A Study on Strength of Toothed Belt*

(5th report, Effect of Pitch Difference on Fatigue Strength of Toothed Belt)

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The pitch difference between toothed belt and pulley exerts great influence on the fatigue strength of toothed belt. We researched experimentally the relationship between pitch difference and fatigue strength for L-belt section of polychloroprene rubber and polyurethane toothed belts.

From this study, the following results have been obtained. There are several features of fracture which relate to pitch difference and belt tension. A high fatigue strength can be expected if the pitch difference is selected to equalize the load distribution.

1. Introduction

Generally a toothed belt is manufactured to have a pitch less than that of a pulley when the belt is softly wound around the pulley, in order to get the normal meshing, by considering the elongation of the load-carrying-cord, the deformation of belt tooth, etc. However this causes an interference of teeth at the slack side of the driven pulley. Therefore it is very important to determine the proper pitch difference depending on the transmitting load in order to increase the fatigue life of belts. In the previous reports^{(1)~(4)} we discussed theoretically and experimentally on the load distribution of toothed belt, the fatigue strength of belts of standard size, and the failure mode. This report deals with the effect of the pitch difference in the belt and pulley teeth on the fatigue lives and shows the proper pitch difference for any belt tension.

2. Strength of Tested Belt and Dimensions of Pulley

2.1 Tensile strength of tested belt

The belts tested are the same as mentioned in the 3rd report, i.e., section L, a polyurethane belt (for short, U belt) and a polychloroprene belt (for short, CR belt) with

width $b=19$ mm and pitch $t_B=9.525$ mm. Wire rope is used as the load-carrying-cord in U belt and glass fiber rope is used in CR belt. The dimensions of belt were the same as shown in the 3rd report. All the specimens were taken from one lot in order to lessen the scatter of results.

In Fig. 1 the tensile test results of the virgin belts are shown. The testing speed was 5 mm/min, and the gauge length was 100 mm. A rack which meshes with the belt teeth was used at the chucking part to lessen the effect of the deformation of the chucking part on the elongation. Two sets of solid and broken lines show the range

mean $\pm 2 \times$ (standard deviation)

which includes 95 % of data. The mean breaking strengths are 455 kgf for U belt and 545 kgf for CR belt, and the standard deviations in strength are 20.3 kgf for U belt and 42.8 kgf for CR belt. Though the tensile strength of CR belt is 90 kgf greater than that of U belt, the scatter of CR belt is larger than that of U belt. This is caused by the characteristics of glass fiber rope.

The shear strength of belt tooth is greatly affected by the ambient temperature. At the room temperature, for

Fig. 1 Stress-strain relation of tested belts

No.	$2R_p$ mm	m_p mm	h_p mm	r_p mm	2α deg.	2β deg.	2ϕ deg.
1	108.03	3.05	2.68	0.85	40.0	3.70	6.30
2	108.25	"	"	"	"	3.72	6.28
3	108.37	"	"	"	"	3.72	6.28
4	108.48	"	"	"	"	3.73	6.27
5	108.72	"	"	"	"	3.74	6.26

No.	Δt mm		c
	Urethane	Rubber	
1	+0.017	+0.022	Urethane belt
2	-0.002	+0.003	0.48 mm
3	-0.013	"	Rubber belt
4	-0.022	-0.018	0.45 mm
5	"	-0.038	"

Fig. 2 Form and dimensions of pulley and pitch difference

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both belts, the maximum shear strength is about 95~110 kgf.

2.2 Dimensions of pulley and method of calculating the pitch difference

Fig. 2 shows the form, dimensions of pulley and the pitch difference between pulley and belt. In the state of the belt being softly wound on the pulley, there are two methods to change the pitch difference; one is to change the distance from the center of the load-carrying-cord to the external radius of pulley, and other is to change the external radius of pulley. In this study we adopted the latter method. By this method the depth of pulley teeth is a little altered by the change of external radius R_p , and as the form and dimensions of tooth groove do not change, the bucklash is assumed to be constant. The pitch difference is given by

$$\Delta t = t_B - 2(R_p + c) \left(\sin \phi + \frac{\pi \beta}{180} \right) \text{ mm},$$

where for positive Δt , the pitch of belt tooth is larger than that of pulley.

