536.423.1

Paper No. 183-9

Fundamental Investigation of Flow in an Agitated Thin Film

Evaporator (3rd Report, Theoretical Analysis)*

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In the previous work we reported an experimental investigation on the flow pattern in an agitated thin film horizontal evaporator without thermal effects. In this work, we theoretically researched an axial flow of the working liquid of low viscosity in the horizontal apparatus neglecting thermal effects and compared the theoretical results with experimental ones.

1. Introduction

In the previous papers⁽¹⁾⁽²⁾ we reported experimental investigations on the flow pattern in an agitated thin film horizontal evaporator without thermal effects. For an axial down flow in agitated thin film vertical evaporators, there is a novel theoretical analysis proposed by Kern and Karakas⁽³⁾. On the other hand, there are few theories concerning the flow in an agitated thin film horizontal evaporator. In this work we theoretically analyze flow patterns in an agitated thin film horizontal evaporator assuming that the viscosity of working liquid is so low that it is possible to predict the height of liquid-fillet by the hydraulic jump theory⁽¹⁾. Our theoretical results are compared with the experimental ones.

2. Nomenclature

B : coefficient of Boussinesq
 D : inner diameter of evaporator cylinder
 dL_θ : variation of L_θ
 dz : variation of z
 g : acceleration due to gravity
 h_e : liquid-film thickness at outlet of evaporator
 h_m : average liquid-film thickness over total length of evaporator
 h_{1m} : average liquid-film thickness in the range where liquid-fillet is formed (see Fig.1)
 h_2 : height of liquid-fillet close to blade
 k^2 : ratio of centrifugal acceleration, which acts on liquid-film close to outlet of evaporator, to that which acts on blade
 L : distance from inlet to outlet of evaporator cylinder

L_θ : circumferential length of liquid-fillet
 $L_{\theta m}$: average of L_θ
 $L_{\theta, z}, L_{\theta, z+dz}$: values of L_θ at z and $z+dz$, respectively
 N : rotational speed of wiping blade (rpm)
 n : number of blades
 Q : volumetric flow rate
 Q_c : maximum of Q_e
 Q_e : volumetric flow rate of liquid-film close to outlet of evaporator
 R : radius of wiping blade
 r : radial distance from rotor axis
 r_h : hydraulic radius
 T : average residence time $=W/Q$
 v_c : critical axial velocity of liquid-film
 v_z, v_{z+dz} : average axial velocities at z and $z+dz$, respectively
 W : liquid holdup
 z : axial co-ordinate
 z_e : disappearance point of the liquid-fillet when $Q < Q_c$
 δ : clearance between cylinder wall and tip of wiping blade $=(D/2)-R$
 θ : circumferential distance
 λ : coefficient of pipe friction
 ν : kinematic viscosity
 π : 3.14159
 τ_0 : shear stress
 ω : angular velocity
 Fr : Froude number $=\sqrt{R/h_{1m}}$
 h_m^* : non-dimensional liquid holdup of liquid-film $=h_m/(g^{-1/3}\nu^{2/3})$
 Re_f : Reynolds number for axial flow in liquid-fillet $=\{4(Q/n)/(h_2+L_\theta)\}/\nu$
 Re_l : film Reynolds number $=(4Q/\pi D)/\nu$
 Re_{lc} : critical Reynolds number $=(4Q_c/\pi D)/\nu$
 Re_N : rotational Reynolds number $=(4\delta\pi DN/60)/\nu$
 T^* : non-dimensional residence time $=T/\{L(g\nu)^{-1/3}\}$
 W^* : non-dimensional liquid holdup $=W/(\pi DLg^{-1/3}\nu^{2/3})$

* Received 4th November, 1977.

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3. Theoretical analysis

3.1 Liquid-fillet model

Assuming that the viscosity of working liquid is so low that the hydraulic jump theory can apply to explain the height of liquid-fillet close to wiping blade, we theoretically analyze the axial flow in agitated thin film horizontal evaporators. A liquid-fillet model shown in Fig.1 is used considering the previous experimental results⁽¹⁾, which are summarized as follows.

1) When the rotational speed of wiping blade $N=400 \sim 1200$ rpm, the ratio of the centrifugal acceleration to the acceleration due to gravity, $R\omega^2/g=8.79 \sim 79.3$, the centrifugal acceleration is so large that the acceleration due to gravity may be negligible.

2) Since we conducted the experiment under the condition of no evaporation, the volumetric flow rate Q at the outlet was equal to the axial flow in an agitated thin film horizontal evaporator and the properties of working liquid did not change in the axial direction.

3) h_{lm} listed in Table 2 increases with an increasing Q and with a decreasing N . Except in the outlet region of evaporator cylinder, h_{lm} scarcely varies in the axial direction.

4) The working liquid in the liquid-film flows axially in the outlet region. If Q is large and the liquid-fillet does not disappear in the evaporator, the working liquid flows axially both in the liquid-film and in the liquid-fillet.

5) The height of liquid-fillet close to a blade h_2 scarcely depends on Q and N , and is approximately constant in the axial direction. The circumferential length of liquid-fillet L_θ decreases in the axial direction.

For simplification, the following assumptions are made to treat the liquid-fillet model based on the experimental results mentioned above.

