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A Study on High-speed Disk Bursting Strength*

BY

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This paper presents a comparison of the spin-test results of both unnotched and notched disks with a center hole.

The unnotched disk bursted after it deformed plastically in quasi-circular shape satisfactorily under approximately steady strain rate. On the other hand, the notched disks deformed in elliptic shape and its change further became greater with an advance of rotating speed because of a severe strain concentration generated around the notch roots and the final bursting at relatively low speed would come quickly. For the unnotched disk the usual average stress theory was adopted as the criterion of burst, but it was not applicable to the notched disks. The measured results of X-ray residual stress in unnotched disk almost coincided with the plastic stress distribution calculated on the basis of the Tresca criterion with the Mises flow rule, but the measured results for the notched disk were different from them.

1. Introduction

In the recent years, the importance of the study on bursting strength of a high-speed rotating disk has been increasingly recognized with the development of gas turbine, centrifugal separator, turbine generator, etc., and so various reports have been published on this field of study.

It is generally considered that the bursting characteristics of a rotating disk vary remarkably according to loading conditions though the bursting phenomenon mainly takes place under the action of centrifugal force in the disk. In most of the papers published previously, the bursting strength of a disk under simple loading has been discussed and studies under cyclic loading^{1,3} or steady loading^{4,5} have been limited. Especially, the test results in the field of low cycle fatigue based on start and stop of rotation of disk are scanty, and there still remains a wide unexplored domain to be studied.

In this series of studies on bursting strength of notched metallic disks subjected to actual loading in the present study, spin-burst tests of both unnotched and notched disks of mild steel were performed under the condition of simple loading. Discussion was made with comparison of the deformation behavior and the bursting strength of both disks.

In a recent paper, attempts have been made to apply the methods of fracture mechanics in the field of spin-burst tests,

which have proved successful in solving the problem of brittle fracture strength such as in a generator rotor. In parallel with this attempt, the study on application of the usual average stress theory to the notched disk is also necessary.

2. Experimental procedure

For the purpose of accelerating a bursting of the disk, an air turbine that supplied the required high-speed rotation was employed. Fig.1 shows the spin-tester used in the present study and disks were spin-tested in a suspending state from the air turbine. Vacuum pump is used to reduce the test chamber pressure to less than 5mmHg for the purpose of reducing the air friction of the disk. A leaden lining of about 55 mm thickness is set up to surround the inside wall of test chamber in order to absorb the kinetic energy of the bursting disk fragments. The rotating speed of air turbine is picked up by a digital tachometer in which a safety trip is provided against over-speed.

The material used in the spin-burst tests was SS41 mild steel, whose mechanical properties under static tensile test are listed in Table 1. Although the tensile specimens were cut from the material plate in rolling direction and in normal direction to rolling, there was no difference between them. Two types of test disks were used; unnotched type and notched type as shown in Fig.2. The notched length l was varied from 5 to 40 mm in 5 mm intervals. In the notched disks, one side surface was polished with emery paper of 0/5 after machine-finishing in order to examine the spreading appearance of Luders'-band. The dimensions of all the disk are listed in Table 2.

The deformation on x and y axes of the disk was measured with sufficient accuracy. Thereby, x -direction was made to agree with the rolling direction of material plate. In the present study, for simplicity, the deformation after spin-test was assumed

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- ① Air turbine, ② Disk fixture, ③ Disk
④ Test chamber, ⑤ Leaden lining,
⑥ Upper cover, ⑦ Observation window
⑧ Center rest holder, ⑨ Vacuum pan
⑩ Exhaust duct, ⑪ Silencer

Fig.1 High-speed disk tester (Spintester)

Fig.2 Shape and dimensions of notched disk

Table 1 Tensile properties of material tested

	lower yield point $\sigma_{0.2}$ kg/mm ²	Ultimate tensile strength σ_B kg/mm ²	Elongation δ %	Young's modulus E kg/mm ²
Rolling direction	25.29	33.60	33.0	2.14×10^4
Cross direction	23.12	33.60	32.2	2.10×10^4

