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Velocity Field of the Flow Around a Circular Cylinder

Started Impulsively*

(2nd Report, Unsteady Boundary Layers)

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A procedure for obtaining true velocities which were estimated from the locations of time-lines visualized by hydrogen bubble technique was developed by taking into account the velocity defect behind a bubble generating wire.

By using this technique, the velocity profiles in unsteady boundary layer on the circular cylinder which was started impulsively in still water were obtained at the Reynolds number of 1200 based on the cylinder diameter. Changes in the displacement thicknesses and shape factors were determined in course of time and were discussed in comparison with the results by the existing theories. Velocity profiles of the boundary layer before and after the occurrence of a reversed flow were also measured. These velocity profiles were in fairly good agreement with Pohlhausen's profile, even at the location where a reversed flow appears. And they seem to change gradually before and after the occurrence of a reversed flow, unlike the velocity profiles at separation points in ordinary steady flows.

1. Introduction

When a circular cylinder starts impulsively with a constant speed in still water, we can observe a certain transient condition of flow in the boundary layer around the cylinder. The boundary layer grows with time elapsed and separates at a certain instant from the surface of the cylinder, causing separation vortices. The detailed mechanism of this type of unsteady flows has been a fundamental subject for the researchers in the fields of both pure science and engineering.

In the previous paper⁽¹⁾, the variation of the outer potential flow field was made clear, relating to the growth of the separation vortices. However, the measurement of the velocity field in the boundary layer, which will make an important contribution to the study of the separation phenomena, has been neglected. In order to clarify the details of the boundary layer, it is necessary to measure the unsteady velocity in an extremely thin layer near the surface of the cylinder. But, there have been few suitable methods for this purpose, and any reliable experimental studies about the unsteady boundary layer have not been carried out. Many theoretic-

cal approaches to this problem have been tried, but there are few studies about the behaviour of the unsteady boundary layer from the initial state to the fully developed one. Only Schuh's solution⁽²⁾ of the momentum integral equation was published as an analytical one. This solution is available only within a short period after the start of a cylinder, because it is based on the assumption that the velocity U on the outer edge of the boundary layer remains unaltered in the process of growth of the vortex region behind the cylinder. Recently, Collins and Dennis⁽³⁾ solved numerically the Navier-Stokes equation and clarified the process of the growth of the vortex region. A detailed description about the boundary layer was not included in their papers.

1. Driving drum 2. Test cylinder
3. Camera 4. Carriage
5. Guide rails 6. Wire
7. Window 8. Water tank
9. Tension pulley
AA : Illuminated layer

Fig.1 Experimental apparatus

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The present paper aims to make clear the development of boundary layers with elapsed time by means of hydrogen bubble technique. To determine accurately the local velocity in the boundary layer by the hydrogen bubble technique, the measurement should be performed at a location near the wire, and the effect of the velocity defect in the wake of the wire should be considered.

A procedure for correction of the velocity defect was established firstly and changes of velocity profiles in the boundary layer in course of elapsed time t were measured by this procedure. The displacement thickness δ_1 , the momentum thickness δ_2 , and the shape factor $H = \delta_1/\delta_2$ of the boundary layer were calculated from the measured velocity profile. The velocity profiles after the occurrence of a reversed flow were also measured and discussed in comparison with Pohlhausen's profile⁽⁴⁾.

The experimental conditions were $Re = U_0 d/\nu = 1200$, $\theta = 36^\circ \sim 130^\circ$, and $\tau = U_0 t/d = 0 \sim 10$, where d is the diameter of the cylinder, U_0 the speed of the cylinder, ν the kinematic viscosity, and θ the azimuthal angle measured from the front stagnation point of the cylinder.

