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Review of the Transient Line Source Technique*

(1st report: Analytical Discussion
on the Influence of Radiation)

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In this paper, the influence of weak radiative heat transfer on measured thermal conductivity by the transient line source technique is discussed. Fundamental equations and non-dimensional parameters (emissivity, ϵ ; Fourier modulus, $t^* = at/(r_1)^2$; optical thickness, $R = \kappa r_1$; non-dimensional radius, $r^* = r/r_1$), which determine the apparent thermal conductivity, λ_{app} , are shown, and the relations between these parameters and radiative heat flux, temperature distribution and λ_{app} are discussed using the results of numerical calculations. The optical condition, $R = 0.01 \sim 0.07$, where the influence of radiation appears maximum for $t^* < 600$, is pointed out. This optical condition corresponds to the measurements of Ccl_4 or toluene for the usual line radius, $r_1 = 12.5 \mu$. As λ_{app} increases rapidly in the range of small t^* ($t^* < 200$), the extrapolation, $t^* \rightarrow 0$, using $\lambda_{app} - t^*$ curves ($t^* < 200$), doesn't give the real thermal conductivity, λ . A method to modify λ_{app} is proposed.

1. Introduction

Transient line heat-source method is one of the most famous techniques to measure thermal conductivities of fluids.⁽¹⁾⁽⁵⁾ Although the method has been tried and improved by many investigators such as C.A. Castro⁽²⁾, J.K. Horrocks⁽³⁾, J.W. Haarman⁽⁴⁾, J.E.S. Venart⁽⁵⁾ and others, it appears that the accuracy of the method still leaves room for improvement by discussing precisely the influences of weak thermal phenomena, which deviate from the basic principle of measurement. The object of this research is to propose a method for modification of an apparent thermal conductivity which includes influences of the above thermal phenomena, and to clarify measuring conditions which reduce the influences. In this paper, as the first report, apparent increase of thermal conductivity under the effect of radiation heat transfer is discussed analytically.

As is generally known, transient line heat-source method determines a thermal conductivity from the transient temperature response of a thin wire after stepwise heating, which is stretched vertically within a sample material, as shown in Fig.1. Defining the symbols r_1 , T , Q , a , t as the radius of the wire, the temperature, the rate of heat supplied to unit length of the wire, the thermal diffusivity of a sample, and the time elapsed after heating respectively, the thermal conductivity, λ_{app} , is determined from the following equation, using measured temperature response of the wire, T , for small $(r_1)^2/at$.

$$\lambda_{app} = Q / \left\{ 4\pi t \left(\frac{dT}{dt} \right)_{r=r_1} \right\} \dots\dots\dots (1)$$

The trend that thermal conductivity is overestimated under the influence of radiation heat transfer in steady parallel-plane method was first pointed out by H. Poltz⁽⁶⁾⁽⁷⁾, and later discussed more precisely by G. Schödel.⁽⁸⁾

Concerning radiation effects in transient line heat-source method, however, neither sufficient discussions nor rational modification have been done, partly because of the difficulty of analysis.

So far as the author knows, N. Mani and J.E.S. Venart⁽⁹⁾ were the first persons who suggested the trend from experimental data, that λ_{app} calculated from Eq.(1) becomes higher than the real value, λ , under the influence of radiation heat transfer in transient line heat-source measurements of materials absorbing IR-radiation. In previous papers^{(10)~(12)}, the author qualitatively clarified the mechanism that the measured thermal conductivity becomes

Fig.1 Temperature field around a line heat source

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higher under the influence of radiation, using integral method. This paper performs precise discussions as follows, taking up a problem of radiation effects, which has been considered "small" and has not been sufficiently discussed:

(1) The basic equation which governs the phenomena, and the equation to calculate radiation heat flux are shown. From these equations, non-dimensional parameters which decide non-dimensional values, $(q_R)^*$, $(T_R)^*$, A (radiation heat flux, deviation of temperature distribution by radiation, apparent increase of thermal conductivity) are revealed.

