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Stability and Self-sustained Oscillations in Nozzle-Flapper Valve with Pipe Line*

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This paper discusses the stability of a system consisting of a nozzle-flapper valve and a pipe line and several features of the self-sustained oscillations occurring in the system. Conclusions are as follows: (1) It is shown that the multiple modes of oscillations may simultaneously become unstable in a system with a long pipe line. (2) It is made clear that the most effective nonlinear characteristic for limiting the amplitude of the unstable oscillation is the collision of the flapper with the nozzle tip. (3) A basic equation of the system including the equivalent nonlinear terms for the collision is proposed and approximately analysed to derive the frequency and the amplitude of the self-sustained oscillations. (4) The selective occurrence of a self-sustained oscillation is also investigated for a system having two unstable modes, and it is concluded that either of the two unstable modes selectively develops to a sustained oscillation.

1. Introduction

A nozzle-flapper valve, which has been extensively used as an amplifier or a sensing element in hydraulic and pneumatic systems, often develops undesirable instability due to the interaction with capacitances, restrictors or pipe line terminating at the valve. A lot of researches^{(1),(2),(3)} have been devoted to the stability of a system consisting of a nozzle-flapper valve and a pipe line, but few investigations have dealt with what modes of oscillation are excited in an unstable system.

Since the working fluid in a long pipe line should be analysed as a distributed-parameter element, the system including the pipe line has infinite degrees-of-freedom. When such a system with infinite degrees-of-freedom becomes unstable, various modes of oscillation may occur depending upon the operating condition. In this case, it is important for stabilization of the system to know the relation between the observed oscillatory modes (possible of realization) and their operating conditions. The purpose of this paper is to elucidate the selective occurrence of a self-sustained oscillation in an unstable nozzle-flapper valve and pipe line system.

2. Basic equations

Figure 1 shows a schematic diagram of a

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nozzle-flapper valve and a pipe line system. The system consists of a nozzle-flapper valve, a pipe line and a constant pressure tank. The dynamic performance of the system is described by equations of motion for the flapper supported by a spring and the oil column in the pipe line, the flow characteristic of the valve and the characteristic of thrust acting on the flapper. The motion of the flapper is represented as

$$m \frac{d^2 X}{dt^2} + d \frac{dX}{dt} + k_f (X - X_n) = F(P, X) \quad \dots\dots\dots (1)$$

where X is the distance between the nozzle tip and the flapper face (hereafter referred to as the flapper deflection), X_n the initial flapper deflection with no thrust acting on it, m the mass of the flapper, d the damping coefficient, k_f the spring constant, $F(P, X)$ the thrust acting on the flapper, P the nozzle back pressure and t' time.

The equation of motion for the oil column in the pipe line is formulated by a simplified difference equation such as Eq. (2) under the assumption: (1) the friction of the pipe can be neglected, (2) the pressure at the upstream end of the pipe line is constant.

$$p(t') + Zq(t') = -p(t'-2T) + Zq(t'-2T) \quad \dots\dots\dots (2)$$

where p and q are perturbations deviating from the steady state pressure P_0 and flow rate Q_0 at the downstream end of the pipe

Fig.1 Schematic of nozzle-flapper valve and pipe line

respectively, namely,

$$p = P - P_0, \quad q = Q - Q_0 \quad (3)$$

$T (= \ell/c)$ the propagation time for a wave travel over the length of the pipe line ℓ , c the velocity of sound in the fluid, $Z (= c\rho/A)$ the characteristic impedance of the pipe line, ρ the density of the working fluid, and A the cross sectional area of the pipe line.

The flow characteristic of the nozzle-flapper valve is given by

$$Q = c_n \pi d_n X \sqrt{\frac{2g}{\gamma} P} \quad (4)$$

where c_n is the discharge coefficient of the valve, d_n the nozzle diameter, γ the specific weight of working fluid and g the acceleration of gravity.

The thrust characteristics of nozzle-flapper valves with some overlap on the nozzle tip and the flapper face are complex in general⁽⁴⁾, while those of the nozzle with no overlap can be approximated by the following simple equation⁽⁵⁾, because in the case of a small flapper deflection, the momentum change of the flow is small and only the pressure in the nozzle chamber is related to the flow force acting on the flapper.

$$F = k_p P, \quad (k_p = \pi d_n^2/4) \quad (5)$$

3. Stability of the steady state

The steady state of the system is determined using Eq.(6), which is obtained by putting $d^2X/dt'^2 = dX/dt' = 0$ and $p = q = 0$ in Eqs.(1) ~ (5).

$$\left. \begin{aligned} k_f(X_0 - X_n) &= k_p P_0, & P_0 &= P_s, \\ Q_0 &= c_n \pi d_n X_0 \sqrt{\frac{2g}{\gamma} P_s} \end{aligned} \right\} \quad (6)$$

where P_s is the constant pressure in the upstream tank.