2. Experimental apparatus and methods

2.1 Apparatus

The experiments were carried out in a towing water tank ($70^D \times 70^W \times 270^L$ cm) which was the same one employed in the previous study⁽⁵⁾. The general arrangement is shown in Fig.1. A circular cylinder (70 mm in diameter, 600 mm in length) attached to the carriage was set vertically in the tank and towed in the direction parallel to the side walls. The flow around the cylinder was visualized in the horizontal plane situating 30 mm beneath the free surface of water. The cylinder was started impulsively with a constant speed U_0 .

2.2 Use of the hydrogen bubble technique

The hydrogen bubble technique as de-

Fig.2 Flow near the surface of the circular cylinder

scribed in detail in the previous papers^(6,7) was employed to measure a very low-speed water flow near the surface of the cylinder (the speed vanished at the surface, the maximum speed being at most 20 mm/s on the outer edge of the boundary layer). A fine tungsten wire $5 \sim 10 \mu$ in diameter was used to generate sufficiently fine bubbles to trace the low-speed flow accurately. The supports of the generating wire were set in the same way as in the previous paper⁽⁸⁾ so as not to disturb the flow near the surface of the cylinder.

Figure 2 shows a flow near the surface of the cylinder, visualized by four groups of time lines. A time line consists of a row of bubbles which were generated from the wire pulsatively and swept away by a flow. The single row of bubbles was seen as a group of time lines, as shown in the picture, because a row of bubbles was illuminated by a mercury vapour lamp flashing at the rate of 120 c/s as seen in the picture. The time lines could be clearly observed close to the surface of the cylinder.

2.3 Measurement of the flow velocity in the boundary layer

When the distance x between the generating wire and the location of the time line in the flow direction was measured, the non-dimensional velocity u/U (U : the velocity on the outer edge of the boundary layer, u : the velocity in the boundary layer) can be obtained approximately from the relation $u/U = x/x_{\max}$. Strictly speaking, the effect of the velocity defect due to

Fig.3 Time-lines in a uniform flow

Fig.4 Relation between the locations of time-lines and elapsed time

the existence of the wire should be taken into account, because the bubbles move in the wake of the wire. In this experiment, the measurements were performed at the location of $200 \sim 300 d_w$ downstream of the wire ($2 \sim 3 \text{ mm}$ in real scale). In such a close region to the wire, u/U cannot be substituted by $x/x_{\max}$, since the velocity defect in the wake of the wire reaches a considerable amount. For this reason, a method of obtaining the true velocity was developed in this paper.

3. Measurement of flow velocity

3.1 Velocity defect in wake of generating wire

Figure 3 shows an example of the uniform flow patterns with a velocity V_T . The flow was visualized by rows of bubbles generated pulsatively and the photograph was taken in a short time exposure. The relation between x_i , the location of the i th time line (illustrated in Fig. 3) and t , the elapsed time is shown in Figure 4, where T_p is the pulse interval, T_D the delay time in photographing, x_i the distance between the wire and the i th time line, and t the time expressed by $t = T_D + T_p(i-1)$. If the velocity at the location of x_i behind the wire is denoted by the symbol V_i , and $\bar{V}_i$ is defined as $\bar{V}_i = x_i/t_i$ or $x_i = \bar{V}_i t_i$, then V_i is given by

$$V_i = \frac{dx_i}{dt} = \bar{V}_i + \frac{d}{dt}(\bar{V}_i t_i) \quad (1)$$

The value of t can be decided by measuring the values of T_D and T_p , and $\bar{V}_i$ is calculated from the values of t_i and x_i on the picture. The velocity in the wake of the wire, or the value of V_i , can be estimated with aid of Eq. (1).