Table 2 Dimensions of rotating disks

Disk	Inside diam. $2r_1$ mm	Outside Diam. $2r_2$ mm	Thickness t mm	Notch length l mm
Unnotched	20	200	2	0
Notched	20	200	2	5, 10, 15, 20, 25, 30, 35, 40

Table 3 Conditions of X-ray diffraction

Characteristic X-ray	Co-K α
Tube voltage	30 kV
Tube current	10 mA
Count range	400 cps
Time constant	4 sec.
Irradiation area	2×3 mm ²
Divergent angle	0.35 deg.
Gonio scan speed	1 deg./min.
Incident angle of X-ray	0, 15, 30, 45 deg.
Chart speed	10 mm/min.

to be approximately equal to one under rotating condition, therefore measurement of the deformation was done after the disk was taken off from the spintester. The strain components of a rotating disk include tangential, radial and axial ones. Since mainly the tangential strain affects fracture the most, the tangential strain was preferentially measured. The strains were determined from the changes in gage length which were continuously marked along the x, y axes at intervals of 5 or 10 mm. Measurement of the gage length was made by using a microscope whose minimum scale was 1/100 mm. Every strain obtained in this way was expressed as nominal strain.

The X-ray measurement of residual stress distribution along radius of rotating disk was performed by the counter-method. The peak position of the diffraction profile was determined by one-eighth value breadth method, and moreover, the stress determined by $\sin^2\phi$ method was employed. The disks for measurement of X-ray residual stress were polished with emery paper and then electro-polished. The conditions of X-ray diffraction are listed in Table 3.

3. Experimental results and discussions

3-1 Deformation and bursting behavior of unnotched disk

Fig.3 shows the variation of tangential strain at inside and outside diameters with rotating speed for unnotched disk. In this figure a plot of the values shows the measured strains at x, y axes of the disk which intersect perpendicularly with each other. The apparent deformation in the disk started at about 26,000 rpm, and then the tangential strain reached about 17 percent on the enlarged inside diameter before the final bursting. On the contrary, the deformation at outside was no more than 1.5 percent. It was observed that the difference between the tangential strains at x axis and y axis was small, and hence there was no remarkable deformation anisotropy of the material tested. Therefore, the unnotched disk deformed plastically in quasi-circular shape. It was also observed that a stoppage of deformation occurred in the period from start of deformation to the bursting, particularly in the stage just before bursting.

Fig.3 Experimental variation of tangential strain at inside and outside diameters with rotating speed for unnotched disk

Fig.4 Measured distribution of tangential strains along radius of unnotched disk

Fig.5 Appearance of fragments of unnotched disk (Nb = 32,000 rpm)

Fig.4 shows the tangential strain distribution on x, y axes of an unnotched disk at 28,500 and 30,000 rpm. It was observed that the strain increased rapidly near the bore and a severe strain concentration generated at the inside diameter of the disk.

Another interesting behavior in the unnotched disk was a warp phenomenon in thickness direction. It was observed that the warp was 3.8 mm on the edge of bore at 28,000 rpm and then it decreased gradually to the original plane surface before bursting. It seems that the warp is caused by a severe strain concentration around the bore. Therefore, it is suggested that a recovery from the warp just before bursting results from relaxation of the large strain gradient along radius of the disk.

Fig.5 shows a photograph of fracture appearance of an unnotched disk after bursting at 32,000 rpm.

3-2 Deformation and bursting behavior of notched disk

Fig.6 shows the variation of enlargement of disk diameters with rotating speed for a disk whose notch length is 15 mm, and data plot was made on x and y axes, respectively. It is noted that the deformation behavior on both axes is contrary to each other; that is, the diameter on x axis tends to be smaller than its original dimension with rotating speed, against a gradual increase of that on y axis. The notched disk deformed in elliptic shape and the change of shape became greater with an advance of rotating speed. In this case, a rapid enlargement of the inside diameter started at about 26,500 rpm. Similar experimental results for a disk having 25 mm notch length are shown in Fig.7, from which it is seen that the disk deformation increases rapidly after a run whose speed is about 1,000 rpm below its bursting speed, just as the tendency in Fig.6. Therefore, comparing the tendencies of Figs.6 and 7 with that of an unnotched disk in Fig. 3, it is found that the unnotched disk deforms always under steady strain rate during the spin-test but the deformation in the notched disk makes a sudden change for the rapid increasing rate after the run just before bursting.

