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Theoretical Analysis on Thermal Behavior of a Disk in Profile Milling*

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In this paper, the influence of heat generated in profile milling on the thermal behavior of a workpiece (a disk) is analyzed theoretically. Applying Duhamel's theorem to the results of an instantaneous heat source problem, the theoretical equations of the thermal behavior for a moving heat source problem are easily derived. This method is confirmed using thermoelastic displacement potential. Comparing the temperature variations observed in an experiment with the theoretical values, the amount of heat input to a disk is estimated. Then, thermal stresses and deformations are computed numerically using actual values of physical constants of the workpiece. The numerical results on deformation are in good agreement with the experimental results, and it is concluded that the theoretical equations obtained are useful for predicting the thermal behavior of a workpiece.

1. Introduction

Recently, automation in machining has been widely developed using computerized numerically controlled machine tools. However, the machining accuracy has not been improved with the advance of automation. One of the major difficulties for this problem is the quantitative analysis of thermal deformation. The thermal deformation is brought about by a complicated combination of the thermal behaviors of the machining system which consists of a machine tool, a tool and a workpiece; therefore, it is difficult to make clear the influence of thermal behaviors of the whole system on the machining accuracy. For this reason, each thermal behavior of a machine tool, a tool and a workpiece has been analyzed independently. Especially, thermal behavior of a workpiece has been usually studied in turning⁽¹⁾ and surface grinding⁽²⁾, and there are no studies in profile milling.

In this paper, the influence of heat generated in profile milling on the thermal behavior of a workpiece (a disk) is analyzed theoretically. If the quantity of thermal deformation of the workpiece were predicted numerically, the tool path could be compensated by a computerized numerically controlled milling machine. This paper deals with a fundamental study to accomplish this purpose. Firstly, theoret-

ical equations of the thermal behavior of a disk due to a moving heat source are obtained. Takeuti et al.⁽³⁾ have analyzed the thermal stresses in a disk due to a moving surface heat source. However, the solution and results are complicated, and it is difficult to obtain equations of thermal displacement using these results. Hsu⁽⁴⁾ has analyzed thermal displacements on a disk due to an immovable continuous heat source using Laplace transformation. This solution is also complicated. In this paper, applying Duhamel's theorem to the results of an instantaneous heat source problem, theoretical equations of the thermal behavior for a moving heat source problem are easily derived. This method is confirmed using thermoelastic displacement potential. Subsequently, comparing the temperature variations observed in an experiment with the theoretical values, the amount of heat input to a disk is estimated. Then, thermal stresses and deformations are computed numerically using actual values of physical constants of the workpiece. Appropriateness of the numerical results on the thermal deformation is verified by comparing with the experimental results.

2. Nomenclature

a : radius of a disk
 l : thickness of a disk
 ρ : density of disk material
 c : specific heat of disk material
 E : modulus of elasticity
 ν : Poisson's ratio
 μ : Lamè's constant $= E/2(1+\nu)$
 α : coefficient of thermal expansion
 K : thermal conductivity of disk material
 κ : thermal diffusivity $= K/c\rho$

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η : surface coefficient of heat transfer
 h : $=\eta/K$
 Q : heat input to a disk
 S : strength of heat source $=Q/cpl$
 ω : angular velocity of heat source
 J_n : Bessel function of order n
 r : radial distance in cylindrical coordinate
 θ : angular distance in cylindrical coordinate
 t : time