3. Experimental Apparatus and Method

A fatigue testing machine of power circulating type for toothed belt was used in this study, which is the same as shown in the 3rd report. The rotational speed of pulley is 2800 rpm. In this study we set a simple equipment to check the belt life in the fatigue machine. If this equipment operates, the driving motor stops automatically.

Fig. 3 shows the equipment to control the belt tension. In order to keep the testing tension constant, a coil spring and an idle roller are set on the tight side of driving belt. If the testing belt has any failure like abrasion or shear on the tooth of belt, the slide block climbs up with a decrease of the spring load, then the limit switch operates and indicates the change of testing tension and the fatigue life of belt. The slack side tension was set at 2~10 kgf in general. However in the case of $\Delta t = -0.038$ mm for CR belt, under a high tight side tension when the belt is going to mesh with driven pulley, the belt tooth is apt to get on the pulley tooth, so that the slack side tension becomes 30 kgf. An experiment was done on Nos. 1, 2, 3, and 4 pulleys for U belt and on Nos. 1, 2, 4, and 5 pulleys for CR belt.

4. Experimental Results and Considerations

4.1 Relation between the pitch difference and fatigue life in U belt

Fig. 4 shows the test results of fatigue life and fracture features for the U belt. On the ordinate the tension on tight side is represented and on the abscissa the number of cycles to failure. For any pitch difference, the fatigue life increases with a decrease of tension on tight side. On the other hand the fatigue strength is greatly affected by the pitch difference. For the pulley No. 1, i.e., for the positive pitch difference, the fatigue strength becomes the least, in other words fatigue lives for positive Δt are one-tenth order

less than the other Δt . Therefore Δt should not be positive. In this case the failure is caused by the shear of belt teeth regardless of tension magnitude at tight side and number of cycles. Shear failures occur on the tooth surface at driving side meshing with the driven pulley; for the high tension the location when the shear failure occurs approaches the tooth bottom, and the crack reaches the opposite side of the tooth, and for the low tension, the shear failure initiates at a relatively central location of tooth surface and the cracks propagate in the normal direction to the surface to reach the load-carrying-cord and propagate along the load-carrying-cord to the opposite tooth surface. On the opposite surface of belt tooth in the running direction meshing with the driving pulley a compressed trace is left but no failure occurs.

In the test results for the pulley No. 2, the T_0 - N curve has the same form as for the pulley No. 1, but the fatigue strength of No. 2 is about 20 kgf higher than that of No. 1 at any number of cycles to failure. The failure features are also the same as for pulley No. 1; i.e., for any tension the shear failure occurs at the contact of tooth surface with driving pulley. The location of crack initiation is also entirely the same for the pulley No. 1.

The test results for the pulley No. 3 show a higher fatigue strength than those for No. 2, and the T_0 - N curve is not different from T_0 - N curves for the pulley No. 1 and 2, but there seems a little difference in the failure features; in high tension range, i.e., at N lower than 10^7 , the failure is a shear which initiates and propagates at the tooth surface contact with driving pulley, but at N higher than 10^7 , the failure is an abrasive wear of the belt tooth and the power transmission becomes impossible on account of that wear.

The test results for the pulley No. 4 show the best fatigue strength at the number of cycles to failure $N = 5 \times 10^7$. However at N higher than $N = 5 \times 10^7$ the fatigue life is not improved but the fatigue strength decreases sharply. The failure feature is the same as for the pulley No. 3.

According to the test results on four kinds of the pitch differences, in the test range and in the range of N lower than 5×10^7 , the maximum fatigue strength occurs for $\Delta t = -0.022$ mm; and in the range of N higher than 5×10^7 , it occurs for $\Delta t = -0.013$ mm.

The scatter of the test results seems to be caused by various factors such as the scatter of the static strength and the dimensions of the individual belts tested, the change of roughness and tooth form of belt by wear, the change of surrounding temperature, the change of tension applied to the belt by the decay of the testing machine, and the error of the loading tension, but we can not determine which factor has the greatest effect on the scatter of the results.