(2) A method to calculate $(q_R)^*$, $(T_R)^*$ and A from the above non-dimensional parameters is shown, and the results of calculations are discussed.

(3) Concerning A , particularly, characteristic trends of dependencies on the above non-dimensional parameters are precisely discussed, and a method to get the real conductivity, λ , by modifying measured conductivities, λ_{app} , is proposed. Dependencies of the amount of modification on various physical properties are also discussed.

Nomenclature

- a : thermal diffusivity, m^2/h
 n : index of refraction
 Q : rate of heat supplied to unit length of line heat-source, $kcal/mh$
 q : heat flux $= q_c + q_R$ $kcal/m^2h$
 q_c : conduction heat flux, $kcal/m^2h$
 q_R : radiation heat flux, $kcal/m^2h$
 $r, r', r'', \bar{r}'$: co-ordinates of radius (Ref. Figs. 2, 3), m
 r_1 : radius of line heat-source, m
 T : temperature, $^{\circ}K$
 T_c : temperature variation of measured material for $q_R=0$, $^{\circ}K$
 T_0 : initial temperature, $^{\circ}K$
 T_R : deviation of temperature caused by radiation heat transfer, $^{\circ}K$
 t : time, h
 ϵ : emissivity of the surface of line heat-source
 $\theta, \bar{\theta}, \phi$: co-ordinates of angle (Ref. Figs. 2, 3)
 κ : absorption coefficient, m^{-1}
 λ : real thermal conductivity, $kcal/mh^{\circ}C$
 λ_{app} : apparent thermal conductivity, $kcal/mh^{\circ}C$
 σ : Stefan-Boltzmann constant, $kcal/m^2h^{\circ}K^4$

(non-dimensional parameters)

$$\begin{aligned}(q_R)^* &= \pi^2 \lambda q_R / \{4\pi^2 \sigma Q (T_0)^3\} \\ R &= \kappa r_1 \\ r^* &= r/r_1, (r')^* = r'/r_1, (r'')^* = r''/r_1, \\ (\bar{r}')^* &= \bar{r}'/r_1 \\ (T_R)^* &= \pi^2 \lambda^2 T_R / \{4\pi^2 r_1 \sigma Q (T_0)^3\} \\ t^* &= at / (r_1)^2 \\ A &= (\lambda_{app} - \lambda) \pi / \{16 r_1 \pi^2 \sigma (T_0)^3\}\end{aligned}$$

2. Analysis of apparent thermal conductivity which includes influence of radiation

2.1 Deviation of temperature distribution by radiation heat transfer

Radiation heat flux in r -direction, q_R , is defined, considering the case where conduction and radiation exist together around a line heat-source shown in Fig. 1. When the deviation of temperature distribution by radiation is small, and the heat capacity of the line heat-source can be neglected, temperature distribution in the vicinity of the line heat-source is given by the solution of Table 1, which shows the basic equation and initial- and boundary-conditions.

The solution of Table 1 is approximated by the following equation.

$$T - T_0 = T_c(r, t) + T_R(r, t) \dots \dots \dots (6)$$

The first term of the right hand side of Eq. (6) denotes the temperature solution for the case where no radiation exists ($q_R=0$), and the second term shows the effect of radiation (modification term). Substituting Eq. (6) into Eqs. (2)-(5) of Table 1, the basic equation and initial- and boundary-conditions which the modification term, T_R , must satisfy become Eq. (7) and Eqs. (3)-(5)', using non-dimensional parameters, $(T_R)^*$, $(q_R)^*$, r^* and t^* .