By this procedure, the relation between V_i/V_T and $x^* = x/d_w$ (d_w : diameter of the generating wire) was obtained as shown in Fig. 5. As seen in this figure, the non-dimensional velocity defect $V^* = 1 - (V_i/V_T)$ can be expressed approximately by the following empirical equation, which is given in Davis' paper⁽⁹⁾.

$$V^* \equiv 1 - (V_i/V_T) = C(x^*)^{-n} \quad (2)$$

This empirical equation can be applicable only when the values of x^* are sufficiently large, since this equation is obtained

Fig. 5 Velocity defect in the wake of a generating wire

by modifying Tollmien's solution⁽¹⁰⁾ for the velocity defect in the far wake region. The exact value of the lowest limit applicable for x^* is unknown, but it may be assumed that Eq. (2) is available for the range of $x^* > 25$ in Fig. 5.

For determination of flow velocity, it is desirable to know the true value of the velocity V_T free from the effect in the wake of the wire. A procedure to determine the value of V_T directly from the measured values at the station of x_i is developed in the following section.

3.2 Correction of velocity defects

If it is assumed that the ratio of the velocity V_i in the wake of the wire to the true velocity V_T in the upstream of the wire, V_i/V_T is expressed as a function of only x^* in the case of $x^* < 25$ as in the case of $x^* > 25$, then from Eq. (2), the following relations can be obtained

$$V_i/V_T = f(x^*) \quad (3)$$

$$f(x^*) \equiv f_1(x^*) \quad \text{for } x^* \leq 25 \quad (3-a)$$

$$f(x^*) = 1 - C(x^*)^{-n} \equiv f_2(x^*) \quad \text{for } x^* \geq 25 \quad (3-b)$$

By use of the relation $V_i = dx_i/dt$, V_T is obtained from Eq. (3) as

$$V_T t/d_w = \int_0^{x^*} [f(x^*)]^{-1} dx^* = \int_0^{25} [f_1(x^*)]^{-1} dx^* + \int_{25}^{x^*} [f_2(x^*)]^{-1} dx^* \quad (4)$$

The second term in the right hand of Eq. (4) can be integrated by use of Eq. (3-b), and if the indefinite integral is expressed by $F(x^*)$, the following expression is obtained,

$$V_T t/d_w = \alpha + F(x^*) - F(25) \quad (5)$$

In the above equation, α denotes a constant given by the definite integral of the first term in the right hand of Eq. (4). An explicit form of the function $f_1(x^*)$ is not known and the value of α must be determined to satisfy Eq. (5) by

Fig. 6 Effect of velocity defect in the wake of the bubble generating wire on the determination of true velocity V_T

use of the relation between x^* and t in the uniform flow. Equation(5) showing the relation between x^* and V_T gives the desired velocity V_T directly from the measured value of x^* , without calculating the velocity V_i , in the wake of the wire.

3.3 Velocity measurement in the boundary layer

In order to measure the velocity in the boundary layer, a single generating wire was set radially from the surface of the cylinder and the time lines were visualized by rows of bubbles generated pulsatively from the wire.

The existence of the generating wire

scarcely influences the whole pattern of a flow around the cylinder, because the diameter of the wire is selected sufficiently small. The influence on the flow is limited only within a very small region around the wire. If it is assumed that the influence extends to a region extending 1000 times the diameter of the wire, the region will still be included within a circle having a radius of 10 mm. The boundary layer to be measured is not steady, but it can be regarded as steady in the short duration in which the fluid particles pass through the small region near the wire.

As shown in the previous paper⁽¹⁾, the

Fig.7 Flow near the surface of the circular cylinder ($Re = 1200$)

defect of velocities in the wake of the wire does not change substantially in the range of $V_T = 10 \sim 20$ mm/s, and therefore it can be assumed that the empirical constants in Eqs. (2) and (5), (C , n , and α) are independent of the flow velocity V_T .