Now let us consider the test results of U belt on the basis of the load distribution of belt teeth.

In Fig. 5 we show the meshing state of the belt and the pulley. The numbers of teeth in the belt and the pulley

Fig. 3 Equipment regulating belt tension

Fig. 4 Fatigue curves of polyurethane belt

are numerically shown in the figure. As the numbers of teeth of both driving and driven pulleys are 36, the belt teeth numbered from 3 to 21 completely mesh with the pulley. As stated already, all the failures of belt are caused by the driven pulley, hence the consideration is limited to the meshing phenomena of the belt teeth with the driven pulley teeth. In this experiment, generally the meshing on the driven pulley begins at teeth numbered 22 to 21 at the slack side and ends at teeth numbered 3 to 1 at the tight side, though there exists a fine difference depending on the pitch difference and tension of belt.

Figure 6 compares the load distributions of pitch differences for the tension when $N=1 \times 10^6$. In this condition, as stated in Fig. 4 the failures are invariably a shear. In the ordinate the load on each tooth is indicated and in the abscissa the number of teeth shown in Fig. 5 is given.

The load is calculated by the method described in the 4th report. One cycle of meshing, i.e., the angle of the No. 3 tooth moving to No. 2, was divided into 5. The values of the elastic modulus E , the compliance $f(K)$, the coefficient of friction μ are the same as those in the 3rd report. The compliance for incomplete meshing part was determined by the method in the 1st report, and the value shown in Fig. 6 is one at complete meshing state. The tension at tight side T_0 is obtained from Fig. 4, and it is different depending on the pitch difference. The tension at slack side $T(N_z)$ was taken as 5 kgf in every case.

For the test 1 (the pulley No. 1), the load begins to be imposed on the teeth numbered 11 to 10, and it is increased rapidly up to No. 3 tooth. The maximum load on the tooth surface occurs at a point about 2 degrees rotated

Fig. 5 Meshing of belt and pulley

No.	Δt mm	T_0 kgf	$T(N_z)$ kgf	E kgf/mm ²	$f(K)$ mm/kgf	μ
1	+0.017	60	5	11,000	0.018	0.45
2	-0.002	100	"	"	"	"
3	-0.013	120	"	"	"	"
4	-0.022	140	"	"	"	"

Fig. 6 Load distribution of polyurethane belt (for $N=1 \times 10^6$)

from tooth No. 3, and it is about 13 kgf. After that point the load on the tooth surface decreases rapidly, and the contact of belt with the pulley comes to an end after the tooth No. 2. The sum of the force components in the circumferential direction of tooth surface load is about 46 kgf, and the difference between $T_0 - T(N_z) = 55 \text{ kgf} - 46 \text{ kgf}$ is 9 kgf, which is transmitted by the friction developed between the top land of pulley teeth and the bottom land of belt teeth, and it does not appear in Fig. 6. For the test No. 2, the load increases gradually from the first tooth as shown in Fig. 6, and becomes maximum at the tooth No. 3. After this maximum point, the load decreases in the same way for the test No. 1. For the tests Nos. 3 and 4, in which Δt is small compared to the tests Nos. 1 and 2, the load curves become concave and have two maximum values. For each test, a fine load variation whose frequency coincides with the tooth pitch appears which is caused by the uncomplete meshing. This phenomenon has already been reported as the experimental results in the 2nd and 4th reports. Though the mean tooth surface load is as large as 4.3~6.0 kgf in these four kinds of belts, the maximum loads on the belt tooth are almost same and they are about 13 kgf for each belt. Also the loads change in the same way on the belt teeth from No. 3 to No. 2. This process of uncomplete meshing on these teeth is considered one factor of shear failure of belt teeth. Comparing the results for the test Nos. 1 and 4, there is found a great difference in the loads of teeth at slack side but this does not affect the fatigue strength for the number of load cycles under 10^6 where the shear failure of the belt teeth occurs.