It is known that the Lüders'-lines in the unnotched disk of mild steel originate at the bore and spread rapidly toward the outside. It is also known that both number and width of Lüders'-lines increase gradually with an advance of rotating speed and plastic deformation occurs in the whole region of the disk^{6,10}. Expansion of the disk will result in a bursting. From this fact, it can be considered that in case of the unnotched disk the plastic deformation occurs uniformly but in the notched disk the deformation occurs in the localized region of radii passing through the notch roots.

Fig.8 shows a spreading appearance of Lüders'-bands on the surface of a disk with two radial notches whose lengths are $l = 25$ mm after having been run at three various

Fig. 6 Experimental variation of enlargement of inside and outside diameters with rotating speed for notched disk (In case of $l = 15$ mm)

Fig. 7 Experimental variation of enlargement of inside and outside diameters with rotating speed for notched disk (In case of $l = 25$ mm)

speeds. As is observed from the figure, the Lüders'-bands began to spread from the notch tips and they reached the outside of the disk before bursting. After having been run at 23,200 rpm, the Lüders'-bands of broomy shape toward the outside of disk appeared on the surface as shown by the negative portions and after run at 24,500 rpm they grew larger with the intact shape as shown by the portions of cross hatching line. Thus, the grown Lüders'-bands, after having been run at 25,500 rpm which is very

Fig. 8 Spreading appearance of Lüders'-bands from notch tips to outside in notched disk (In case of $l = 25$ mm)

close to the bursting speed of 25,700 rpm, branched out in three directions as shown by the hatched regions in the figure. Though in one disk having 5 mm notch length a few slender Lüders'-bands appeared additionally at 29,000 rpm near the bursting speed, in another unnotched disk a similar appearance to one in Fig. 8 was observed. From these facts, it seems that the plastic deformation in the unnotched disk is concentrated locally at neighboring region of the radii passing through each notch tip.

Fig. 9 shows the tangential strain distribution along two rectangular diameter x , y axes of a notched disk having $l=15$ mm at different speeds which are advanced at intervals of 500 rpm from 25,500 rpm at which the disk deformation begins to aggravate. As is seen from the figure, a severe strain concentration appears around the notch roots. Comparing Fig. 9 with Fig. 4, it is seen that the strain concentration around the notch roots at early stage of the rotating speed is smaller than that around the bore of the unnotched disk, whereas before bursting it is considerably larger than that of an unnotched disk. Accordingly, it is considered from the comparison that the increasing rate of the tangential strain around the notch roots becomes remarkable with an advance of rotating speed and the strain concentration becomes more severe especially in a disk whose notch length increases with a decrease of effective diameter section of the disk. The similar strain distribution of a disk whose notch length is the greatest ($l = 40$ mm) is shown in Fig. 10. It is evident from the figure that just before bursting the tangential strain at the notch tip has reached about 15 percent and the strain concentration is more remarkable.

In the case of a notched disk, it is inferred that the bursting is to take place along the x axis, since the plastic deformation is concentrated locally at neighboring region of the diameter passing through the notch tips. In fact, from the fracture appearance of a disk having 15 mm notch

Fig.9 Measured distribution of tangential strains along radius of notched disk (In case of $l = 15$ mm)

Fig.10 Measured distribution of tangential strains along radius of notched disk (In case of $l = 40$ mm)

length shown in Fig.11, it is demonstrated that the disk has broken in two fragments along the diameter section contained in the notch roots. As is seen from Fig.6, the bursting instantaneously takes place with about 10 percent enlargement of the inside diameter at a time when the rotating speed becomes 27,800 rpm.