Fig.1 Coordinate system

3. Analysis

The model used for analysis is a disk, as shown in Fig.1, which is insulated on both flat faces. The initial temperature of this disk is set at 0°C , and it is assumed that there is radiation from its side to the medium of 0°C . The temperature distribution in this model due to an instantaneous line heat source of length l

and strength S , which appears at the point $P_0(a, 0)$ at time $t=0$, is written as⁽⁵⁾⁽⁶⁾

$$\hat{T} = \frac{S}{\pi} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\epsilon_n p_{nm}^2 J_n(p_{nm}r) e^{-\epsilon_n p_{nm}^2 t} \cos n\theta}{[a^2(p_{nm}^2 + h^2) - n^2] J_n(p_{nm}a)} \dots (1)$$

where $\epsilon_0=1$, $\epsilon_1=\epsilon_2=\dots=2$, and p_{nm} denotes the m -th positive root of the equation

$$p_{nm} a J_{n+1}(p_{nm} a) - (ah + n) J_n(p_{nm} a) = 0 \dots (2)$$

The stress and displacement at the point P , for example $\hat{\sigma}_{rr}$ and $\hat{u}_r$, are given as⁽⁶⁾

$$\hat{\sigma}_{rr} = \frac{2\mu m_0 S}{\pi} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\epsilon_n e^{-\epsilon_n p_{nm}^2 t} \cos n\theta}{[a^2(p_{nm}^2 + h^2) - n^2] J_n(p_{nm}a)} \left[\frac{n(1-n)}{r^2} J_n(p_{nm}r) - \frac{p_{nm}}{r} J_{n+1}(p_{nm}r) \right] + 2b_0 + 2b_1 r \cos \theta + \sum_{n=2}^{\infty} \{ n(n-1) A_n r^{n-2} - (n+1)(n-2) b_n r^n \} \cos n\theta \dots (3)$$

$$\hat{u}_r = \frac{m_0 S}{\pi} \frac{\cos \theta}{a(ah+1)} - \frac{m_0 S}{\pi} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\epsilon_n e^{-\epsilon_n p_{nm}^2 t} \cos n\theta}{[a^2(p_{nm}^2 + h^2) - n^2] J_n(p_{nm}a)} \times \left[\frac{n}{r} J_n(p_{nm}r) - p_{nm} J_{n+1}(p_{nm}r) \right] + \frac{1}{E} \left[2(1-\nu) b_0 r + (1-3\nu) b_1 r^2 \cos \theta + \sum_{n=2}^{\infty} \{ n(1+\nu) A_n r^{n+1} - (n+\nu-2+2\nu) b_n r^{n+1} \} \cos n\theta \right] \dots (4)$$

where

$$m_0 = (1+\nu)\alpha \dots (5)$$

$$\left. \begin{aligned} b_0 &= \frac{\mu m_0 S}{\pi} \sum_{m=1}^{\infty} \frac{p_{0m} e^{-\epsilon_n p_{0m}^2 t} J_1(p_{0m}a)}{a^3(p_{0m}^2 + h^2) J_0(p_{0m}a)} \\ b_n &= \frac{2\mu m_0 S}{\pi} \sum_{m=1}^{\infty} \frac{p_{nm} e^{-\epsilon_n p_{nm}^2 t} J_{n+1}(p_{nm}a)}{a^{n+1}[a^2(p_{nm}^2 + h^2) - n^2] J_n(p_{nm}a)}, \quad n \geq 1 \\ A_n &= \frac{2\mu m_0 S}{\pi} \sum_{m=1}^{\infty} \frac{e^{-\epsilon_n p_{nm}^2 t}}{a^n[a^2(p_{nm}^2 + h^2) - n^2]} \left[2 + p_{nm} a \frac{J_{n+1}(p_{nm}a)}{J_n(p_{nm}a)} \right], \quad n \geq 2 \end{aligned} \right\} \dots (6)$$

According to the thermoelasticity theory, the solutions of an instantaneous heat source problem can be written as Green's functions. Therefore, Duhamel's theorem can be applied to a continuous heat source problem. This method has been applied to analyzing the temperature distribution⁽³⁾, but nobody has ever applied it to analyzing the stress, etc. Here, this method is confirmed using thermoelastic displacement potential. Considering the purpose of this paper, a disk as shown in Fig.1 is analyzed.

