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A Study on Facilities Location with Tie Set *

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This paper deals with the problem of facilities location, sometimes called plant layout.

Some facilities location algorithms are developed based upon such graphic theory as spanning tree, planar graph and dual graph. In this paper, nodes are defined as facilities and arrows are defined as materials flow, the information of intraplant movement of materials being described by use of oriented graph.

An algorithm, which finds out "the minimum backward materials flow location" is developed to rationalize the intraplant facilities location, on the assumption that the facilities locate in a straight line.

The feature of this algorithm is in the ability to select the optimal solution, only by using easy matrices manipulation.

For verification of it, a trial of this algorithm in regard to 6 actual facilities data is made.

1. Introduction

Production is made up of 3M (Man, Machine, Materials) and Manegement, which supports a systematic combination of 3M. From the point of view of "flow", there are seven "flow" in the production. They are as follows;

- (1) Flow of materials
- (2) Flow of men
- (3) Flow of machines
- (4) Flow of materials and men
- (5) Flow of materials and machines
- (6) Flow of men and machines
- (7) Flow of materials, machines and men

The production has always one flow at least of 3M. The production can not dispense with "flow". Therefore "flow" is a very important thing for consideration on Factory Planning and in particular Facilities location is affected by the nature, the quantity of products, process of manufacturing and various factors. But there are four typical patterns, in the facilities location, as follows;⁽¹⁾

- (1) Fixed location
- (2) Line layout
- (3) Functional location
- (4) Composite location

The greater part of intraplant facilities location is " (4) Composite location ".

A theme of this paper is how to combine the advantages of different patterns of location. There are so many cases in the multi-item production. How to combine effectively the properties of functional

location and line layout ? Is this actually possible ? The flow of materials, which is mentioned above, is handled by many traditional approaches to solve this kind of problem.

In this paper, nodes are defined as facilities and arrows are defined as flows, the information of intra-plant movement of materials being described by use of oriented graph. In this paper is described a new facilities location algorithm using the property of Tie Set matrix, and is made a trial of this algorithm in regard to the actual data.

2. The domain of this paper

The situation of the problem is as follows. It is not good for production to locate the facilities in the form of functional location and line layout, because of the property of product and machines. The type of location (straight-, U-, L-type) is decided by the space and shape of a proposed site. It is no easy matter to decide it, so we discuss about the straight type of facilities location, because it is basic to all others.

The criterion of estimation about the efficiency of facilities location is to minimize the total backward materials flow and make the production smooth.

3. Outline of algorithm

The definitions of principal terms are as follows;⁽²⁾⁽³⁾

Tree	(A_T) : Subgraph taken from arrow of tree from original graph.
Incidence matrix	: Matrix corresponding to node of graph as row and label of arrow as column.
Cut Set	(Q_a) : Set of minimal branches needed for open elimination in divi-

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ding graph into two parts.

Tie Set (Ba) : Set of minimal branches needed for start elimination in dividing graph into two parts.

In this paper, an algorithm which minimizes "backward materials flow" is developed on the assumption that facilities locate in a straight line. The backward materials flow is caused by "Tie Set". It is indispensable condition for nonexistence of backward materials flow that the materials flow have no Tie Set. If you want to select "the minimum backward materials flow location", you have to set the minimum arrow which is a part of the Tie Set backward.

We give an outline of algorithm using a five facilities example.

Fig.1 Oriented graph for materials flow

Fig.1 shows an oriented graph of materials flow among five machines (M_i , $i = 1 \sim 5$). This graph has one Tie Set ($M_1 \rightarrow M_2 \rightarrow M_3 \rightarrow M_4 \rightarrow M_5 \rightarrow M_1$). The whole of isomorphic graph of Fig.1 always has a Tie Set, in other words, a backward materials flow.

Fig.2 Straight type facilities location

Fig.2 described one of the straight types of location for Fig.1. There is a backward materials flow ($M_5 \rightarrow M_1$). In this case, we can not locate facilities without a backward materials flow.