Basic equation:

$$\begin{aligned}\frac{1}{r^*} \frac{\partial}{\partial r^*} \left\{ r^* \frac{\partial (T_R)^*}{\partial r^*} \right\} \\ = \frac{\partial (T_R)^*}{\partial t^*} + \frac{1}{r^*} \frac{\partial}{\partial r^*} \{ r^* (q_R)^* \} \dots \dots \dots (7)\end{aligned}$$

Initial conditions:

$$(T_R)^* = 0, (q_R)^* = 0 \quad (t^* = 0, r^* > 1) \dots (3)'$$

Boundary conditions:

$$\begin{cases} (T_R)^* = 0, (q_R)^* = 0 \\ (t^* > 0, r^* \rightarrow \infty) \dots \dots \dots (4)' \\ -\frac{\partial (T_R)^*}{\partial r^*} + (q_R)^* = 0 \\ (t^* > 0, r^* = 1) \dots \dots \dots (5)' \end{cases}$$

On the other hand, radiation heat flux, q_R , can be calculated from temperature distribution, T , when the radius of line heat-source, emissivity and thermal properties of the sample material are known. For example, radiation heat flux at S -surface (unit area perpendicular to r -direction) of Fig. 2 is calculated, integrating throughout the whole temperature field ⁽¹³⁾ to sum up radiation energy which actually arrives on S -surface after absorption through the sample material leaving the surface of the line heat-source or small volume-elements of the sample material, dv . When temperature deviation by radiation heat

+ For $r_1 \rightarrow 0$ T_c of Eq. (6) becomes the well-known solution ⁽¹⁾ :

$$\begin{aligned}T_c &= (-Q/4\pi\lambda) E_3(-r^2/4at) \\ -E_3(-X) &= \int_X^\infty (e^{-u}/u) du\end{aligned}$$

transfer is small ($T_c \gg T_R$), the temperature distribution to calculate q_R is approximated by the following equation, using r'' of Fig.2.

$$T \approx T_0 - \frac{Q}{4\pi\lambda} E_i \left\{ \frac{-(r'')^2}{4at} \right\}$$

Therefore, considering that $T_0 \gg (T - T_0)$ is satisfied in usual measurements, non-dimensional radiation heat flux, $(q_R)^*$, which appears in Eqs.(3)'-(5)' and in Eq.(7) is given by the following equation, if for example, diffuse reflection occurs on the surface of the line heat-source.

$$\begin{aligned} (q_R)^* = & -R \int_0^\pi \int_0^\infty E_i \left[\frac{-(r'')^2}{4t^*} \right] (\cos \theta) \int_0^{\pi/2} (\cos \phi) \exp \{-R(r')^*/\cos \phi\} d\phi d(r')^* d\theta \\ & - R \int_0^\delta \int_0^{(\bar{r}')^*} E_i \left[\frac{-(r'')^2}{4t^*} \right] (\cos \theta) \int_0^{\pi/2} (\cos \phi) \exp \{-R(r')^*/\cos \phi\} d\phi d(r')^* d\theta \\ & - \varepsilon \int_0^\delta E_i \left(\frac{-1}{4t^*} \right) (\cos \theta) \int_0^{\pi/2} (\cos \phi)^2 \exp \{-R(\bar{r}')^*/\cos \phi\} d\phi d\theta \\ & - (1-\varepsilon) \int_0^\delta \frac{4}{\pi} (q_R)^* \int_0^{\pi/2} (\cos \phi)^2 \exp \{-R(\bar{r}')^*/\cos \phi\} d\phi d\theta \dots\dots\dots (8) \end{aligned}$$

where,

$$\begin{aligned} (q_R')^* = & -R \int_{\pi/2}^\pi \int_0^\infty E_i \left[\frac{-(r'')^2}{4t^*} \right]_{r^*=1} (\cos \theta) \int_0^{\pi/2} (\cos \phi) \exp \{-R(r')^*/\cos \phi\} d\phi d(r')^* d\theta \dots\dots\dots (9) \\ \bar{\theta} = & \sin^{-1} \left(\frac{1}{r^*} \right) \quad \{ (r'')^2 = (r^*)^2 + (r')^2 - 2r^*(r')^* (\cos \theta) \} \\ \{ (\bar{r}')^2 = & (r^*)^2 - 2r^*(\bar{r}')^* (\cos \theta) = 1 \} \end{aligned}$$