Assuming that the velocity defect in the wake of the generating wire in the unsteady boundary layer is the same as in a steady uniform flow with a velocity of about 20 mm/s, the value of the non-dimensional velocity u/U can be estimated by substituting the measured values of x^* and t into Eq. (5). The empirical constants C , n , and α also depend on the manner of bubble generation (e.g. the diameter, the number, etc.). And the constants could be determined in a uniform flow where bubbles were generated from the wire in the same manner as in the case of measuring the velocity in the boundary layer in each test. Figure 6 shows several relations between V_T and x^* obtained by use of these empirical constants C , n , and α . The curve with the constants $C=2.0$, $n=0.52$, and $\alpha=40$, corresponds to the flow in Fig. 5. The other two curves were obtained respectively under different experimental conditions. There exists an approximate relation of $u/U = x/x_{\max}$, making a straight line. The broken line in this figure shows an example of the relation at $x_{\max}=300$ obtained with the constants of $C=0.52$, $n=2.0$, and $\alpha=40$.

The velocity profiles in the boundary layer could be obtained by correcting the effect of the velocity defect for the values of $50 < x^* < 350$. The results will be described in the following section.

4. Experimental results and discussion

4.1 Velocity profile in the boundary layer

Figure 7 shows an example of the flow near the cylinder visualized by the hydrogen bubble technique. The developments of boundary layer as time elapses are seen in these pictures. As shown in Photo I, the boundary layer at the location of $\theta = 36^\circ$ developed immediately and soon attained the value as in a steady state, and any remarkable change in profile is observed in course of time as seen in the pictures. But at the location $\theta = 92^\circ$, the boundary layer grows remarkably with time and the growth continues up to the time $\tau = 10$, as seen in Photo II (a)~(d). At the section $\theta = 106^\circ$, the development of boundary layer with time becomes more quick as is seen in Photo III and a reversed flow appears near the surface of the cylinder at the instant $\tau = 2.2$ (cf. Photo III (d)).

The velocity profiles in the boundary layer are shown in Fig. 8, in which the velocity profiles with and without corrections of the velocity defect in the wake of the wire are compared. The effect of the non-dimensional time $\tau = U_0 t / d$ is shown as a parameter. The open and closed cir-

cles in the figure show respectively the values with and without corrections of the velocity defect. The values of u/U with correction are larger than those without correction by 5~7% at the higher value, showing a thicker velocity profile. The solid line in this figure represents the solution of the boundary layer equation for a steady flow around the cylinder by use of Blasius series⁽¹¹⁾. The velocity profile obtained experimentally almost agrees with this solution at the instant $\tau = 1.0$, after which ($\tau > 1.0$), the thickness of the boundary layer increases beyond the values obtained by Blasius series, and the shearing stress τ_w at the surface of the cylinder decreases with the elapsed time. From the measured profile at the instant $\tau = 10$, it may be inferred that the velocity profile in the steady state is remarkably different from that in the unsteady state as is indicated by the solid line in the figure.

In order to make clear the details of the development of the boundary layer, the velocity profile was measured at several azimuthal angles θ . The displacement thickness δ_1 , the momentum thickness δ_2 , and the shape factor H (the ratio δ_1/δ_2) of the boundary layer were determined by integrating numerically these velocity profiles. The results will be described in the following sections.

4.2 Displacement thickness δ_1 and shape factor H

If the displacement thickness δ_1 and the shape factor H of the boundary layer are given, it may be said that the character of the boundary layer is decided quantitatively⁽¹²⁾. The displacement thickness shows the amount of the distance over which the stream lines of the outer potential flow are displaced away from the surface owing to the development of the

Fig. 8 Velocity profiles in the boundary layer

boundary layer, and the shape factor is a significant factor to decide the shape of the velocity profile.

The relation between non-dimensional displacement thickness $(\delta_1/d)\sqrt{Re}$ and non-dimensional time τ at the location of $\theta=92^\circ$ is shown in Fig.9 in which the relation between shape factor H and τ is also plotted. The displacement thickness δ_1 and the shape factor H increase with the elapsed time, and even at the time $\tau=10$, they are still in an increasing stage.