Fig.11 Appearance of fragments of notched disk ($l = 15$ mm, $N_b = 27,800$ rpm)

3-3 Application of average stress theory

It is wellknown that the average stress theory is based on a view point that the disk should centrifugally fail when the average value of tangential stress on a diameter section is equal to the ultimate tensile strength of the material. It is generally said⁸ that, according to this theory, in the case of an unnotched disk of isotropic ductile materials the calculated results are in fairly good agreement with the experimental ones. The previous experimental data³ of the authors obtained from the spin-test of an unnotched disk of SS41 mild steel also agreed approximately with the theoretical value. In general, since this theory is not only simple but also convenient for practical use, the design technique based there on has been widely applied in the field of high-speed rotating machines.

The average tangential stress at angular velocity ω for a uniform-thickness disk with a center hole or bore is given by the following formula,

$$\bar{\sigma}_t = \gamma \omega^2 (r_2^3 - r_1^3) / 3g(r_2 - r_1)$$

where γ is the specific weight of material used, r_1 and r_2 are inside and outside diameters, respectively, and g is the gravitational acceleration. For the disk with two diametrically opposite radial notches at the bore, each of length l , Fig.2, the net average tangential stress given by D.H. Wine and B.M. Wundt^{9,10} becomes

$$\bar{\sigma}_{th} = \gamma \omega^2 (r_2^3 - r_1^3) / 3g(r_2 - r_1 - l)$$

Fig.12 shows the variation with rotating speed of average tangential stress, where one of the ordinates $\bar{\sigma}_{th}$ is the net average tangential stress for a disk whose notch length is $l = 15$ mm. Since the ultimate tensile strength of material tested is $\sigma_B = 33.6$ kg/mm², the predicted bursting speed obtained by applying the value in Fig.12 is 32,150 rpm for the unnotched disk as indicated in the figure. Consequently, in the case of the unnotched disk, it seems to be quite all right to consider that there is approximately no difference between the actual bursting speed obtained from the

Fig.12 Variation with rotating speed of average tangential stress for unnotched and notched disks

spin-test, that is $N_b = 32,000$ rpm, and the predicted one. On the other hand, in the case of a notched disk, the theoretical bursting speed obtained from Fig.12 is 29,350 rpm, and the difference between the experimental value and the calculated one can not be neglected. This is true with other notched disk with various notch lengths. Fig.13 shows a comparison of the experimental results of bursting speed which is affected by notch length with the calculated results based on the net average tangential stress $\bar{\sigma}_{th}$. As is seen from this figure the bursting strength of disks decreases linearly with the notch length, but a notable difference from 1,000 to 1,500 rpm is found between the calculated and the experimental results. It may be concluded that the average stress theory can be adopted as the fracture criterion for the unnotched disk but not always for the notched disk. Therefore, for the notched disk a modification of the net average tangential stress theory must be made. It is interesting to note that the transition from the data point of the unnotched disk to ones of the notched disk in a region of smaller notch lengths in Fig.13 is associated with a study on the modification described above.

3-4 Residual stress distribution measured by X-ray

It is known that the analytical results of plastic deformation for a rotating disk based on the Tresca criterion and the Mises flow rule used in plasticity theory are in considerably good agreement with the experimental ones³.

Fig.13 Comparison between experimental results of bursting speed affected by notch length and theoretical ones for notched disk

Now, from this point of view, the distribution of tangential residual stresses calculated by the theoretical analysis and the measured results of that by X-ray obtained after operating at 31,500 rpm for an unnotched disk are shown in Fig.14, where in determining the residual stress calculated, the authors have used an approximation which, as is noted from the figure, can be given as the difference between the plastic stress obtained by using the analytical method of Tresca-Mises mentioned above and the elastic stress derived by assuming the material tested to be a perfectly elastic body. And also in this figure, the ordinate tangential stresses are expressed in a non-dimensional form dividing them by the average tangential stress. Here, the elastic stress, using Poisson's ratio $\nu = 0.3$, was calculated by the following expression

$$\frac{\sigma_t}{\bar{\sigma}_t} = \frac{3(3+\nu)}{8} \frac{r_2 - r_1}{r_2^3 - r_1^3} \{ (r_2^2 + r_1^2) - \left(\frac{1+\nu}{3+\nu} \right) r^2 + \frac{r_2^2 \cdot r_1^2}{r^2} \}.$$