Linearizing Eq.(4) by small perturbations from steady state values

$$x_f = X - X_0, \quad p = P - P_0, \quad q = Q - Q_0 \quad (7)$$

as

$$q = c_p p + c_x x_f, \quad c_p = \left(\frac{\partial Q}{\partial P} \right)_{P=P_0, Q=Q_0} = \frac{1}{2} \frac{Q_0}{P_0}, \quad c_x = \left(\frac{\partial Q}{\partial X} \right)_{P=P_0, Q=Q_0} = \frac{Q_0}{X_0} \quad (8)$$

and arranging Eqs.(1), (2), (5), and (8) with respect to x_f , a linear difference-differential equation is derived, whose dimensionless form is

$$\frac{d^2 x}{dt^2} + 2\zeta \frac{dx}{dt} + x + \mu \left(\frac{d^2 \bar{x}}{dt^2} + 2\zeta \frac{d\bar{x}}{dt} + \bar{x} \right) + \eta_0 (x - \bar{x}) = 0 \quad (9)$$

where

$$\left. \begin{aligned} x &= x_f/X_0, & \bar{x} &= x(t-2\tau), & t &= \omega_n t', & \tau &= \omega_n T, \\ \omega_n &= \sqrt{k_f/m}, & \zeta &= d/2m\omega_n, & \mu &= (1-\lambda)/(1+\lambda), \end{aligned} \right\} \quad (10)$$

$$\lambda = c_p Z, \quad \eta_0 = k_p Z c_x / (k_f (1+\lambda))$$

Substituting the trial solution, $x = x_0 e^{st}$, into Eq.(9), the characteristic equation of Eq.(9) is derived as

$$s^2 + 2\zeta s + (1 + \eta_0) + \mu (s^2 + 2\zeta s + (1 - \eta_0/\mu)) e^{-2\tau s} = 0 \quad (11.a)$$

To facilitate the application of Nyquist's stability criterion, Eq.(11.a) is rearranged as

$$1 + G(s) e^{-2\tau s} = 0 \quad (11.b)$$

$$G(s) = \mu \frac{s^2 + 2\zeta s + (1 - \eta_0/\mu)}{s^2 + 2\zeta s + (1 + \eta_0)} \quad (11.c)$$

The stability of a system including a dead time is decided by the positional relation between the critical point $(-1, 0j)$ and the vector $G(j\Omega)e^{-2j\tau\Omega}$ which is obtained by rotating the vector $G(j\Omega)$ clockwise by an angle of $2\tau\Omega$, where Ω is a frequency corresponding to the intersecting point of a $G(j\omega)$ locus and the unit circle, i.e., a gain-crossover frequency. $G(j\omega)$ loci are classified as in Fig.2 depending on the existence of intersections of $G(j\omega)$ loci with the unit circle, which coincides with the existence of real roots of the equation $|G(j\omega)| = 1$, that is,

$$\omega^4 - K_1 \omega^2 + K_2 = 0, \quad K_1 = 2 + \frac{2\eta_0}{1-\mu} - 4\zeta^2, \quad K_2 = 1 + \frac{2\eta_0}{1-\mu} \quad (12)$$

The roots of Eq.(12) are

$$\Omega_{1,2} = \sqrt{(K_1 \pm \sqrt{D})/2}, \quad D = K_1^2 - 4K_2 \quad (13)$$

If $D < 0$, $G(j\omega)$ loci are entirely contained in the unit circle, that is, $G(j\omega)e^{-2j\tau\omega}$ loci never enclose the critical point $(-1, 0j)$. In this case, therefore, the system is stable regardless of the length of the pipe line. If $D > 0$, a $G(j\omega)$ locus and the unit circle intersect at two points and the corresponding gain-crossover frequencies are Ω_1 and Ω_2 ($\Omega_1 < \Omega_2$). The critical condition of stability $G(j\Omega_1)e^{-2j\tau\Omega_1} = -1$ is rewritten as

$$\angle G(j\Omega_1) - 2\tau\Omega_1 = -(2n-1)\pi$$

$$\angle G(j\Omega_i) = \angle \left(\mu \frac{1 - \eta_0/\mu - \Omega_i^2 + 2\zeta j\Omega_i}{1 + \eta_0 - \Omega_i^2 + 2\zeta j\Omega_i} \right), \quad i=1, 2, \quad n=1, 2, \dots \quad (14)$$