Suppose the temperature distribution due to an instantaneous line heat source of strength S , which appears at the point P_0 , is $\hat{T}$. Then the thermoelastic displacement potential $\hat{\phi}$ is given as⁽⁷⁾

$$\hat{\phi}(r, \theta, t) = m_0 \kappa \int_0^t \hat{T}(r, \theta, t') dt' \dots (7)$$

We assume that a line heat source of strength $S dt$ moves around the periphery of the disk from the starting point P_0 with constant angular velocity ω . Let the temperature distribution of the disk and the thermoelastic displacement potential in this case be denoted as T_ω , ϕ , respectively, and we have

$$\hat{\phi}(r, \theta, t) = m_0 \kappa \int_0^t T_\omega(r, \theta, t') dt' \dots (8)$$

By Duhamel's theorem⁽³⁾, we have

$$T_\omega(r, \theta, t) = \int_0^t \hat{T}(r, \theta - \omega t', t - t') dt', \dots (9)$$

therefore, substituting Eq.(9) in Eq.(8), we obtain

$$\begin{aligned} \phi(r, \theta, t) &= m_0 \kappa \int_0^t \int_0^{t_1} \hat{T}(r, \theta - \omega t_1, t_2 - t_1) dt_2 dt_1 \\ &= m_0 \kappa \int_0^t \int_{t_1}^t \hat{T}(r, \theta - \omega t_1, t_2 - t_1) dt_2 dt_1 \\ &= \int_0^t \left[m_0 \kappa \int_0^{t-t_1} \hat{T}(r, \theta - \omega t_1, t_0) dt_0 \right] dt_1 \end{aligned}$$

Consequently, from Eq.(7), we obtain

$$\phi(r, \theta, t) = \int_0^t \hat{\phi}(r, \theta - \omega t_1, t - t_1) dt_1 \dots (10)$$

Finally, the components of stress and displacement are given as

$$\begin{aligned} u_r &= \frac{\partial \phi}{\partial r} = \frac{\partial}{\partial r} \int_0^t \hat{\phi}(r, \theta - \omega t_1, t - t_1) dt_1 \\ &= \int_0^t \frac{\partial}{\partial r} \hat{\phi}(r, \theta - \omega t_1, t - t_1) dt_1 \\ &= \int_0^t \hat{u}_r(r, \theta - \omega t_1, t - t_1) dt_1 \dots (11) \end{aligned}$$

and so on. When the angular velocity ω is zero, the above equations represent the case when an immovable continuous line heat source is applied to the fixed point P_0 . Therefore, in general the components of the stress and displacement of a disk due to a continuous heat source can be obtained as

$$\sigma_{ij} = \int_0^t \dot{\sigma}_{ij}(r, \theta - \omega t', t - t') dt', \quad i, j = r, \theta \dots \dots \dots (12)$$

$$u_i = \int_0^t \dot{u}_i(r, \theta - \omega t', t - t') dt', \quad i = r, \theta \dots \dots \dots (13)$$

From Eq.(10), this method can be applied to the thermoelastic displacement potential as well. Actually the function Φ obtained by Hsu⁽⁴⁾ using Laplace transformation can be obtained easily by substituting the result in Ref.(6) into Eq.(10).

If a surface heat source of arc length $2a\beta$ (2β denotes central angle) moves around the periphery of a disk with

constant angular velocity ω starting from the point P_0 , the stress and displacement of the disk can be obtained using Eqs.(12) and (13) as

$$\bar{\sigma}_{ij} = \int_{\theta_0-\beta}^{\theta_0+\beta} a \sigma_{ij}(r, \theta - \theta_0, t) d\theta_0 \dots \dots \dots (14)$$

$$\bar{u}_i = \int_{\theta_0-\beta}^{\theta_0+\beta} a u_i(r, \theta - \theta_0, t) d\theta_0 \dots \dots \dots (15)$$