Fig.2 Co-ordinates system

Table 1 Basic equation and initial- and boundary-conditions

Basic Equation	$\left\{ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) = \frac{1}{a} \frac{\partial T}{\partial t} + \frac{1}{\lambda r} \frac{\partial}{\partial r} (r q_R) \dots\dots (2) \right.$
Initial Condition	$\left\{ T = T_0, q_R = 0 \quad (t=0, r > r_1) \dots\dots\dots (3) \right.$
Boundary Condition	$\left\{ T = T_0, q_R = 0 \quad (t > 0, r \rightarrow \infty) \dots\dots\dots (4) \right.$
Conditions	$\left\{ -\lambda \frac{\partial T}{\partial r} + q_R = \frac{Q}{2\pi r} \quad (t > 0, r = r_1) \dots\dots (5) \right.$

And the sample material is assumed to be grey and non-scattering.

The first and the second terms of Eq. (8) are the radiation term from the sample material. The third term is radiation energy from the line heat-source, and the fourth term is the reflection component at the surface of the line heat-source.

Defining $(q_R)_B^*$ and $(q_R)_M^*$ as the limiting values of $(q_R)^*$ for the cases where emissivity of the line heat-source, ε , becomes $\varepsilon=1$ (black body), or $\varepsilon=0$ respectively, Eq.(8) becomes

$$(q_R)^* = (q_R)_M^* + \varepsilon \{ (q_R)_B^* - (q_R)_M^* \} \dots\dots (10)$$

If the mode of reflection at the surface of the line heat-source is not diffuse, $(q_R)_M^*$ has a different value, of course, but integral expression like Eq.(8) is also possible and Eq.(10) is applicable even for such a case.

Fig.3 Co-ordinates system to calculate $(q_R)^*$

The solution of Eq.(7) for $\epsilon=0\sim 1$ is given by the following equation:

$$(T_R)^* = (T_R)^M + \epsilon \{ (T_R)^B - (T_R)^M \} \dots (11)$$

,where $(T_R)^B$ and $(T_R)^M$ are defined as the solutions of Eq.(7) for $\epsilon=1$ and for $\epsilon=0$, respectively.

2.2 Increase of apparent thermal conductivity by the influence of radiation

When $(r_1)^2/at$ is small, from Eqs.(1) (6),

$$\lambda_{app} = \frac{Q}{4\pi t \{ (\partial T_C / \partial t)_{r=r_1} + (\partial T_R / \partial t)_{r=r_1} \}} \dots (12)$$

The thermal conductivity, λ , for $q_R=0$ ($T_R=0$)
++ is

$$\lambda = \frac{Q}{4\pi t (\partial T_C / \partial t)_{r=r_1}} \dots (13)$$

From Eqs.(12)(13), considering $T_C \gg T_R$,

$$\frac{\lambda_{app}}{\lambda} \doteq 1 - \frac{4\pi\lambda t}{Q} \left(\frac{\partial T_R}{\partial t} \right)_{r=r_1}$$

The above equation is written in non-dimensional form;

$$A = \frac{(\lambda_{app} - \lambda)\pi}{16r_1^2 \sigma (T_0)^3} = - \left\{ \frac{\partial (T_R)^*}{\partial t^*} \right\}_{r^*=1} t^* \dots (14)$$

Defining A_B and A_M as values of A for $\epsilon=1$ and $\epsilon=0$, the following equation is obtained from Eqs.(11)(14).

$$A = A_M + \epsilon(A_B - A_M) \dots (15)$$

Equation(14) is modified for relatively large $(r_1)^2/4at$, and is shown in the supplement. In Eq.(15), A_M depends on the mode of reflection at the surface of the line heat-source.