In the figure, the mark " ∇ " denotes Schuh's solution⁽²⁾ for the displacement thickness δ_1 . This is a solution of the integral equation for the unsteady boundary layer, and it does not take into account the change of the velocity at the outer edge of the boundary layer. The velocity at the outer edge of the boundary layer is assumed to be $U=2U_0\sin\theta$ which will be realized only at a short instant after the start of motion of the cylinder.

As seen in Fig.9, the experimental values of $(\delta_1/d)\sqrt{Re}$ agree with Schuh's solution only in a short period of $\tau < 0.4$ after the start of motion. When the time is elapsed and $\tau > 0.4$, the experimental values deviate considerably from Schuh's solution. When $\tau=1.0$, the experimental values agree with those of Schuh's results solved for $\tau \rightarrow \infty$ (as shown by the broken line in the figure), and when $\tau > 1.0$, they have much higher values.

The solution by Schuh is obtained by assuming that the velocity at the outer edge of the boundary layer is $U=2U_0\sin\theta$, and hence, the value of $dU/d\theta$ reduces nearly to zero at the location of $\theta=90^\circ$. For this reason the value of $\Lambda=(2\delta^2/dv)dU/d\theta$ reduces nearly to zero, giving a constant shape factor H for all the elapsed time τ , since H is related uniquely in terms of Λ . This value of H is indicated by a broken line in the figure. The measured values of H are in agreement with Schuh's results when $\tau < 0.8 \sim 1.0$, and in the time when $\tau > 1.0$, the measured values exceed the values of Schuh's solution. A similar tendency is seen in the experimental values of δ_1 .

These results coincide well with the

Fig.9 Variation of displacement thickness δ_1 and shape factor H with elapsed time

change of the velocity distribution at the outer edge of the boundary layer as was described in the previous paper⁽¹⁾. When $\tau < 0.5 \sim 0.8$, the measured velocity can be expressed by $U=2U_0\sin\theta$ the relation of which was deduced from the potential flow theory. When $\tau > 1.0$, the measured velocity distribution is far from that expressed by the equation $U=2U_0\sin\theta$. These facts confirm the accuracy of the velocity measurements in the boundary layer.

The velocity measurements above were performed at the location of several azimuthal angles θ . Figure 10 shows the relation between the non-dimensional displacement thickness $(\delta_1/d)\sqrt{Re}$ and the azimuthal angle θ with elapsed time τ as a parameter. In the period of $\tau < 0.4$, immediately after the start of the cylinder, the measured values of $(\delta_1/d)\sqrt{Re}$ are in fairly good agreement with Schuh's solution as illustrated by solid lines. But in the period of $\tau > 1.0$, the agreement becomes poor. The measured values of $(\delta_1/d)\sqrt{Re}$ at any azimuthal angle θ are larger than those of Schuh's solution for $\tau \rightarrow \infty$, and the deviation increases with the angular position θ . In the region in which θ exceeds 80° the increasing rate of δ_1 against θ is remarkable. A state near the fully developed boundary layer flow seems to appear when $\tau > 10$, as is conjectured in Fig.9 or Fig.10.

Fig.10 Development of displacement thickness

Fig.11 Variation of shape factor, H

In Fig.11, the shape factor H is plotted against azimuthal angle θ with τ as a parameter. The solid line in the figure shows Schuh's solution. The measured values of H show a similar tendency to the curves of δ_1 .

4.3 Change of boundary layer due to reversed flow

In order to study the details of the boundary layer before and after the occurrence of a reversed flow, the velocity profile was measured at the locations of the azimuthal angles $\theta=106^\circ$ and 130° , where a reversed flow would be expected near the surface of the cylinder in course of time.

4.3.1 Shape factor H

The relation between the shape factor H and elapsed time τ is shown in Fig.12, in which the coefficient of shearing stress assumed by the method described latter is also plotted. The arrows in the figure indicate the instant of the time $\tau_{(s=0)}$ when a reversed flow appears initially in the boundary layer. The values of H measured at the location of $\theta = 106^\circ$ and 130° at the beginning of reversed flow $\tau_{(s=0)}$ are nearly equal to 3.7, and any abrupt change in H is not observed at this instant. This means that the character of the boundary layer is not changed substantially by the occurrence of a reversed flow*.