Also the plastic stress was calculated in the following form suggested by A.M.Wahl³⁻⁵ and others, where $\phi(r)$ is a function determined by the disk-principal-stress ratio of $\sigma_r/\sigma_t = x$; by taking $(2x-1)/(2-x) = \phi(r)$, it is expressed as follows

$$[\phi(r)]^n = \frac{\exp \left[\int \frac{x \phi(r)}{r} dr \right]}{r(1-x+x^2)^{(n-1)/2} (1-x/2)}$$

Fig.14 Comparison between calculated distribution of tangential residual stresses and measured results of that by X-ray at 31,500 rpm for unnotched disk

Fig.15 Measured distribution of X-ray residual stresses along radius of notched disk

In this, the value of n is given as an exponent which defines in plastic range the stress-strain relation obtained from a simple tension test using the disk material. The calculation of plastic stress was made using the value of $n = 4.71$, in this study. As is shown in Fig.14, the calculated residual stress indicated by the broken line is nearly zero on the mean radius ($r=50\text{mm}$) of disk. Therefore, the residual stress of disk is a compressive stress in the inside region and turns to a tensile stress in the outside region. Though the number of measured points is not sufficient, within the limits of this experiment, it is considered that those values nearly coincide with theoretical ones. Consequently, it can be suggested again that the plastic stress distribution calculated on the basis of the Tresca criterion with the Mises flow rule for an unnotched disk is quite appropriate.

Next, the measured results of X-ray residual stress in a rotating disk with two radial notches whose lengths are 15 mm and a comparison with that in an unnotched disk will be made. The measured distributions of X-ray residual stresses along radius of a notched disk obtained after operating at 22,000 and 25,000 rpm are shown in Fig.15, in which the ordinate $\sigma_{tr}/\bar{\sigma}_t$ is non-dimensionalized by the quotient of the average tangential stress $\bar{\sigma}_t$ for an unnotched disk, and further the theoretical distribution of X-ray residual stresses for an unnotched disk is also replotted as a broken line in this figure. What can be found on comparing the stress-distribution curves of both notched and unnotched disks is as follows. Namely, the experimental

curves of a notched disk at either rotating speed are very different from the theoretical ones of an unnotched disk, and in the notched disk a large shift of the curve toward the outside diameter occurs when the rotating speed of disk approaches that of its bursting.

4. Conclusions

In order to compare the deformation behavior and bursting strength of both unnotched and notched disks of SS41 mild steel, spin-tests of both disks were performed under the condition of simple loading. The results obtained in this study are summarized as follows:

- (1) The unnotched disk deformed plastically in quasi-circular shape, and no remarkable deformation anisotropy of the material tested was observed. A severe strain concentration appeared at the inside diameter of the unnotched disk with an advance of rotating speed and a warp of the disk was observed. However, the disk gained its original plane surface after the run just before bursting, because of relaxation of a large strain gradient along radius.
- (2) A plastic deformation of the notched disks appeared in the local region on both sides of two radii passing through each notch root, but the region of two another radii perpendicularly intersecting with those radii was kept in elastic state even very close to the bursting speed, and preferably in this region a little compressive deformation was measured. For this reason, the notched disk deformed in elliptical shape with an advance of rotating speed.

(3) The unnotched disk deformed plastically with a uniform strain rate during the spin-test and the final bursting occurred. On the other hand, the notched disk deformed with a small strain rate and the deformation increased rapidly just before bursting.

(4) The usual average stress theory was adapted for the unnotched disk but the net average tangential stress theory was not adopted as the fracture criterion of the notched disks.

(5) From the X-ray measurement, it was found that the residual stress on the mean radius of the unnotched disk was nearly zero but not zero for the notched disk. A large shift of the distribution curve toward the outside diameter was observed when the rotating speed of the notched disk came up to the bursting speed.

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