4. Mathematical preparation ^{(4)~(6)}

Nodes are defined as facilities and arrows are defined as materials flow, while the information of materials flow is described as oriented graph (Fig.1). But this is too difficult to calculate, so it is described as matrix.

Here, the mathematical process from the From-to chart (Fig.3), which is the information of materials flow among facilities, to the Tie Set matrix is shown.

In Fig.3, "0" means that a materials flow does not exist, and "1" means that a materials flow exists.

	M_1	M_2	M_3	M_4	M_5
M_1		1	0	1	0
M_2	0		1	0	1
M_3	0	0		1	0
M_4	0	0	0		0
M_5	1	0	1	0	

Fig.3 From-to chart

Fig.4 Labeled oriented graph

The oriented graph (Fig.4) is made up of a materials flow which is described on Fig.3.

$$A_a = \begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline M_1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 \\ M_2 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ M_3 & 0 & 0 & 0 & 0 & -1 & -1 & 1 \\ M_4 & 0 & 0 & -1 & 0 & 0 & 0 & -1 \\ M_5 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \end{array}$$

Fig.5 Incidence matrix (A_a)

An incidence matrix (Fig.5) is made of labels, which are seen in Fig.4. The sign (+, -) depends on the direction of arrows. No.1 arrow described the materials flow from M_1 to M_2 , and so on. An arbitrary row of incidence matrices has a subordinated relation to other rows. So change the incidence matrix into a reduced incidence matrix. Reduced incidence matrix is made of a set of arbitrary Tree parts, (A_t) which has a degree of regular matrix, and a Co-tree part (A_l). The reduced incidence matrix involves same information of the incidence matrix, and it is changed to the form $A = A_t, A_l$ by replacement of row.

The set of arrows [2, 4, 6, 7] is selected as "Tree" in Fig.6.

$$A = (A_t \quad A_l) = \begin{array}{c|ccccccc} & 2 & 4 & 6 & 7 & 1 & 3 & 5 \\ \hline M_1 & -1 & 0 & 0 & 0 & 1 & 1 & 0 \\ M_2 & 0 & 1 & 0 & 0 & -1 & 0 & 1 \\ M_3 & 0 & 0 & -1 & 1 & 0 & 0 & -1 \\ M_4 & 0 & 0 & 0 & -1 & 0 & -1 & 0 \end{array}$$

Fig.6 Reduced incidence matrix (A)

$$Q_f = [U \ Q_p] = \begin{bmatrix} 2 & 4 & 6 & 7 & 1 & 3 & 5 \\ 1 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Fig.7 Fundamental Cut Set matrix (Q_a)

Fundamental Cut Set matrix (Fig.7) is calculated by use of the reduced incidence matrix and Eq.(1).

$$Q_f = [U, Q_p] \quad Q_p = A_t^{-1} \cdot A_1 \\ \therefore Q_f = [U, (A_t^{-1} \cdot A_1)] \quad \text{---(1)}$$

Eq.(2) is conducted by right crossing theorem about Cut Set (Q_a) and Tie Set (B_a).

$$\begin{aligned} Q_a \cdot B_a &= 0 & B_a \cdot Q_a^t &= 0 \\ A_a \cdot B_a^t &= 0 & B_a \cdot A_a^t &= 0 \end{aligned} \quad \text{---(2)}$$

Fundamental Tie Set (B_f) and Tie Set (B_a) can be calculated by use of Eq.(2), as follows;

$$\begin{aligned} Q_f \cdot B_f^t &= 0 \\ [U, Q_f] \cdot [B_p, U] &= 0 \\ Q_p &= -B_p^t \\ A \cdot B_p^t &= 0 \\ [A_t, A_1] \cdot [B_p, U]^t &= 0 \\ B_p^t &= -A_t^{-1} \cdot A_1 \\ Q_p &= A_t^{-1} \cdot A_1 = -B_p^t \end{aligned} \quad (3)$$

Fundamental Tie Set (B_f) is $[B_p, U]$, which is shown in Fig.8. And the Tie Set is shown in Fig.9.