Fig.2 Vector loci of $G(j\omega)$

Fig.3 Unstable regions of the linearized system
 $P_s = 30 \text{ kgf/cm}^2$, $X_0 = 0.1 \text{ mm}$, $\zeta = 0.1$

or

$$\tan 2\tau Q_i = \frac{2\zeta Q_i \eta_0 (1 + 1/\mu)}{(1 - \eta_0/\mu - Q_i^2)(1 + \eta_0 - Q_i^2) + (2\zeta Q_i)^2} \quad (15)$$

τ and the corresponding pipe length ℓ ($= c\tau/\omega_n$) calculated from Eq.(15) give the stability boundary. Figure 3 shows the boundary curves on a stability diagram plotted in the $\ell - \omega_n$ plane, in which the shadowed portions indicate the unstable region. The number noted on each sub-region divided by the stability boundaries means the number of conjugate pairs of unstable characteristic roots $s_i = \sigma_i \pm j\omega_i$ ($\sigma_i > 0$) obtained from Eq.(11), each pair of which corresponds to an oscillatory mode $[\omega_i]$. In the sub-region numbered n , n modes of oscillation simultaneously become unstable for a given length of the pipe line.

Figure 4 indicates the variation of frequencies in each oscillation mode in terms of pipe length. The frequencies are the imaginary part of the characteristic roots. In the figure, solid lines correspond to unstable roots and dashed lines to stable roots. The value of ω_n in Fig.4 corresponds to that shown by a dashed line in Fig.3, and the marks A, B, A', and C in Fig.3 correspond to those in Fig.4. The lowest frequency $[\omega_0]$ in Fig.4 is the mode of oscillation corresponding to the flapper motion, which is always stable regardless of pipe length.

4. Nonlinear effects in the nozzle-flapper valve

In an unstable system, the initial perturbations corresponding to unstable characteristic roots develop to a certain mode of sustained oscillation having a definite amplitude. The behaviour of this oscillation must be analysed by taking account of nonlinearity.

In this section, the most effective nonlinearity for limiting the amplitude of self-excited oscillation is investigated.

Figure 5 shows a schematic diagram of the experimental apparatus. A flapper (4) is attached to one end of the core shaft of a differential transformer (3), the other end of which is fixed to the center of a leaf spring (1). The flapper position is adjusted by two screws (2). A pressure transducer (6) is mounted on the pipe line (8)

Fig.4 Frequencies of the linearized system

in the vicinity of the nozzle (5). The upstream end of the pipe line is terminated at a constant pressure tank (9), to which the working fluid is supplied. A stopper (7) is installed to limit the flapper motion within a certain range of deflections. In the beginning, experiments are carried out without a stopper and it is confirmed that the amplitude for unstable modes of oscillation increases until the flapper hits the nozzle tip and that the amplitude subsequently maintains a definite magnitude. This implies that the main factor limiting the amplitude of self-excited oscillation is the collision of the flapper with the nozzle tip.

Further experimental verification is carried out under the condition that the nonlinearity of the stopper becomes prominent by restricting the flapper deflection by the stopper within the range where the flow characteristics of the valve can be regarded as approximately linear. The experimental results are marked in Fig.6 by the symbols O (without a stopper) and ● (with a stopper). In both cases the results agree well with theoretical values. This indicates that the nonlinearity due to the collision is essential for limiting the amplitude of self-excited oscillation. The nonlinearity is divided into two equivalent effects: one is a nonlinear restoring force and the other is a nonlinear damping force. First, the former effect is investigated.

The flapper restoring force including the nonlinear effect of the stoppers positioned symmetrically to the flapper is approximately represented by the following formula with respect to the flapper deflection x_f .

$$f = k_f x_f + B x_f^3 \quad (16)$$

Taking account of this nonlinear spring force, Eq.(6) is written as

$$\frac{d^2 \bar{x}}{dt^2} + 2\zeta \frac{d\bar{x}}{dt} + (1 + \eta_0)x + \frac{4}{3}\beta \bar{x}^3 + \mu \left\{ \frac{d^2 \bar{x}}{dt^2} + 2\zeta \frac{d\bar{x}}{dt} + \left(1 - \frac{\eta_0}{\mu}\right)\bar{x} + \frac{4}{3}\beta \bar{x}^3 \right\} = 0 \quad (17)$$

where $\beta = 3BX_0^2/4k_f$.