As mentioned above, by substituting Eqs.(1),(3),(4) etc. into Eqs.(9),(12) and (13) respectively, the temperature and components of stress and displacement are obtained as

$$T_n = \frac{S}{\pi} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\varepsilon_n p_{nm}^2 C_{nm} J_n(p_{nm} r)}{[a^2(p_{nm}^2 + h^2) - n^2] J_n(p_{nm} a)} \dots \dots \dots (16)$$

$$\sigma_{rr} = \frac{2\mu m_0 S}{\pi} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\varepsilon_n C_{nm}}{[a^2(p_{nm}^2 + h^2) - n^2] J_n(p_{nm} a)} \left\{ \frac{n(1-n)}{r^2} J_n(p_{nm} r) - \frac{p_{nm}}{r} J_{n+1}(p_{nm} r) \right\} \\ + 2b_0' + 2b_1' r + \sum_{n=2}^{\infty} \{n(n-1) A_n' r^{n-2} - (n+1)(n-2) b_n' r^n\} \dots \dots \dots (17)$$

$$\sigma_{\theta\theta} = \frac{2\mu m_0 S}{\pi} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\varepsilon_n C_{nm}}{[a^2(p_{nm}^2 + h^2) - n^2] J_n(p_{nm} a)} \left\{ \left[\frac{n(n-1)}{r^2} - p_{nm}^2 \right] J_n(p_{nm} r) \right. \\ \left. + \frac{p_{nm}}{r} J_{n+1}(p_{nm} r) \right\} + 2b_0' + 6b_1' r + \sum_{n=2}^{\infty} \{n(1-n) A_n' r^{n-2} + (n+1)(n+2) b_n' r^n\} \dots \dots \dots (18)$$

$$\sigma_{r\theta} = \frac{2\mu m_0 S}{\pi} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\varepsilon_n D_{nm}}{[a^2(p_{nm}^2 + h^2) - n^2] J_n(p_{nm} a)} \left\{ \frac{n(n-1)}{r^2} J_n(p_{nm} r) - \frac{n p_{nm}}{r} J_{n+1}(p_{nm} r) \right\} \\ + 2b_1' r + \sum_{n=2}^{\infty} \{n(1-n) A_n' r^{n-2} + n(n+1) b_n' r^n\} \dots \dots \dots (19)$$

$$u_r = \frac{m_0 S}{\pi} \frac{\sin \theta - \sin(\theta - \omega t)}{\omega a(h+1)} - \frac{m_0 S}{\pi} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\varepsilon_n C_{nm}}{[a^2(p_{nm}^2 + h^2) - n^2] J_n(p_{nm} a)} \\ \times \left[\frac{n}{r} J_n(p_{nm} r) - p_{nm} J_{n+1}(p_{nm} r) \right] + \frac{1}{E} \left[2(1-\nu) b_0' r + (1-3\nu) b_1' r^2 \right. \\ \left. + \sum_{n=2}^{\infty} \{n(1+\nu) A_n' r^{n-1} - (n+2\nu-2+2\nu) b_n' r^{n+1}\} \right] \dots \dots \dots (20)$$

$$u_\theta = \frac{m_0 S}{\pi} \frac{\cos \theta - \cos(\theta - \omega t)}{\omega a(h+1)} + \frac{m_0 S}{\pi} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{\varepsilon_n D_{nm}}{[a^2(p_{nm}^2 + h^2) - n^2] J_n(p_{nm} a)} \frac{n}{r} J_n(p_{nm} r) \\ + \frac{1}{E} \left[(5+\nu) b_1' r^2 + \sum_{n=2}^{\infty} \{-n(1+\nu) A_n' r^{n-1} + (n+2\nu+4) b_n' r^{n+1}\} \right] \dots \dots \dots (21)$$