$$B_f = \begin{bmatrix} 2 & 4 & 6 & 7 & 1 & 3 & 5 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & -1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Fig.8 Fundamental Tie Set matrix (B_f)

$$B_a = \begin{array}{c|ccccccc} & 2 & 4 & 6 & 7 & 1 & 3 & 5 \\ \hline \text{Tie-Set} & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & -1 & -1 & 0 & 1 & 0 \\ 3 & 0 & -1 & -1 & 0 & 0 & 0 & 1 \\ 4 & 0 & 1 & 1 & 1 & 1 & -1 & 0 \\ 5 & 1 & 0 & 1 & 0 & 1 & 0 & -1 \\ 6 & 1 & 1 & 0 & 1 & 0 & 1 & -1 \\ 7 & 0 & 0 & 0 & 1 & 1 & -1 & 1 \end{array}$$

Fig.9 Tie Set matrix (B_a)

All Tie Sets (B_a) are given by above-mentioned procedure, only with use of the From-to chart information. If there is

any Tie Set in the oriented graph, we can not locate facilities without a backward materials flow.

5. Outline flow of developed algorithm

On the basis of mathematical preparation, we develop a facilities location algorithm, using the property of " Tie Set " to desire a solution to minimize the backward materials movement.

The outline of developed algorithm is as follows;

- Step 1. Grasp the materials flow among facilities
Grasp the materials flow among facilities in the form of a From-to chart.
- Step 2. Make an incidence matrix (A_a)
Change the materials flow information which is in the form of a From-to chart into incidence matrix (A_a).
- Step 3. Make a reduced incidence matrix (A)
To eliminate an arbitrary row from the incidence matrix (A_a), we make a reduced incidence matrix (A).
- Step 4. Change the incidence matrix into the form of $A = A_t, A_1$
Separate the incidence matrix into Tree part (A_t) and Co-tree part (A_1). And make a new matrix in the form of $A = A_t, A_1$.
- Step 5. Make a fundamental Tie Set (B_f)
Calculate the fundamental Cut Set by use of Eq.(1). Using the right crossing theorem about Cut Set (Q_a) and Tie Set (B_a), Eq.(2) and Eq.(3), calculate the fundamental Tie Set (B_f).
- Step 6. Make a Tie Set (B_a)
Make a Tie Set (B_a) using a combination of fundamental Tie Set (B_f).
- Step 7. Select the existing Tie Set (E)
Check the sign (+, -) of every Tie Set (B_a), which is calculated by " Step 6 ", and select the existing Tie Set (E).
- Step 8. Select arrows which eliminate the existing Tie Set (E)
Considering one Tie Set, if we eliminate an arbitrary arrow, this Tie Set will be gone and the backward materials flow will be gone, too. We continue this step for every existing Tie Set (E).
- Step 9. Make a matrix about eliminating arrows
We describe arrow, which are select by " Step 8 ", in the form of matrix.
Here the term of the eliminating arrow means a backward flow.
- Step 10. Add the quantity information of materials flow
If any Tie Sets and any matrices exist that means we can not allocate facilities without a backward materials flow. And, we have to set the smallest quantity arrow backward. Therefore, in this step, we add the quantity information.
- Step 11. Select a case, which minimizes the total materials flow
Using the information of " Step.10 ", select a case, which minimizes the total

materials flow.

Step 12. Sketch the selected facilities location

Following the information of "Step 11" sketch the selected facilities location which has the smallest backward flow.

Using the above-mentioned 12 steps, we can select a facilities location, which minimizes the backward materials flow, for the straight type of location.

6. Practical selection

For verification, we make a trial of this developed algorithm in regard to the actual data. Take a case of 6 facilities (A~F) problem as an example. An input information about materials flow among facilities is given by a From-to chart (Fig. 10).

	A	B	C	D	E	F
A		9	8	0	0	17
B	0		0	11	2	0
C	3	0		0	14	0
D	10	9	0		0	5
E	16	4	0	0		0
F	0	5	2	0	0	

Fig.10 From-to chart for 6 facilities

Fig.10 is a multi-graph, for instance between facilities A and C, and so on. The multi-graph is complex for matrices manipulation. So it is modified to Fig.11, which considers the backward movement.