The equation equivalently linearized by the describing-function method is

Fig.5 Outline of the experimental apparatus

$$\frac{d^2x}{dt^2} + 2\zeta \frac{dx}{dt} + [k(a) + \eta_0]x + \mu \left[\frac{d^2\bar{x}}{dt^2} + 2\zeta \frac{d\bar{x}}{dt} + \left(k(a) - \frac{\eta_0}{\mu} \right) \bar{x} \right] = 0 \quad (18)$$

where $k(a) = 1 + \beta a^2$ and a is the amplitude of sinusoidal deflection of x . The characteristic equation for Eq.(18) is

$$s^2 + 2\zeta s + [k(a) + \eta_0] + \mu \left[s^2 + 2\zeta s + \left(k(a) - \frac{\eta_0}{\mu} \right) \right] e^{-2\tau s} = 0 \quad (19)$$

Substituting $s = \omega j$, the equations of frequencies and amplitudes of self-excited oscillation are derived.

$$\tan \tau \omega = \left[-\frac{\eta_0(1+\lambda)}{2\lambda^2} \pm \sqrt{\left(\frac{\eta_0(1+\lambda)}{2\lambda^2} \right)^2 - \left(\frac{2\zeta\omega}{\lambda} \right)^2} \right] / 2\zeta\omega \quad (20)$$

$$a^2 = (\omega^2 - 1 + 2\zeta\omega\lambda \tan \tau \omega) / \beta$$

Theoretical results calculated from Eq.(20) are plotted in Figs.6 and 7. The stability of stationary oscillations obtained by Eq.(20) is determined by the sign of the real part σ/σ_a of ds/da where $s = \sigma \pm j\omega$ is the characteristic root of Eq.(19). In Figs.6 and 7, stable solutions are indicated by solid lines and unstable ones by dashed lines. The theoretical maximum frequency $\omega_{\max}$ reaches about 13 times the level of the flapper natural frequency ω_n , but according to experimental values in Fig.6, the frequency is only 2.5 ω_n at most. The difference between them is too large to affirm the theoretical results. This implies that one has to take account of not only the nonlinear restoring force of the stopper, but also the damping effect due to impact.

Energy dissipation due to the inelastic collision of the flapper and the stopper results in a damping effect on oscillations. The dissipation energy E_c by collisions during a period of oscillation is estimated as Eq.(21), under the assumption that the velocity of the flapper under collision is approximated by the mean velocity $u_m = 4\varepsilon/T$, where 2ε is the distance between the stoppers, and T the period of oscillation.

$$E_c = 2 \left[\frac{m}{2} (1-e^2) u_m^2 \right] = \frac{4m(1-e^2)\varepsilon^2\omega_0^2}{\pi^2} \quad (21)$$

where e is the coefficient of restitution, $\omega_0 (= 2\pi/T)$ the frequency of oscillation, and m the mass of the flapper. On the other hand, the dissipation energy E_d during a period due to a nonlinear damping force

$$f_D = \xi \left| \frac{dx_f}{dt} \right| \frac{dx_f}{dt} \quad (22)$$

is

$$E_d = \int_0^{2\pi/\omega_0} f_D \left(\frac{dx_f}{dt} \right) dt = \frac{8}{3} \xi \varepsilon^3 \omega_0^2 \quad (23)$$

Since both E_c and E_d are proportional to ω_0^2 , the damping effect of collision can be approximated by Eq.(22) and the equivalent coefficient of ξ is derived by putting $E_c = E_d$

$$\xi = \frac{3m(1-e^2)}{2\pi^2\varepsilon} \quad (24)$$

Adding the equivalent nonlinear damping force f_d to Eq.(17), the following nonlinear difference-differential equation is derived.

$$\frac{d^2x}{dt^2} + \left(2\zeta + \xi_0 \left| \frac{dx}{dt} \right| \right) \frac{dx}{dt} + (1 + \eta_0)x + \frac{4}{3}\beta x^3 + \mu \left[\frac{d^2\bar{x}}{dt^2} + \left(2\zeta + \xi_0 \left| \frac{d\bar{x}}{dt} \right| \right) \frac{d\bar{x}}{dt} + \left(1 - \frac{\eta_0}{\mu} \right) \bar{x} + \frac{4}{3}\beta \bar{x}^3 \right] = 0 \quad (25)$$