2.3 Non-dimensional parameters governing the phenomena

As is evident in Eq.(8), $(q_R)^B$ and $(q_R)^M$ are functions of R , r^* and t^* . And from Eqs.(3)-(5)' and Eq.(7), $(T_R)^B$ and $(T_R)^M$ are also functions of R , r^* and t^* . Therefore, from the form of Eq.(14), A_B and A_M of Eq.(15) are decided by giving R and t^* . The process to calculate A_B and A_M is as follows:

- (1) R and t^* are given.
- (2) Substituting $\epsilon=1$ and $\epsilon=0$ into Eq.(8), $(q_R)^B$ and $(q_R)^M$ are calculated as functions of r^* .
- (3) Using Eqs.(3)-(5)' and Eq.(7), solutions of $(T_R)^B$ and $(T_R)^M$ are calculated, for $(q_R)^* = (q_R)^B$ and $(q_R)^* = (q_R)^M$, as functions of r^* .
- (4) From Eq.(14), A_B and A_M are calculated for $(T_R)^* = (T_R)^B$ and $(T_R)^* = (T_R)^M$.

Making preliminary calculations of A_B and A_M for various values of R and t^* by the above process, A for an arbitrary measuring condition is given by Eq.(15), and

++ In this paper, author considers reasonable measuring conditions, and the thermal conductivity, λ , defined by Eq.(13) is treated as the real thermal conductivity in the sense that "it doesn't include radiation effects".

an apparent thermal conductivity, λ_{app} , is easily modified to give the real conductivity, λ , which doesn't include radiation effects.

2.4 A for optically thick ($R \rightarrow \infty$) and thin ($R \rightarrow 0$) limits

When the absorption coefficient of a sample material is very large, putting $R \rightarrow \infty$ in Eqs.(8)(9), $(q_R)^*$ converges to zero; $(q_R)^* \rightarrow 0$. That is, λ_{app} converges to real value, λ , regardless of the surface emissivity, ϵ .

In contrast with this, when the absorption coefficient of a sample material is very small, putting $R \rightarrow 0$ in Eqs.(8)(9), the following equations are obtained.

$$(q_R)^* = -\epsilon \int_0^\theta E_t \left(\frac{-1}{4t^*} \right) (\cos \theta) \times \int_0^{\pi/2} (\cos \phi)^2 d\phi d\theta = -\frac{\pi\epsilon}{4r^*} E_t \left(\frac{-1}{4t^*} \right) \dots (16)$$

For the case, $\epsilon=0$, in Eq.(16), $(q_R)^*=0$ is satisfied, and λ_{app} converges to λ , similarly to the case of $R \rightarrow \infty$.

Generally speaking, when R increases gradually from zero, A gradually increases from the value which corresponds to Eq.(16), and turns to decrease after it reaches the maximum value, and finally becomes zero ($A=0$). This trend also appears in the result of calculations shown in Chapter 3.3.

3. Results of calculation and discussions

3.1 Calculation of $(q_R)^*$

$(q_R)^*$ of Eq.(8) was calculated varying the non-dimensional parameters; R , ϵ and t^* . The first and second terms of Eq.(8) were integrated numerically using small volume-elements, dV' , which are divided by concentric cylindrical surfaces of the line heat-source (r'' ; constant), radial plane surfaces passing the center line (β ; constant) and surfaces perpendicular to the center line (z ; constant). +++ Integration in z -direction was done using a table prepared in advance. Sizes of the volume elements, both in r'' - and β -directions, were decided inequally, so that the elements became finer near the line heat-source. The finest volume element was approximately $0.0012r_1 \times 0.042r_1$.

Concerning the third and fourth terms of Eq.(8), numerical integration was also done dividing the surface of line heat-source into small surface elements.