where

$$\left. \begin{aligned} b_0' &= \frac{\mu m_0 S}{\pi \kappa} \sum_{m=1}^{\infty} \frac{1 - e^{-\varepsilon p_{0m}^2 t}}{a^2 p_{0m} (p_{0m}^2 + h^2)} \frac{J_1(p_{0m} a)}{J_0(p_{0m} a)} \\ b_n' &= \frac{2\mu m_0 S}{\pi} \sum_{m=1}^{\infty} \frac{p_{nm} C_{nm}}{a^{n+1} [a^2(p_{nm}^2 + h^2) - n^2]} \frac{J_{n+1}(p_{nm} a)}{J_n(p_{nm} a)}, \quad n \geq 1 \\ b_n'' &= \frac{2\mu m_0 S}{\pi} \sum_{m=1}^{\infty} \frac{p_{nm} D_{nm}}{a^{n+1} [a^2(p_{nm}^2 + h^2) - n^2]} \frac{J_{n+1}(p_{nm} a)}{J_n(p_{nm} a)}, \quad n \geq 1 \\ A_n' &= \frac{2\mu m_0 S}{\pi} \sum_{m=1}^{\infty} \frac{C_{nm}}{a^n [a^2(p_{nm}^2 + h^2) - n^2]} \left[2 + p_{nm} a \frac{J_{n+1}(p_{nm} a)}{J_n(p_{nm} a)} \right], \quad n \geq 2 \\ A_n'' &= \frac{2\mu m_0 S}{\pi} \sum_{m=1}^{\infty} \frac{D_{nm}}{a^n [a^2(p_{nm}^2 + h^2) - n^2]} \left[2 + p_{nm} a \frac{J_{n+1}(p_{nm} a)}{J_n(p_{nm} a)} \right], \quad n \geq 2 \end{aligned} \right\} \dots \dots \dots (22)$$

and

$$\left. \begin{aligned} C_{nm} &= \frac{1}{\kappa^2 p_{nm}^4 + n^2 \omega^2} [\kappa p_{nm}^2 \cos n(\theta - \omega t) - n \omega \sin n(\theta - \omega t) + e^{-\varepsilon p_{nm}^2 t} (n \omega \sin n\theta - \kappa p_{nm}^2 \cos n\theta)] \\ D_{nm} &= \frac{1}{\kappa^2 p_{nm}^4 + n^2 \omega^2} [\kappa p_{nm}^2 \sin n(\theta - \omega t) + n \omega \cos n(\theta - \omega t) - e^{-\varepsilon p_{nm}^2 t} (n \omega \cos n\theta + \kappa p_{nm}^2 \sin n\theta)] \end{aligned} \right\} \dots \dots \dots (23)$$

4. Estimation of heat input Q in machining experiments

While a disk of radius 65 mm and thickness 10 mm was being worked by profile milling under the conditions shown in Table 1, the temperature at the points P_1 , P_2 , and P_3 in Fig.2 was measured using CC thermocouples. The point P_0 is the starting point. The measured results are shown in Fig.3. On the other hand, the temperature variations at these points were computed using Eq.(16) under the unit heat input. Then, comparing the computational results with the experimental results, heat input Q was estimated. The parameter values used for numerical computations are

shown in Table 2. As the result, the heat input Q is estimated as 3.5 to 4.5 cal/s as shown in Fig.3. Therefore we postulate that $Q=4.0$ cal/s for the following numerical computations.

Table 1 Working conditions

machine tool	NC milling machine
tool	end milling cutter with two teeth ($\phi 20$ mm)
cutting speed	21400 mm/min
depth of cut	0.75 mm
feed rate	100 mm/min
width of cut	10 mm
workpiece	S35C
material

Fig.2 Measured points of temperature

Table 2 Parameter values used for computations

radius of a disk	$a = 6.50 \times 10$ mm
thickness of a disk	$l = 1.00 \times 10$ mm
density	$\rho = 7.86 \times 10^{-6}$ kg/mm ³
specific heat	$c = 1.14 \times 10^{-1}$ kcal/kg·°C
modulus of elasticity	$E = 2.10 \times 10^4$ kg/mm ²
Poisson's ratio	$\nu = 3.00 \times 10^{-1}$
coefficient of thermal expansion	$\alpha = 1.12 \times 10^{-5}$ 1/°C
thermal conductivity	$K = 1.39 \times 10^{-5}$ kcal/mm·s·°C
thermal diffusivity	$\kappa = 1.55 \times 10$ mm ² /s
heat transfer coefficient	$\eta = 3.00 \times 10^{-9}$ kcal/mm ² ·s·°C
angular velocity of heat source	$\omega = 2.56 \times 10^{-2}$ 1/s