The characteristic equation of the equivalently linearized equation of Eq.(25) by the describing function method is

$$s^2 + [2\zeta + \Delta(a)]s + K(a) + \eta_0 + \mu \left[s^2 + [2\zeta + \Delta(a)]s + K(a) - \frac{\eta_0}{\mu} \right] e^{-2\tau s} = 0 \quad (26)$$

where a is the amplitude of oscillation, $\Delta(a) = 8\xi_0 a \omega / 3\pi$ and $\xi_0 = \xi X_0 / m$. Substituting $s = \omega j$ into Eq.(26), the frequency and the amplitude of self-excited oscillations occurring in the unstable system under the influence of the nonlinear restoring and damping forces equivalent to the collision are determined from the following two equations.

$$\left. \begin{aligned} & [4\zeta^2\omega^2\beta\lambda^2 + 64\xi_0^2\omega^4\lambda^3\{\lambda(1-\omega^2) + \eta_0(1+\lambda)\}/9\pi^2] \tan^4\tau\omega + 4\zeta\omega\beta\eta_0\lambda^2(1+\lambda) \tan^2\tau\omega \\ & + [8\zeta\omega^2\beta\lambda^2 + \beta\eta_0^2(1+\lambda)^2 + 64\xi_0^2\omega^4\lambda\{\eta_0(1+\lambda) + 2(1-\omega^2)\lambda\}/9\pi^2] \tan^2\tau\omega \\ & + 4\zeta\omega\beta\eta_0(1+\lambda) \tan \tau\omega + 64\xi_0^2\omega^4(1-\omega^2)/9\pi^2 + 4\zeta^2\omega^2\beta = 0 \\ & a^2 = \left\{ \omega^2 - 1 - \frac{\eta_0(1+\lambda) \tan \tau \omega}{1 + \lambda^2 \tan^2 \tau \omega} \right\} / \beta \end{aligned} \right\} \quad (27)$$

Fig.6 Frequency of self-excited oscillation $P_S = 30 \text{ kgf/cm}^2$, $X_0 = 0.1 \text{ mm}$, $\zeta = 0.1$, $\omega_n = 856 \text{ rad/s}$

Fig.7 Frequency of self-excited oscillation (theoretical results taking account of nonlinear restoring force)

Theoretical results from Eq.(27) are shown in Fig.8 together with experimental results. In the calculation, the coefficient of restitution of steel to steel is used. As seen in the figure, theoretical results agree well with experimental results. In addition, the theoretical maximum frequency $\omega_{\max}$ almost agrees with the experimental value. These results apparently indicate that a collision of the flapper with the nozzle tip or the stopper has the effect of limiting the amplitude of oscillations, and the nonlinear damping effect due to the collision is remarkable.

5. Selective occurrence of self-excited oscillation

As shown in Fig.3, there exist a number of unstable modes of oscillation in a system with a long pipe line. But it is experimentally demonstrated that, even in such a case only one mode of oscillation develops into a self-excited oscillation and two or more modes are never sustained simultaneously. To clarify the mechanism of this selective occurrence of self-excited oscillation, we shall analyse a system which has two unstable modes of oscillation, for example, as in the interval between B and A' or between C and D in Fig.3. In order to analyse this situation, the trial solution of Eq.(25) must be represented as the sum of two sinusoidal waves corresponding to the two unstable modes⁽⁶⁾ and it must be separated into the components of sine and cosine when it is substituted into the nonlinear term of Eq.(25). However, it is difficult to do this on account of nonlinear damping term $\xi |dx_f/dt'| dx_f/dt'$. Therefore, for convenience of the analysis, we replace the term $\xi |dx_f/dt'| dx_f/dt'$ approximately with $4\xi' (dx_f/dt')^3/3$. Here the factor ξ' is determined equivalently by equating the dissipated energy during a period of oscillation by each nonlinear damping force*. As a result, the equivalent coefficient ξ' is given as

$$\xi' = \frac{8\xi}{3\pi\epsilon\omega_0} \quad (28)$$

In computation, we use $(\Omega_1 + \Omega_2)/2$ as ω_0 , where Ω_1 and Ω_2 are the gain-crossover frequencies shown in Fig.4 and half of the clearance ϵ between both stoppers is the amplitude. The validity of such an approximation is verified later.