Time-dependencies of $(q_R)^*$ distribution for $R=0.0076$ are shown in Fig.4 and

+++ The integration in r'' direction was performed in the range $r'' < (r'')_{max}$, where $(r'')_{max} = r_T + r_A$. r_T was defined such that temperature variation of a sample material becomes less than 0.1% of the temperature variation of the line heat-source for $r'' > r_T$. And r_A was the thickness of the sample material through which the intensity of radiation decreases to 1% ($r_A \approx 4.6$). For example, $(r'')_{max} < 1.020r_1$ is obtained for $t^*=0\sim 600$ and $R>0.005$.

Fig.4 Time-dependency of $(q_R)^*$ distribution Fig.5 Time dependency of $(q_R)^*$ distribution Fig.6 Relation between $(q_R)^*$ distribution and ϵ Fig.7 Relation between $(q_R)^*$ distribution and R Fig.8 Relation between $(q_R)^*$ distribution and R

Fig.5. In Fig.4, it is observed that $(q_R)^*$ decreases monotonously with an increasing r^* for small t^* , however, different trends appear for large t^* , that is, as r^* increases, $(q_R)^*$ decreases initially, and gradually turns to increase, and again turns to decrease (the difference of heat energy between plus- and minus-directions of r^* becomes maximum at the place relatively apart from the line heat-source, for large t^*). This is because a large cylindrical space of sample material surrounding the line heat-source is warmed for large t^* and becomes to radiate heat energy more actively, and therefore, radiation energy in minus-direction of r^* increases in a region very close to the line heat-source. The above trends are not so apparent for $R=0.0076$ and $\epsilon=1.0$ (Fig.5), because radiation energy from the surface of line heat-source is large in this case.

Figure 6 shows the relation between $(q_R)^*$ distribution and ϵ , for $R=0.044$ and $t^*=500$. Equation (10) is satisfied in Fig.6. When r^* is large, $(q_R)^*$ is not influenced by ϵ . This critical value of r^* seems to decrease, as R increases. The distributions of $(q_R)^*$ for $\epsilon=0.0$ and $\epsilon=1.0$ [$(q_R)^*_M, (q_R)^*_B$] are shown in Fig.7 and Fig.8. Using Figs.7 and 8, $(q_R)^*$ for $\epsilon=0 \sim 1$ is calculated by Eq.(10). In Fig.7, the above mentioned trend, that $(q_R)^*$ becomes maximum at the place apart from the line heat-source, is observed. It is clear in Fig.8, that, for the case of $R \rightarrow 0$, radiation of heat from the sample material converges to zero, and $(q_R)^*$ decreases monotonously with an increasing r^* . And the difference between $(q_R)^*_M$ and $(q_R)^*_B$ becomes smaller, as R increases.

3.2 Calculation of $(T_R)^*$

Solutions for Eq.(7) and Eqs.(3)'-(5)' were calculated by the finite difference method of the explicit type⁽¹⁴⁾, using $(q_R)^*$ calculated by the method shown in Chapter 3.1. In the finite difference calculation, temperature field was divided inequally by concentric cylindrical surfaces of the line heat-source, so that the mesh size becomes finer near the line heat-source. The finest mesh size was approximately $\Delta r = (1/4)r_1$.

Fig.9 Time-dependency of $(T_R)^*$ distribution

Fig.10 Relation between $(T_R)^*$ distribution and R

Fig.11 Time-dependency of $(T_R)^*_{r*=1}$

Fig.12 Time-dependency of A

Results of calculations are shown, for example, in Fig.9 and Fig.10. It is understood from Fig.9, how heat is transferred by radiation from the vicinity of the line heat-source to the surrounding slightly distant place (heat is transferred to the place around $r^*=100$, in the case of Fig.9). It is also clear in Fig.10, that the absolute value of $(T_R)^*$ ($=|(T_R)^*|$) increases gradually, when R increases from zero, but when R becomes larger, the absolute value turns to decrease to the contrary, and finally converges to zero, for $R \rightarrow \infty$. Time-dependency of $(T_R)^*_{r*=1}$ (the value of $(T_R)^*$ for the line heat-source) is shown in Fig.11.