	A	B	C	D	E	F
A		9	5	-10	-16	17
B			0	2	-2	-5
C				0	14	-2
D					0	5
E						0
F						

Fig.11 Modified From-to chart

Fig.11 gives renewed input data for the algorithm. The labels of arrows are shown in Fig.12.

Fig.13 is a reduced incidence matrix of Fig.12, following the above mentioned "Steps 2 and 3".

Reduced incidence matrix is transformed into the form of $A = A_t \cdot A_l$ (Step 4). In this example, a set of arrows [a, f, k, j, i] is selected for Tree (A_t), but any Tree will be all right. Then a new reduced incidence matrix is shown in Fig.14.

Fig.12 Labeled oriented graph

$$A =$$

	a	b	c	d	e	f	g	h	i	j	k
A	1	1	-1	-1	1	0	0	0	0	0	0
B	-1	0	0	0	0	1	-1	-1	0	0	0
C	0	-1	0	0	0	0	0	0	1	-1	0
D	0	0	1	0	0	-1	0	0	0	0	1
E	0	0	0	1	0	0	1	0	-1	0	0

Fig.13 Reduced incidence matrix (A)

$$A =$$

$$(A_t \cdot A_l) =$$

	a	f	k	j	i	b	c	d	e	g	h
A	1	0	0	0	0	1	-1	-1	1	0	0
B	-1	1	0	0	0	0	0	0	0	-1	-1
C	0	0	0	-1	1	-1	0	0	0	0	0
D	0	-1	1	0	0	0	1	0	0	0	0
E	0	0	0	0	-1	0	0	1	0	1	0

Fig.14 Reduced incidence matrix, form of $A = A_t \cdot A_l$

Fundamental Tie Set (B_f), which is seen in Fig.15, is calculated by Eq.(1), (2) and the preceding flow.

$$B_f =$$

	a	f	k	j	i	b	c	d	e	g	h
1	1	1	1	1	0	-1	0	0	0	0	0
1	1	1	0	0	0	0	1	0	0	0	0
1	1	1	1	1	1	0	0	1	0	0	0
1	1	1	1	0	0	0	0	0	-1	0	0
0	1	1	1	1	1	0	0	0	0	1	0
0	1	1	1	0	0	0	0	0	0	0	1

Fig.15 Fundamental Tie Set matrix (B_f)

Tie Set (B_a) can be calculated by using the combination of fundamental Tie Set (Step 6). The existing Tie Set (E) which can be selected by checking the sign (+, -), following Step 7, is shown in Fig. 16.

$E =$

	a	f	k	j	i	b	c	d	e	g	h	Tie Set
1	1	1	0	0	0	0	1	0	0	0	0	1
2	1	1	1	1	1	0	0	1	0	0	0	2
3	0	1	1	1	1	0	0	0	0	1	0	3
4	0	1	1	0	0	0	0	0	0	0	1	4
5	0	0	0	0	1	1	0	1	0	0	0	5
6	0	0	1	1	0	0	0	1	1	0	0	6
7	1	1	0	0	1	1	1	1	0	0	0	7
8	0	1	0	0	1	1	1	0	0	1	0	8
9	0	1	0	0	0	0	1	0	1	0	1	9
10	0	1	0	1	0	1	1	1	1	1	0	10
11	0	1	0	0	1	1	1	1	1	0	1	11

Fig.16 Existing Tie Set (E)

Tie Set number, which are given along right section, serve the convenience of explaining. In this example, there are eleven Tie Sets. Therefore, when the facilities are located in a straight type, a backward materials flow is sure to occur. Then go ahead with Steps 8 and 11 to set the arrows backward. The existing Tie Set matrix (E) is calculated as in Fig.16.

Tie Set 7 = Tie Set 1 + Tie Set 5

Tie Set 11 = Tie Set 5 + Tie Set 9

If Tie Sets 1, 5, 9 are checked, it will be necessary to check, Tie Sets 7 and 11, because of overlapping. Checking the overlap of every Tie Set in Fig.16, will leave 9 (1,2,3,4,5,6,8,9,10) Tie Sets. We should handle only nine Tie Sets.