Replacing the term of $\xi_0 |dx/dt| dx/dt$ of Eq.(25) with $4\xi'_0 (dx/dt)^3/3$, we obtain

$$\begin{aligned} \frac{d^2x}{dt^2} + \left\{ 2\zeta + \frac{4}{3}\zeta'_0 \left(\frac{dx}{dt} \right)^2 \right\} \frac{dx}{dt} + (1 + \eta_0)x + \frac{4}{3}\beta x^3 \\ + \mu \left[\frac{d^2\bar{x}}{dt^2} + \left\{ 2\zeta + \frac{4}{3}\zeta'_0 \left(\frac{d\bar{x}}{dt} \right)^2 \right\} \frac{d\bar{x}}{dt} + \left(1 - \frac{\mu_0}{\mu} \right) \bar{x} + \frac{4}{3}\beta \bar{x}^3 \right] = 0 \end{aligned} \quad (29)$$

where $\xi'_0 = \xi'_0 X_0^2 \omega_n / m$. In the present case discussing the mechanism of the selective

occurrence of self-excited oscillation in a system having two unstable modes of oscillation, we must analyse the process in which either of the two unstable modes develops into a stationary self-oscillation with a definite amplitude and a frequency as the result of interaction between the two unstable modes⁽⁶⁾. A trial solution of Eq.(29) is assumed as

$$x = a_1 \cos(\omega_1 t + \phi_1) + a_2 \cos(\omega_2 t + \phi_2) \quad (30)$$

where a_1 , a_2 , ϕ_1 , and ϕ_2 are slowly varying functions of time and the subscripts 1, 2 indicate the two unstable modes of oscillation respectively ($\omega_1 < \omega_2$). Substituting Eq.(30) into Eq.(29) and applying the method of harmonic balance of Kryloff-Bogoliuboff⁽⁷⁾, the following equations are obtained

$$\left. \begin{aligned} 2\omega_i \frac{da_i}{dt} &= f(\omega_i) a_i - \xi'_0 (a_i^2 \omega_i^2 + 2a_j^2 \omega_j^2) a_i \omega_i \\ 2\omega_i a_i \frac{d\phi_i}{dt} &= -g(\omega_i) a_i + \beta (a_i^2 + 2a_j^2) a_i, \quad i=1, 2 (i \neq j) \end{aligned} \right\} \quad (31)$$

where

$$\left. \begin{aligned} f(\omega_i) &= -2\zeta \omega_i - \frac{\eta_0(1+\lambda) \tan \tau \omega_i}{1+\lambda^2 \tan^2 \tau \omega_i} \\ g(\omega_i) &= \omega_i^2 - 1 - \frac{\eta_0(1+\lambda) \tan \tau \omega_i}{1+\lambda^2 \tan^2 \tau \omega_i} \end{aligned} \right\} \quad (32)$$

When Eq.(31) includes both nonlinearities of the restoring and damping force due to the collision, it is very difficult to solve them and to clarify the mechanism of the selective occurrence of the self-excited

(a) Frequency

(b) Amplitude

Fig.8 Frequency and amplitude of self-excited oscillation (theoretical results taking account of both nonlinear restoring and damping effects and experimental results) $P_S = 30 \text{ kgf/cm}^2$, $X_0 = 0.1 \text{ mm}$, $\zeta = 0.1$, $\epsilon = 0.05 \text{ mm}$, $\omega_n = 856 \text{ rad/s}$

* The calculation is carried out as follows:

$$\int_0^{2\pi/\omega_0} \xi' \left| \frac{dx_f}{dt'} \right| \left(\frac{dx_f}{dt'} \right)^2 dt' = \int_0^{2\pi/\omega_0} \frac{4}{3} \xi'_0 \left(\frac{dx_f}{dt'} \right)^4 dt',$$

where $x_f = \epsilon \sin \omega_0 t'$.

ed oscillation. Thus, taking into account the fact pointed out in the previous section, namely, that the nonlinear damping effect by collision more noticeably limits development of oscillations, we first analyse a system in which the nonlinear restoring force is neglected, i.e., $\beta = 0$ in Eq.(31).

The steady solutions of Eq.(31) are obtained by putting $da_1/dt = d\phi_1/dt = 0$ ($i = 1, 2$). The frequencies are obtained as

$$g(\omega_i) = \omega_i^2 - 1 - \frac{\gamma_0(1+\lambda)\tan^2\tau\omega_i}{1+\lambda^2\tan^2\tau\omega_i} = 0 \quad (33)$$

From the following algebraic equations derived by putting $da_1/dt = da_2/dt = 0$ in Eqs.(31), four sets of solutions shown in Table 1 are obtained.

$$\left. \begin{aligned} \zeta_0'(a_1^2\omega_1^2 + 2a_2^2\omega_2^2)a_1\omega_1 &= f(\omega_1)a_1, \\ \zeta_0'(2a_1^2\omega_1^2 + a_2^2\omega_2^2)a_2\omega_2 &= f(\omega_2)a_2 \end{aligned} \right\} \quad (34)$$