3.3 Calculation of A

A is calculated by Eq.(14), using $(T_R)^*$ obtained in Chapter 3.2. A_M and A_B , which correspond to $\epsilon=0$ and 1.0 , respectively, are shown in Figs.12(A)(B). Because A_M and A_B show steep variation in the range of small t^* ($t^* < 200$), extrapolation of A_M and A_B to $t^* \rightarrow 0$, using values for $t^* > 200$, does not converge to zero, but to a positive value. This fact means that in the line heat-source method, the usual extrapolation technique ($t \rightarrow 0$) for measured apparent thermal conductivities, λ_{app} , gives a conductivity higher than the real conductivity, and cannot take off the influence of radiation sufficiently. This trend becomes obvious for large R . The mechanism which causes this characteristic trend was explained in former reports⁽¹⁰⁾⁽¹¹⁾.

When R increase from zero (0) to infinity (∞), A_M and A_B initially increase, and turn to decrease after a while, and finally become zero. The range of R , which makes A_M and A_B maximum, is approximately $R=0.01 \sim 0.07$ for $t^* < 600$. When thermal conductivities of carbon tetrachloride or toluene are measured by a line heat-source of the radius $r_1=12.5 \mu$, for example, the values of R are 0.0076 and 0.044 , respectively, and therefore, they correspond to the above range of R , where the influence of radiation is most significant.

3.4 Method to modify λ_{app}

The real thermal conductivity, λ , is obtained by modifying the time-dependency of measured thermal conductivity, λ_{app} , which includes radiation effect, through the following process:

- (1) R is calculated from the radius of the line heat-source, r_1 , and the absorption coefficient, κ , of the sample material. And time, t , for each measured datum is converted to non-dimensional parameter, t^* .
- (2) A for the above t^* is decided by Eq. (15), from the emissivity of the surface of the line heat-source, ϵ , using A_M, A_B curves in Figs.12(A)(B) for the value of R calculated in (1).
- (3) Using A given in (2), $(\lambda_{app} - \lambda)$ is calculated from the initial temperature, T_0 , of the sample material, the index of refraction, n , the radius of the line heat-source, r_1 , and the Stefan-Boltzmann constant, σ . Measured data are modified by $(\lambda_{app} - \lambda)$.

Table 2 Dependencies of $(\lambda_{app}-\lambda)$ on various parameters

Parameter which increases monotonously	Variation of the amount of modification $(\lambda_{app}-\lambda)$
$R=\kappa r_1$	Increases initially, becomes the maximum around $R=0.01\sim 0.07$, and decreases after that.
$t^*=at/(r_1)^2$	Increases steeply in the range, $t^*<200$, and increases monotonously after that, except for the case of large R .
ϵ	Rate of increase is proportional to ϵ . The rate becomes smaller for large R .
T_0	Increases in proportionately with $(T_0)^3$.
n	Increases in proportionately with n^2 .

Dependencies of the amount of modification, $(\lambda_{app}-\lambda)$, on various parameters are shown qualitatively in Table 2.

4. Conclusions

The following conclusions were obtained concerning the transient line heat-source method, which is extremely useful to measure thermal conductivities of fluids, discussing the influence of weak radiation heat transfer and the method for modification of it.