Figure 7 shows the load distribution of belt tooth for the tension at the fatigue life of 10^8 . Failures are in two types, i.e., shear and wear, depending on the pitch difference. For the test No. 1, only four to five teeth contribute to load transmission, also at the position from tooth No. 3 to No. 2, over 40% of transmitting load, i.e., 5.5 kgf in 13 kgf acts, and this is supposed to cause a shear failure of belt teeth. In the test No. 3, the tension at tight side is the greatest and the fatigue strength is the greatest. The load distribution for the test No. 3 has two maximum values at both tight and slack sides whose values are about 4 kgf. The failure is a wear, which is considered to be caused in the process of meshing of belt teeth with the driven pulley from the tooth No. 22 to No. 21 at slack side. The same tooth surface load as in the test No. 2 acts at tooth No. 3 at the tension side, but a shear failure does not occur, the reasons for which are that the load distributions in the tooth No. 3 to No. 2 are a little different from each other, and that the pressure angle of belt tooth comes to be enlarged by the progress of wear and the interference at the end of meshing

No.	Δt mm	T_0 kgf	$T(N_z)$ kgf	E kgf/mm ²	$f(K)$ mm/kgf	μ
1	+0.017	15	2	11,000	0.018	0.65
2	-0.002	35	"	"	"	"
3	-0.013	60	"	"	"	"
4	-0.022	25	"	"	"	"

Fig. 7 Load distribution of polyurethane belt (for $N=1 \times 10^8$)

becomes small. In the test No. 4, the load acts on four to five teeth at the slack side, and after the tooth No. 16 the belt teeth do not contact with belt pulley. The failure is a wear, the reason for which is the same as in the test No. 3. Over 50 % of the transmitted load 23 kgf is transmitted by the friction between the bottom land of belt tooth and the top land of the pulley tooth.

Comparing these four kinds of belt with different pitch differences, the No. 3 belt is found to have the most idealized load distribution, and the highest fatigue strength.

4.2 Relation between the pitch difference and fatigue life in CR belt

The fatigue curves for CR belts with four pitch differences are shown in Fig. 8. In the range of the experiments, the fatigue life increases linearly with a decrease of the tension at tight side, and the greater the absolute pitch difference, the higher the fatigue strength. The failures are of a greater variety than in the U belt. The belt No. 1 has the lowest fatigue strength, and the failure is the same as in U belt. This shows that the positive pitch difference should not be adopted. In the belt No. 2 the tight side tension is 20 to 30 kgf higher than in the belt No. 1 at the same number of cycles, but the result is not improved. In the belt No. 4 the fatigue strength is 20 kgf higher, and the belt No. 5 has the highest fatigue strength. However in this case when the belt is slightly wound on the pulley, the belt tooth comes to run on the pulley tooth at the slack side, so that the slack side tension increases, and therefore the transmitting load decreases.

Figure 9 shows the load distribution of belt tooth for

Fig. 8 Fatigue curves of polychloroprene rubber belt

No.	Δt mm	T_0 kgf	$T(N_2)$ kgf	E kgf/mm ²	$f(K)$ mm/kgf	μ
1	+0.022	130	5	5,000	0.030	0.3
2	+0.003	160	"	"	"	"
4	-0.018	180	"	"	"	"
5	-0.038	210	"	"	"	"

Fig. 9 Load distribution of polychloroprene rubber belt (for $N=1 \times 10^6$)

the tension at the fatigue life 10^6 of CR belt. In this case as the failure of the belt tooth is caused by the driving pulley, the load distribution is also shown for the driving pulley side. The tight side tension is 130~210 kgf, as shown in Fig. 8, and the slack side tension is set at 5 kgf. For all the belts Nos. 1~5, a load of over 10 kgf acts at tooth No. 2 which is not yet in the state of complete meshing. At this position, the interference $\lambda(K)$ between the top of belt tooth and the bottom of pulley tooth reaches 0.62 mm, and this meshing state continues to the position of tooth No. 3. The tooth surface load becomes maximum 22 to 24 kgf at the tooth No. 3, thereafter the load of belt tooth decreases, and the rates of decrease are different depending on the pitch difference as shown in Fig. 9. Therefore the occurrence of failure is considered to depend on the interference $\lambda(K)$ of teeth in the transporting process from tooth No. 2 to No. 3. In this case, the difference in the load distribution of belt tooth at the slack side does not affect the failure mode of belt.