The problem which set of solutions is physically realized must be decided from the stability of these sets of solutions. The stability criterion may be obtained by investigating the development and decay of the amplitudes a_1 and a_2 in Eqs.(31). Here we introduce new variables $X_i = \omega_i a_i$ ($i = 1, 2$) and represent the steady solutions of Eq.(31) as X_{i0} and perturbations deviating from them as x_i . Rewriting Eq.(31) with x_i and then neglecting the higher order terms than the second in x_i , the following equations are obtained.

$$\left. \begin{aligned} \frac{dx_1}{dt} &= \frac{1}{2} (f(\omega_1) - \zeta_0'(3X_{10}^2 + 2X_{20}^2))x_1 - 2\zeta_0'X_{10}X_{20}x_2 \\ \frac{dx_2}{dt} &= -2\zeta_0'X_{10}X_{20}x_1 + \frac{1}{2} (f(\omega_2) - \zeta_0'(2X_{10}^2 + 3X_{20}^2))x_2 \end{aligned} \right\} \quad (35)$$

The stability of each set of stationary solutions can be examined by the method of Poincaré⁽⁸⁾. The results are summarized in Table 1. From this table, it is seen that a stationary oscillation occurs in the cases (ii) and (iii), where either of the two unstable modes develops and neither mode develops simultaneously.

On the basis of the above discussion, the frequency and the amplitude of the developed mode of oscillation are obtained and they are plotted in Fig.9 (a) and (b). In this figure, solid lines indicate theoretical frequencies and amplitudes calculated from Eq.(33) and Eq.(34), and chain

lines with arrows and the indications a, b, c, and d show the mode shift points where a stable mode of self-oscillation becomes unstable and is switched over to another stable mode of self-excited oscillation. The curves AA', BB', and CC' indicate the frequencies of self-excited oscillation corresponding to the unstable mode $[\omega_1]$, $[\omega_2]$, and $[\omega_3]$ respectively. As the result of the stability analysis of the stationary self-excited oscillations shown in Table 1, the following conclusion is obtained for the selective occurrence of self-excited oscillation: that is, the stationary oscillation of the mode $[\omega_1]$ is stable in the

(a) Frequency

(b) Amplitude

Fig.9 Selective occurrence of self-excited oscillation (theoretical and numerical results taking account of nonlinear damping force)

$P_S = 30 \text{ kgf/cm}^2$, $X_0 = 0.1 \text{ mm}$, $\zeta = 0.1$
 $\varepsilon = 0.05 \text{ mm}$, $\omega_n = 856 \text{ rad/s}$, $\beta = 0$

Table 1 Stationary solutions and their stability

Classification of stationary solutions	Amplitudes		Stability
(i) Equilibrium point	$a_1 = 0$	$a_2 = 0$	Unstable if $f(\omega_1) > 0$ and $f(\omega_2) > 0$
(ii) Only an oscillation of the mode $[\omega_1]$ occurs.	$a_1 = \sqrt{\frac{f(\omega_1)}{\zeta_0'\omega_1^2}}$	$a_2 = 0$	Stable if $2f(\omega_1) > f(\omega_2)$
(iii) Only an oscillation of the mode $[\omega_2]$ occurs.	$a_1 = 0$	$a_2 = \sqrt{\frac{f(\omega_2)}{\zeta_0'\omega_2^2}}$	Stable if $2f(\omega_2) > f(\omega_1)$
(iv) Two oscillations of the modes $[\omega_1]$ and $[\omega_2]$ occur simultaneously.	$a_1 = \sqrt{\frac{2f(\omega_2) - f(\omega_1)}{3\zeta_0'\omega_1^2}}$	$a_2 = \sqrt{\frac{2f(\omega_1) - f(\omega_2)}{3\zeta_0'\omega_2^2}}$	Always unstable

* Here, $f(\omega_1) > 0$ and $f(\omega_2) > 0$ because two modes corresponding to the frequencies ω_1 and ω_2 are unstable in the linearized system [Eq.(9)].

interval Ab in Fig.9, but unstable in bA', that of the mode $[\omega_2]$ is stable in ad but unstable in Ba and dB', and that of the mode $[\omega_3]$ is stable in cC' but unstable in Cc. Although there exist two stable modes of oscillation over the intervals a'b and c'd, it does not imply that both are sustained simultaneously. Either of the two modes is selected and develops into a stationary self-excited oscillation depending upon the initial condition. Theoretical results of the selective occurrence of self-excited oscillations are similar to experimental results in Fig.8.

