^2} \lambda_i^2 n \end{pmatrix} \quad (24)$$

where $A_i = \alpha_i C_i$, $B_i = \beta_i C_i$.

The boundary conditions for both ends are,

$$\left. \begin{array}{l} x=0 \quad S_x = Q_x + \frac{M_{x\phi}}{a} = \sum_{n=0}^{\infty} A_n \cos n\phi, \quad T_x = N_{x\phi} - \frac{M_{x\phi}}{a} = 0, \quad N_x = 0, \quad M_x = 0 \\ x=l \quad S_x = 0, \quad T_x = 0, \quad N_x = 0, \quad M_x = 0 \end{array} \right\} \quad (25)$$

Solving Eq. (23), N_n , M_n , S_n , T_n for fixed n , i are given by

$$\left. \begin{array}{l} N_{ni} = \left\{ \frac{D}{a} (\lambda_i \alpha_i + \nu \beta_i n + \nu) - \frac{K}{a^3} \lambda_i^3 \right\} \exp [(\lambda_i x - \eta_i R_i(\lambda_i) l) / a] C_i \cos n\phi \\ M_{ni} = \frac{K}{a^3} (\lambda_i^2 - \nu n^2 - \lambda_i \alpha_i - \nu \beta_i n) \exp [(\lambda_i x - \eta_i R_i(\lambda_i) l) / a] C_i \cos n\phi \\ T_{ni} = \left\{ \frac{D}{a} \frac{1-\nu}{2} (-\alpha_i n + \lambda_i \beta_i) + \frac{K}{a^3} \frac{3}{2} (1-\nu) (\lambda_i \beta_i + \lambda_i n) \right\} \exp [(\lambda_i x - \eta_i R_i(\lambda_i) l) / a] C_i \sin n\phi \\ S_{ni} = \frac{K}{a^3} \left\{ \lambda_i^3 - (2-\nu) \lambda_i \eta_i^2 - \lambda_i^2 \alpha_i - \frac{1-\nu}{2} \alpha_i n^2 - \frac{3-\nu}{2} \lambda_i \beta_i n \right\} \exp [(\lambda_i x - \eta_i R_i(\lambda_i) l) / a] C_i \cos n\phi \end{array} \right\} \quad (26)$$

Solving for complex unknown quantities C_i from Eq. (25), the influence factors for the cylindrical shell are calculated by Eqs. (22).

2.7 Stress analysis of the cylindrical shell

Using the influence factors of the asymmetric disk and the cylindrical shell, the pressure distributions are solved by Eq. (1). Expanding these pressure distributions as Fourier series, the radial deformations, and stress and moment resultants are calculated by the same methods discussed above.

3. Numerical analysis

For a numerical example, we deal with a retaining ring ($l/a = 1.35$, $h/a = 0.146$) shrunk onto an asymmetric disk ($k_1 = 9.05$, $k_2 = 1.49$, $\mu = 6.57$, $q = 8$). Fig. 5 shows the nondimensional radial deformations of the cylindrical shell divided by the shrinkage. To verify the analytical method, two models are made (these configurations are shown in Fig. 6), and the radial deformations of the cylindrical shell are measured by use of the outer-micrometer. The experimental data are shown in Fig. 5 as solid circles. The experimental and calculated radial deformations agree with each other and the cylindrical shell deforms as an elliptical form.

The contact pressure and its Fourier series approximation with 100 terms are shown in Fig. 7. The pressure of shrink fit attains the maximum value at the end of the sectors.

The axisymmetric stress and moment resultants N_ϕ, M_ϕ for the zero-th order of the Fourier series are shown in Fig. 8. N_ϕ takes the maximum value at the end of shrink fit and then decreases rapidly in the x -direction. M_ϕ equals zero at both ends of the cylindrical shell because of the boundary conditions and it takes the maximum value at $x/a = 0.24$.

The asymmetric stress and moment resultants are as follows.

The distribution of N_n in Fig. 9 shows that the asymmetric membrane stress σ_n (max. 0.1 kg/mm^2) is negligible compared with the axisymmetric bending stress (max. $\sigma_n = \pm 30 \text{ kg/mm}^2$). The distribution of N_ϕ in Fig. 10 shows that the asymmetric membrane stress σ_ϕ

is same as σ_n unlike the axisymmetric one (max. 51 kg/mm^2). The value of $N_{n\phi}$ in Fig. 11 is negligible (max. 0.2 kg/mm^2) compared with the asymmetric bending stress $\tau_{n\phi}$ (max. 7.5 kg/mm^2) mentioned later. N_ϕ and $N_{n\phi}$ oscillate in ϕ -direction at the shrink fit end $x=0$, but decrease uniformly.

The distribution of M_n in Fig. 12 shows that the value of the bending stress σ_n (max. $\pm 0.5 \text{ kg/mm}^2$) is small compared with the axisymmetric one. The distribution of M_ϕ in Fig. 13 shows that the maximum bending stress σ_ϕ is $\pm 11.3 \text{ kg/mm}^2$ and this value must be added to the axisymmetric one. The distribution of $M_{n\phi}$ in Fig. 14 shows that the maximum shearing stress $\tau_{n\phi}$ is $\pm 7.5 \text{ kg/mm}^2$. From these results, the dominant stresses of the cylindrical shell are found to be axisymmetric membrane stress and bending stress by N_ϕ, M_ϕ and the asymmetric bending stresses by $M_\phi, M_{n\phi}$. The maximum hoop stress at the outer surface of the cylindrical shell is: the axisymmetric membrane stress $\sigma_\phi = 51 \text{ kg/mm}^2$ + asymmetric bending stress $\sigma_\phi = 11.3 \text{ kg/mm}^2$ its value being 1.22 times the membrane stress.

4. Conclusions

In this paper, we deal with the asymmetric problem of a cylindrical shell shrunk onto an asymmetric disk at its one end. Two-dimensional elasticity theory, classical shell theory and the point matching approximation method are used for this problem. In this problem, the shell is shrunk onto the asymmetric disk where sectors and ribs are pressed onto the circular disk, and then there are two contact surfaces. This approximation method has not yet been applied for this kind of problem. The obtained results are summarized as follows.

- (1) The stress function developed by Matsumoto for sectors and Flugge's equations for the cylindrical shell are used, and the calculated asymmetric radial deformations are in good agreement with the experimental data.
- (2) The stress and moment resultants for the cylindrical shell show that the dominant stresses are axisymmetric membrane stress and bending stress by N_ϕ, M_ϕ .

- M_s and asymmetric bending stresses by M_ϕ , $M_{s\phi}$.
- (3) The maximum hoop stress of the cylindrical shell is the axisymmetric hoop stress $\sigma_\phi = 51 \text{ kg/mm}^2$ plus the bending stress $\sigma_\phi = 11.3 \text{ kg/mm}^2$.

5. References

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Fig.5 Calculated and experimental radial deformations of the cylindrical shell

Fig.6 Configuration of the experimental model

Fig.10 Asymmetric stress resultant N_ϕ

Fig.12 Asymmetric moment resultant M_x

Fig.13 Asymmetric moment resultant M_ϕ

Fig.14 Asymmetric moment resultant $M_{x\phi}$

Fig.7 Fourier series expansion of the shrink fit pressure

Fig.8 Axisymmetric stress resultant N_ϕ and moment resultant M_x

Fig.9 Asymmetric stress resultant N_x

Fig.11 Asymmetric stress resultant $N_{x\phi}$