The next step is selection of arrows. Here is given an example about Tie Sets 1, 4, 5. The arrows of them are as follows;

Tie Set 1 = [a, f, c]

Tie Set 4 = [f, k, h]

Tie Set 5 = [i, b, d]

Elimination of these Tie Sets can be done by leaving out an arrow from each of them. the arrows which are left out from Tie Sets will be set backward.

The combinations of arrows for elimination are as follows;

aki, akb, akd, ahi, ahb, ahd,

cki, ckb, ckd, chi, chb, chd,

fi, fb, fd

After eliminating [a, k, i] arrows, there still remain two Tie Sets, which are seen in Fig.17.

	a	f	k	j	i	b	c	d	e	g	h	Tie Set
9	0	1	0	0	0	0	1	0	1	0	1	
10	0	1	0	1	0	1	1	1	1	1	0	

Fig.17 Remaining Tie Sets

The combinations of arrows to eliminate two Tie Sets are as follows;

[f, jh, bh, c, dh, e, gh]

To eliminate Tie Sets 9 and 10, you have to leave out only one combination of arrows from among the combinations [a, k, i] and [f, jh, bh, c, dh, e, gh], which

are described below;

akif, akijh, akibh, akic,

akidh, akigh, akie

If you eliminate one of them, the whole Tie Set will be gone.

Continue the above-mentioned procedure about the other combinations of arrows. Following Step 9, make a matrix of eliminated arrows. Then following Step 10, add the quantity information to every arrow and calculate the total backward materials flow. the result is shown in Fig.18. Following Step 11, select a case, which minimizes backward materials flow.

The smallest one is No.48, which is formed by [f, j, b] arrows. If you set them backward, the total backward materials flows will be 9 (= 2+2+5). And this location is the optimal solution for this example.

movement of materials	a	f	k	j	i	b	c	d	e	g	h	total backward movement
1	V	V	V		V							30
2	V		V	V	V							35
Cut No.												
23	V		V		V	V					V	35
24	V		V		V					V	V	35
25	V				V			V			V	44
26	V				V							
48	V		V		V							9
49	V				V	V		V				23
50	V				V			V				24
51	V							V				19

Fig.18 Set of arrows to eliminate all of Tie Sets

Using the above-mentioned steps, we gain the optimal solution for the facilities location. The information of materials flow is described by a From-to chart (Fig.10) and an oriented graph Fig.12). We should set [f, j, b] arrows in a backward materials flow, which is selected by Step 11.

A sketch of the selected solution is given in Fig.19.

Fig.19 A sketch of the selected solution

In Step 11 (Fig.18), we enumerate all combinations of eliminated arrows, and select the most desirable one. Then we can select the optimal solution to locate facilities in a straight type.

7. Conclusions

In this paper, we develop an algorithm to select the location of minimum backward materials flow, only with use of a From-to chart, which gives the most basis information of materials flow among facilities for location problem.

The developed algorithm is described in " 5. Outline flow of developed algorithm ". And in " 6. Practical selection ", we make a trial of the algorithm on a 6 facilities problem for verification.

The developed algorithm has 12 steps, we can calculate easy matrices manipulation from 1 to 6 steps, as explained in " 4. Mathematical preparation ". After 7 step, we can manipulate easily, too. That is, we have only to check the direction of Tie Set and calculate the total backward materials flow.

A special feature of the developed algorithm is in ability to select the optimal solution.

In " 6. Practical selection ", we handle a problem of 6 facilities, which are different from each other. In the case of selecting plural facilities from same kind of facilities, we regard the same facilities as one node, just like in functional location. Then apply the developed algorithm to select a provisional optimum solution. Second step is to think two more facilities as different nodes from each other. And compare the solutions to select the real optimum one.

We set the assumption that facilities locate in a straight type. For other type locations, L- or U-type for example, we make a straight type location at first and then modify it to a required form.

In future, we will try to develop another algorithm for another objective function, using Cut Set, Hamilton's Cycle and other properties of graphic theory.

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