In the past no experimental results relating to the change of H at the time of a reversed flow generation were published. For reference, therefore, the theoretical values of H which were obtained for the separation point in a steady flow will be used.

* In an ordinary steady flow, the character of the boundary layer around a circular cylinder is quite different between the upstream and downstream sections of the separation points.

Fig.12 Variation of shape factor, H and coefficient of shearing stress, C_f with time at the location where a reversed flow appears

Many approximate solutions for the boundary layer have been proposed by several authors but the values of H at the separation point are not constant but scattered within the range of $3.50 \sim 4.04^{(13)}$. For a steady flow, Tani obtained exact solutions for five cases in which the velocities at the outer edge of the boundary layer were assumed to be systematical in different ways, and showed that each value of H at the separation points in the assumed flow ranges from 3.53 to 3.83⁽¹⁴⁾. The value of $H=3.7$ at the separation beginning obtained in this unsteady flow experiment is substantially the same as the theoretical one of H at the separation points in many existing solutions for steady boundary layers.

The effects of the velocity defects in the wake of the bubble generating wire on the shape factor H are shown in Fig.13, where the corrected values of H for the wake effect are plotted in the ordinate and the uncorrected ones in the abscissa. The same kind of plot for the values of $(\delta_1/d)\sqrt{Re}$ is shown in Fig.14. The suffix "m" in each figure is used to denote the values without correction. The values of $(\delta_1/d)\sqrt{Re}$, and of H with correction are smaller than those without correction by $4 \sim 5\%$ as is seen in these figures. When the respective values of $(\delta_1/d)\sqrt{Re}$ and H become large, the amounts to be corrected also become large. The details of the corrections which should be made for large values of H and $(\delta_1/d)\sqrt{Re}$ should be discussed more closely. In such a case, the above assumption of the velocity defect in the boundary layer is not valid, because the velocity profile becomes "thinner" in shape and the time lines to be measured will concentrate near the wire.

The value of H at the time of the occurrence of a reversed flow in this experiment is decided to be about 3.7, and this corresponds to the value obtained by Thwaites⁽¹³⁾ for the shape factor at the separation point in a steady flow. This numerical value ($H=3.7$), however, may in-

Fig.13 Displacement thickness with and without corrections of the velocity defect in the wake of the generating wire

clude some errors for the above reason.

4.3.2 Velocity profile and coefficients of shearing stress

The measured velocity profiles in the boundary layer were compared with Pohlhausen's velocity profile. In Pohlhausen's one, it is assumed that u/U is a polynomial of the fourth degree in terms of y/δ with $\Lambda = (2\delta^2/dv)du/d\theta$ as a parameter (δ : boundary layer thickness). This profile has often been used as an approximate velocity profile of the boundary layer in either steady or unsteady flow. By integrating Pohlhausen's profile, the relation between the shape factor H and Λ can be estimated as

$$H = \left(\frac{3}{10} - \frac{\Lambda}{120} \right) / \left(\frac{37}{315} - \frac{\Lambda}{945} - \frac{\Lambda^2}{9072} \right) \dots \dots \dots (6)$$

When the value of Λ is calculated by substituting the measured value of H into

Fig.14 Shape factor, H with and without correction of the velocity defect in the wake of the generating wire

Fig.15 Variation of velocity profiles in the boundary layer with elapsed time at the location where a reversed flow appears