Figure 11 shows the load distribution of each belt at $N=10^6$. As the failure is a shear on the belt tooth meshing with the driven pulley, the figure shows the load distribution at the driven pulley side. The tendency of load distribution is not so different from that in Fig. 9, but the load at tight side is about half of that in Fig. 9. The failure is caused in the meshing process from the tooth No. 3 to No. 2 where the load is about 12 kgf.

Approximately equal load distribution is observed in the belt No. 5, $\Delta t = -0.038$ mm, which has the highest fatigue strength.

4.3 Comparison of fatigue strengths of both belts

For the number of cycles to failure under $N=10^6$, the fatigue curves of U belt are steeper than those of CR belt, and the same results are given in the 3rd report. This difference is caused by the failure features. The inner surface of CR belt is covered with the nylon cloth in order to protect the rubber from failure. This decreases the coefficient of friction to half of that for U belt, and the wear characteristics of CR belt are better than those of U belt. However in the high tension range, the interference

Fig. 10 Interference at meshing of driving pulley

No.	Δt mm	T_0 kgf	$T(N_2)$ kgf	E kgf/mm ²	$f(K)$ mm/kgf	μ
1	+0.022	60	5	5,000	0.030	0.3
2	+0.003	75	"	"	"	"
4	-0.018	100	"	"	"	"
5	-0.038	125	"	"	"	"

Fig. 11 Load distribution of polychloroprene rubber belt (for $N=1 \times 10^6$)

at the beginning of meshing becomes very high, and the nylon cloth of internal lining is easy to fracture. When this nylon cloth is fractured, the rubber exposed becomes liable to wear and the fatigue life rapidly decreases. In the range of low tensions over $N > 10^6$, for example $\Delta t = 0$, CR belt can transmit 1.5 to 2 times the tension of U belt.

In the experimental range, the proper pitch difference of CR belt can be $\Delta t = -0.013$ mm. For high tension, the pitch difference $\Delta t = -0.022$ mm yields a good result, but there occurs a wear caused by the interference with the driven pulley and in the low tension range the fatigue strength rapidly decreases. However for $\Delta t = -0.013$ mm at very low tension we can not expect a long fatigue life corresponding to the tension. This is presumed from the facts that the failure at $N = 8 \times 10^7$ is a wear for the tight side tension $T_0 = 64$ kgf, that the tooth surface load at the tooth No. 21 is lowered to only 0.5 kgf for the tight side tension 30 kgf, and the tendency of the fatigue curve and the failure feature of $\Delta t = -0.022$ mm at $N = 10^8$. Therefore at the tight side tension 30 kgf and $N = 10^8$ for $\Delta t = -0.013$ mm, the belt is supposed to fail by wear. In order to avoid a wear failure, it is necessary to make the pitch difference a little smaller from -0.013 mm to 0 mm, to lessen the interference at the beginning of the meshing, and to equalize the load distribution.

Now, we have discussed on the experimental results of L type U belt and CR belt winding on the 36 teeth pulley with friction angle 180° . When both belts are given the tight side and the slack side tensions, the belt which has a high fatigue strength has a pitch difference with a comparatively equal load distribution. In the case of the same pitch difference, for different numbers of the pulley teeth

or for different winding angles, the belts have the same tendency of the load distribution, therefore, the most proper pitch difference is not likely to be affected by the number of pulley teeth or the winding angle. Also the slack side tension should be as small as possible, because of the effect on the bearing of pulley. Considering from the view point of failure feature, we can obtain good results in the case of smooth load distribution and in the case when the wear appears on the belt tooth in the running direction. As a problem in future, we should take up and study the interference at the beginning and end of meshing of belt and pulley, which is a factor affecting the fatigue life of belt.

5. Conclusion

In this report, we state the experimental results about the effect of the pitch difference of belt teeth and pulley teeth on a polyurethane belt and a polychloroprene belt. If the pitch difference is so selected as to equalize the load distribution, a high fatigue strength can be obtained. Therefore, when we design the pulley, a negative pitch difference can be created by increasing the outer diameter of pulley a little for high tension, and by decreasing it for low tension.

References

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