Further, frequencies and amplitudes of self-excited oscillations obtained from Eq.(26) for $\beta = 0$ are plotted together in Fig.9. Here the frequencies are calculated from the first equation of Eq.(27) by putting $\beta = 0$, but the amplitudes are obtained from the equation

$$a = \frac{3\pi}{8\zeta_0\omega^2} \frac{((1-\lambda)(1-\omega^2) - \eta_0(1+\lambda)) \tan \tau \omega}{1 + \lambda^2 \tan^2 \tau \omega} - 2\zeta_0\omega \dots (36)$$

which is directly calculated from Eq.(26). these results indicate the validity of replacing the term $\xi_0 |dx/dt| dx/dt$ equivalently with $\xi_0' (dx/dt)^3$. The frequencies calculated from Eq.(33) and Eq.(27) completely agree with each other, but the amplitudes in both cases indicate slight differences. As long as we restrict the discussion to qualitative consideration, it may be concluded that an equivalent replacement of the nonlinear damping force is permissible. Furthermore, in order to confirm validity of the approximate analysis, Eq.(26) for $\beta = 0$ is numerically solved by the Runge-Kutta-Gill method, and the exact numerical solutions, i.e., the frequencies and the amplitudes of self-excited oscillation and the situations of the selective occurrence of self-excited oscillation numerically obtained, are compared with those of the approximate solution. The symbol O in Fig.9 (a), and (b) indicates the numerical results, and the shadowed portions on the solid lines represent the region where the corresponding modes of self-excited oscillation diminish. As seen in the figure, the numerical results are qualitatively in good agreement with the

approximate solutions.

So far, the equivalent nonlinear term of the restoring force caused by a collision between the flapper and the stoppers has been neglected to simplify the analysis of the selective occurrence of self-excited oscillation. In order to clarify the properties of self-excited oscillation occurring in the system affected by both nonlinear terms i.e., the nonlinear restoring force and the nonlinear damping force, Eq. (29) is numerically analysed by the Runge-Kutta-Gill method. The results are shown in Fig.10, where the symbol O indicates the numerical solutions and the shadowed portions indicate the regions where the corresponding stationary self-excited oscillations are unstable. The solid curves are the approximate results computed from Eq.(27), which has been derived under the assumption that only one mode of oscillation is excited. The numerically obtained results on the selective occurrence of the self-excited oscillation agree well qualitatively with the case of putting $\beta = 0$. It is concluded that even though there exist simultaneously multiple unstable modes in the system, only one mode of oscillation develops into a stationary oscillation.

6. Conclusions

In this paper, the stability of a nozzle-flapper valve and pipe line system and the property of self-excited oscillation occurring in this system have been studied. The results are summarized as follows:

- (1) The stability region of the system and the number of unstable characteristic roots are analytically revealed.
- (2) The main factor to limit the amplitude of self-excited oscillation is a collision of the flapper with the nozzle tip or the stoppers. The effects of the collision are represented equivalently by a nonlinear restoring force and a nonlinear damping force.
- (3) Even though the system has a number of modes, only one mode among them develops into a stationary oscillation having a definite amplitude and frequency.

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References

- (1) Feng, T. Y., Trans. of the ASME, Ser.D, 81-3 (1959), 275.
- (2) Tominari, J., Seisan Kenkyu (in Japanese), 12-4 (1960-4), 189.
- (3) Hyodo, Y., Journal of the JSME, 66-532 (1963-5), 613.
- (4) Hayashi, S., et al., Trans. of the ASME, Ser.I, 97-1 (1975-3), 39.
- (5) Ito, T., et al., Bull. of the JSME, 10-41 (1967-10), 793.
- (6) Hayashi, S., Bull. of the JSME, 14-71 (1971-5), 438.
- (7) Kryloff, N. & Bogoliuboff, N., Introduction to Nonlinear Mechanics (1947), 8, Princeton University Press.
- (8) Stoker, J. J., Nonlinear Vibrations (1950), 36, Interscience.

Fig.10 Selective occurrence of self-excited oscillation (numerical result taking account of both nonlinear restoring and damping force)
 $P_s = 30 \text{ kgf/cm}^2$, $X_0 = 0.1 \text{ mm}$, $\zeta = 0.1$
 $\epsilon = 0.05 \text{ mm}$, $\omega_n = 856 \text{ rad/s}$