- (1) When conduction and radiation heat transfer are coupled in a sample material, radiation heat flux, temperature variation by radiation and the apparent increase in measured thermal conductivity can be expressed in the following forms, using non-dimensional quantities:

$$(q_R)^* = (q_R)^M + \epsilon \{ (q_R)^B - (q_R)^M \} \dots\dots(10)$$

$$(T_R)^* = (T_R)^M + \epsilon \{ (T_R)^B - (T_R)^M \} \dots\dots(11)$$

$$A = A_M + \epsilon (A_B - A_M) \dots\dots\dots(15)$$

- (2) $(q_R)^B$ and $(q_R)^M$ are decided by giving non-dimensional parameters, R , r^* and t^* . For diffuse reflection, they are calculated by Eqs.(8)(9). $(T_R)^M$ and $(T_R)^B$ are obtained from Eqs.(3)-(5)' and Eq.(7). The method to calculate $(q_R)^*$, $(T_R)^*$, and the results of calculation were discussed in Chapters 3.1, 3.2.
- (3) When R and t^* are known, A_M and A_B are calculated by the procedure explained in Chapter 2.3. Results are shown in Fig.12.
- (4) When R increases from zero to infinity, A initially increases, but after a while decreases to the contrary, and finally converges to zero. A becomes maximum for $R=0.01\sim 0.07$. This range of R corresponds to the case where carbon tetrachloride or toluene is measured by a line heat-source of radius $r_1=12.5\mu$.
- (5) When t^* increases, A shows steep variation in the range, $t^*<200$. Therefore, extrapolation towards the limit, $t^*\rightarrow 0$, using apparent thermal conductivities, λ_{app} , measured in $t^*>200$, only gives a

thermal conductivity higher than the real conductivity. A method to get the real thermal conductivity from measured data of apparent thermal conductivities including radiation effect, was proposed in Chapter 3.4.

- (6) Table 2 summarizes the dependencies of the amount of modification, $(\lambda_{app}-\lambda)$, on various parameters.

Using the method shown in this paper, it is possible to modify the influence of radiation, which was difficult to analyze and to which sufficient consideration was not given in the past, and it is also possible to improve the accuracy of the measurement.

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Supplement

Equation (14) is modified in this supplement for relatively large $(r_1)^2/4at$.

When $(r_1)^2/4at$ is large, expansion of T_C shown in comment + becomes,

$$T_C \approx \frac{-Q}{4\pi\lambda} \left\{ \gamma + \ln \frac{(r_1)^2}{4at} - \frac{(r_1)^2}{4at} \right\} \dots\dots\dots(17)$$

And the third term of right hand side cannot be neglected, where $\gamma=0.5772\dots$.

Then, Eq.(13) is modified using Eq. (17).

$$\lambda = \frac{Q}{4\pi t} \left\{ 1 - \frac{(r_1)^2}{4at} \right\} \frac{1}{(\partial T_C / \partial t)_{r=r_1}} \dots\dots\dots(18)$$

Therefore, apparent thermal conductivity, λ_{app} , which is obtained from temperature variation (T in Eq.(6)) that includes the influence of radiation, is

$$\lambda_{app} = \frac{Q}{4\pi t} \left\{ 1 - \frac{(r_1)^2}{4at} \right\} \times \frac{1}{(\partial T_C / \partial t)_{r=r_1} + (\partial T_R / \partial t)_{r=r_1}} \dots\dots\dots(19)$$

Considering $T_C \gg T_R$ in Eqs.(18)(19),

$$\lambda_{app}/\lambda \approx 1 - \frac{4\pi t \lambda}{Q} \left\{ 1 + \frac{(r_1)^2}{4at} \right\} \left(\frac{\partial T_R}{\partial t} \right)_{r=r_1}$$

Therefore, Eq.(14) is modified as,

$$A = - \left(1 + \frac{1}{4t^*} \right) \left\{ - \frac{\partial (T_R)^*}{\partial t^*} \right\}_{r^*=1} t^* \dots \dots (20)$$

Considering, for example, a case to measure toluene by a line heat-source of radius $r_1 = 12.5 \mu$, Eq.(20) means that modification factors $\{1 + (1/4t^*)\} = 1.004$ and 1.008 must be multiplied to Eq.(14) for $t = 0.1$ sec and 0.05 sec, respectively.

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