Fig.3 Temperature variation of the workpiece

In Fig.3, the difference between the experimental results and the computational results is increased gradually after the tool passes the measured points. The interpretation for this phenomenon is that a

part of heat input is lost in warming up the support, which is clamped with a vise, located at the center of the disk, and there occurs a heat outflow from this support to the machine tool.

5. Numerical computations and discussion

For simplicity, the following dimensionless quantities will be used for the following numerical computations.

$$\left. \begin{aligned} \bar{T}_w &= \frac{T_w}{\theta_1}, \quad \bar{\sigma}_{rr} = \frac{\sigma_{rr}}{\theta_2}, \quad \bar{\sigma}_{\theta\theta} = \frac{\sigma_{\theta\theta}}{\theta_2} \\ \bar{\sigma}_{r\theta} &= \frac{\sigma_{r\theta}}{\theta_2}, \quad \bar{u}_r = \frac{u_r}{d}, \quad \bar{u}_\theta = \frac{u_\theta}{d} \\ \bar{\gamma} &= \frac{\gamma}{a}, \quad \alpha_{nn} = p_{nn}a, \quad \bar{\tau} = \frac{\kappa t}{a^2} \\ H &= ah, \quad \omega_d = \frac{\omega a^2}{\kappa} \end{aligned} \right\} \dots\dots(24)$$

where

$$\theta_1 = \frac{S}{\pi \kappa}, \quad \theta_2 = \frac{\mu m_0 S}{\pi \kappa}, \quad d = \frac{m_0 S a}{\pi \kappa}, \quad D = \frac{\mu}{E}. \quad (25)$$

Thermal behavior of a disk, which is assumed to be worked by profile milling under the same conditions as in previous section, is computed numerically using Eqs. (16) through (21). The combination of $n=0, 1, 2, \dots, 30$ and $m=1, 2, \dots, 20$ is used for

Fig.4 Temperature distribution

Fig.5 Stress distribution σ_{rr} Fig.6 Stress distribution $\sigma_{\theta\theta}$

the double series. The computations are performed by the computer NEAC2200/700 in the Computation Center of Osaka University.

Fig.4 illustrates the temperature distribution on the diameters located at $\theta=0, \pi$ and $\theta=\pi/2, 3\pi/2$ on the disk, where the position of the tool is varied around the periphery. The temperature variation near the tool position is enormous, while the temperature at the center rises slowly. Figs.5, 6 and 7 show the stress distributions along the diameters $\theta=0, \pi$ and $\theta=\pi/2, 3\pi/2$ in relation to the tool position. The patterns of these stress distributions are

uniform after the tool passes the position at $\pi/2$. Therefore, if the axes of coordinates were fixed at the heat source, the stress distributions would be almost constant independently of time. The stress is approximately larger in order of $\sigma_{r\theta}$, σ_{rr} and $\sigma_{\theta\theta}$; especially $\sigma_{\theta\theta}$ is fairly large in the neighborhood of the tool position. Fig. 8 illustrates the thermal deformations of the axes $\theta=0, \pi$ and $\theta=\pi/2, 3\pi/2$ in relation to the tool position. The thermal deformation ΔR is considered as

$$\Delta R(r, \theta, t) = u_r(r, \theta, t) - u_r(0, \theta, t) \quad (26)$$

where $u_r(0, \theta, t)$ is obtained in the limit of $r \rightarrow 0$ in Eq.(20) as

$$u_r(0, \theta, t) = \frac{m_0 S}{\pi} \frac{\sin \theta - \sin(\theta - \omega t)}{\omega a(a h + 1)} - \frac{m_0 S}{\pi} \sum_{n=1}^{\infty} \frac{p_{1n} C_{1n}}{[a^2(p_{1n}^2 + h^2) - 1] J_1(p_{1n} a)} \quad (27)$$