Eq.(6), Pohlhausen's profile corresponding to the measured velocity profile can be given. In Fig.15, Pohlhausen's profile obtained in this way is shown by solid lines in comparison with the measured velocities. The measured velocities are plotted by the symbols of circles. Within the range where the reversed flow in the boundary layer could not be seen, namely, the value of H is smaller than about $3.5 \sim 3.6$ ($\Lambda = -12.0 \sim -12.5$), the measured values agree well with Pohlhausen's profiles. Pohlhausen's approximate solution has been said to be inaccurate near the separation point⁽¹⁵⁾. But it was confirmed experimentally that the measured velocity profiles can be closely approximated by Pohlhausen's profile in the case of $H < 3.5 \sim 3.6$, as shown in the figure. In the case of $H = 4.35$, when a reversed flow can be seen clearly near the surface of the cylinder, the velocity profile becomes "thinner" in shape. In such a case, especially in the region of the reversed flow, the motion of bubbles is not exactly the flow motion, since the velocity is so small that the effect of the buoyancy on the flow cannot be neglected and the reversed flow velocity cannot be measured precisely. However, the location where the flow direction changes can be measured with a considerable accuracy. The broken line in Fig.15 shows the velocity profile inferred from the data obtained near the surface of the cylinder. The upper part of this profile coincides well with Pohlhausen's profile, and even in the reversed flow region, the difference between these two profiles is considerably small. Consequently, it may be concluded that Pohlhausen's profile is in fairly good agreement with the real velocity profile in the boundary layer developed around the cylinder started impulsively.

The coefficient of the shearing stress C_f was estimated by use of Pohlhausen's profile with the experimental values of H . The shearing stress $C_f = \tau_w / \rho U_0^2$ (τ_w : shearing stress on the surface of the cylinder) in Pohlhausen's profile is expressed by,

$$c_f \sqrt{Re} = \left(2 + \frac{\Lambda}{6} \right) \left(\frac{37}{315} - \frac{\Lambda}{945} - \frac{\Lambda^2}{9072} \right) \times \left(\frac{U}{U_0} \right) H / \left((\delta_1/d) \sqrt{Re} \right) \dots \dots \dots (7)$$

If the values of H are determined by u/U , the values of Λ can be calculated from Eq.(6). Thus, the shearing stress coefficient C_f can be determined by substituting the measured values of H , U/U_0 and $(\delta_1/d) \sqrt{Re}$ into Eq.(7). The relation between the shearing stress coefficient $C_f \sqrt{Re}$ and the non-dimensional time τ at the locations of $\theta = 106^\circ$ and 130° is shown also in Fig.12. The measured values of $C_f \sqrt{Re}$ indicated by solid lines are in fairly good agreement with Collins-Dennis' numerical solution⁽³⁾, indicated by the marks "V" in the figure. This fact shows that the shearing stress can be reasonably estimated from the measured values of H by the above method.

5. Conclusions

The results of the present experiment are summarized as follows.

(1) A procedure to estimate true velocities from the location of time lines visualized by hydrogen bubbles was developed by taking into account the velocity defect behind a bubble generating wire. This procedure is available to measure the velocity profile in the unsteady boundary layer including a reversed flow around a cylinder.

(2) When the non-dimensional time $\tau = U_0 t/d$ is smaller than $0.2 \sim 0.4$, the behaviour of a developing boundary layer is in fairly good agreement with that assumed from Schuh's solution, and when $\tau=1$, it is approximated as the solution for $\tau \rightarrow \infty$. When $\tau > 1$, the thickness of the boundary layer at the location of $\theta > 80^\circ$ increases more as time elapses. This increase continues over a time interval longer than $\tau=10$.

(3) The value of H increases after the beginning of the motion and a reversed flow appears when the measured value of H attains about 3.7, but any abrupt change in H does not appear at this instant nor thereafter. The measured value of H is approximately the same as the value obtained by theoretical studies at the separation point in a steady flow.

(4) Pohlhausen's velocity profile in which the measured values of H are used agrees well with the measured velocity profile within the range of $H < 3.5$ ($\Lambda > -12$). The agreement is maintained also in the flow including a reversed flow in the boundary layer, except in the region extremely near the surface of the cylinder.

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