Therefore, we obtain

$$\begin{aligned} \Delta R = & \frac{m_0 S}{\pi \kappa} \sum_{n=1}^{\infty} \frac{(1 - e^{-p_{0n}^2 t}) J_1(p_{0n} r)}{a^2 p_{0n} (p_{0n}^2 + h^2) J_0(p_{0n} a)} \\ & - \frac{m_0 S}{\pi \kappa} \sum_{n=1}^{\infty} \frac{C_{1n}}{[a^2(p_{1n}^2 + h^2) - 1] J_1(p_{1n} a)} \left[\frac{2}{r} J_1(p_{1n} r) - 2 p_{1n} J_2(p_{1n} r) - p_{1n} \right] \\ & - \frac{2 m_0 S}{\pi} \sum_{n=2}^{\infty} \sum_{m=1}^{\infty} \frac{C_{nm}}{[a^2(p_{nm}^2 + h^2) - n^2] J_n(p_{nm} a)} \left[\frac{n}{r} J_n(p_{nm} r) - p_{nm} J_{n+1}(p_{nm} r) \right] \\ & + \frac{1}{E} \left[2(1-\nu) b_0 r + (1-3\nu) b_1 r^2 + \sum_{n=2}^{\infty} \{ n(1+\nu) A_n r^{n-1} - (n+n\nu-2+2\nu) b_n r^{n+1} \} \right] \quad (28) \end{aligned}$$

Fig.7 Stress distribution $\sigma_{r\theta}$

Fig.8 Variation of thermal deformation

Since the dimensionless quantity H is small in this case, the deformation of every axis increases monotonously in Fig.8. Fig.9 shows the thermal deformation of the disk at the tool position on the periphery. The maximum deformation of the disk in this experiment is estimated as $8.1 \mu\text{m}$. The deviation from the circularity was measured by Talyrond after profile milling, and the maximum thermal deformation of $7-9 \mu\text{m}$ was observed. This result indicates that the prediction of the thermal behavior using the theoretical equations in Section 3 is relatively reliable. Figs.10 and 11 illustrate the influence of the dimensionless quantities H and ω_d on the thermal deformation, respectively, where $\Delta \bar{R}$ denotes the dimensionless quantity given as $\Delta \bar{R} = \Delta R/d$.

6. Conclusions

The thermal behavior of a disk in profile milling is analyzed theoretically. The results obtained can be summarized as follows:

(1) Applying Duhamel's theorem to the results of an instantaneous heat source problem, theoretical equations of thermal behavior for a continuous heat source

Fig.9 Thermal deformation at the tool position

Fig.10 Influence of H on the thermal deformation ($\omega_d = \pi$, $\nu = 1/3$)

Fig.11 Influence of ω_d on the thermal deformation ($H = 0.5$, $\nu = 1/3$)

thermal behavior using the theoretical equations (16) through (21) is relatively reliable.

problem can be easily obtained.

(2) The above method can be also applied to analyzing the thermoelastic displacement potential as well.

(3) The stress and displacement of a disk due to a continuous line heat source can be expressed concisely by Eqs.(17) through (21).

(4) The stress and displacement of a disk due to a continuous surface heat source can be easily obtained as Eqs.(14) and (15).

(5) The thermal stress is approximately larger in order of $\sigma_{r\theta}$, σ_{rr} and $\sigma_{\theta\theta}$; especially $\sigma_{\theta\theta}$ is fairly large in the neighborhood of the tool position.

(6) The maximum deformation of the work in this experiment is estimated as $8.1 \mu\text{m}$. This value is in pretty good agreement with the experimental result. This result shows that the prediction of

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