1) We use the relative co-ordinate neglecting the effect of the curvature of cylinder wall. The centrifugal acceleration, which acts on the liquid-fillet, is so large that the acceleration due to gravity is negligible.

2) The working liquid properties do not change in the axial direction.

3) h_{lm} depends on Q and N , and variation of h_{lm} in the axial direction is neglected. The liquid-film thickness in the outlet region decreases from h_{lm} to h_e which is the thickness at the outlet of evaporator.

4) In the outlet region, the working liquid flows in the liquid-film. If the volumetric flow rate Q is less than the maximum flow rate in the liquid-film Q_c , the liquid-fillet disappears in the evaporator. Then the working liquid in the liquid-fillet flows to the liquid-film through clearance δ . If $Q > Q_c$, Q_c flows in the liquid-film and $(Q-Q_c)$ flows in the liquid-fillet. In the region where h_{lm} is constant, all the working liquid flows

only in the liquid-fillet.

5) The liquid-fillet cross-section is a right-angled triangle^{*1}. Its radial height and circumferential length are expressed as h_2 , L_θ , respectively.

3.2 The momentum equation for the axial flow in the liquid-fillet

The liquid-fillet plays an important role by such effects as cross-sectionally mixing with the liquid-film, forming a uniform liquid-film and thrusting the working liquid axially in agitated thin film horizontal evaporators. The momentum equations for the axial flow in the liquid-fillet (Fig.1) are derived from a balance of the forces acting on the fluid in the axial direction.

The pressure forces act on the liquid-fillet at the axial positions z and $z+dz$. The shear forces act on the liquid-fillet at the surfaces of both wiping blade and cylinder wall. A difference between the pressure force and the shear force acting on the liquid-fillet causes the working liquid to flow in the axial direction. The difference of momentum per unit time is given by $\rho(Q/n) \cdot (v_{z+dz} - v_z)$, where ρ is the density of working liquid, n is the number of blades, and v_z , v_{z+dz} are the average axial velocities at the axial positions z and $z+dz$, respectively.

At the axial positions z and $z+dz$, the external forces based on the static

Fig.1 Liquid-fillet model

*1 In the previous paper⁽¹⁾ we presented a liquid-fillet model with a rectangular cross-section. However, the residence time and the variation of L_θ in the axial direction calculated from the theory based on that model were not in good accord with experimental ones. In that experiment⁽¹⁾, it was seen that the height of liquid-fillet varied from h_2 at a point close to a blade to h_{lm} . Considering these results, we use a liquid-fillet model with a triangle cross-section in this work.

pressures are given by $(1/6) \cdot (D/2 - h_2/3) \cdot \omega^2 h_2^2 L_{\theta, z}$ and $(1/6) \cdot (D/2 - h_2/3) \omega^2 h_2^2 L_{\theta, z+dz}$, respectively. In these expressions ω is the angular velocity of wiping blade and $r = D/2 - h_2/3$ is the gravity of the cross

sectional area of liquid-fillet $h_2 L_{\theta}/2$. The summation of shear forces acting on blade and cylinder surfaces is $\tau_0 \{h_2 + (L_{\theta, z} + L_{\theta, z+dz})/2\} dz$. The momentum equation for the axial flow of liquid-fillet is obtained as

$$\rho \frac{Q}{n} (v_{z+dz} - v_z) = \frac{\rho(D/2 - h_2/3) \omega^2 h_2^2}{6} (L_{\theta, z+dz} - L_{\theta, z}) - \tau_0 \left(h_2 + \frac{L_{\theta, z} + L_{\theta, z+dz}}{2} \right) dz \quad (1)$$

Putting $L_{\theta, z+dz} = L_{\theta} + (dL_{\theta}/dz)dz$, the left side term of Eq.(1) gives

$$v_{z+dz} - v_z = \frac{2Q}{n h_2 L_{\theta}} \frac{-(dL_{\theta}/dz)dz}{\{L_{\theta} + (dL_{\theta}/dz)dz\}} \quad (2)$$

Then, Eq.(1) is rearranged as

$$\frac{-2\rho Q^2 (dL_{\theta}/dz) dz}{n^2 h_2 L_{\theta} \{L_{\theta} + (dL_{\theta}/dz)dz\}} = \frac{-\rho(D/2 - h_2/3) \omega^2 h_2^2 (dL_{\theta}/dz) dz}{6} - \tau_0 \left(h_2 + L_{\theta} + \frac{1}{2} \frac{dL_{\theta}}{dz} dz \right) dz \quad (3)$$

Assuming $L_{\theta} \gg (dL_{\theta}/dz)dz$, Eq.(3) is represented as

$$\frac{dL_{\theta}}{dz} = \frac{\tau_0 (h_2 + L_{\theta})}{\rho(Q^2/h_2 L_{\theta}^2) [2/n^2 - (D/2 - h_2/3) \omega^2 h_2^2 L_{\theta}^2 / 6Q^2]} \quad (4)$$

$Re_f = v_z 4r_h / \nu$, which is the Reynolds number for axial flow in the liquid-fillet based on the equivalent diameter $4r_h$ and the average axial velocity v_z , gives

$$Re_f = \{ (4Q/n) / (h_2 + L_{\theta}) \} / \nu \quad (5)$$

, where the hydraulic radius $r_h = (h_2 L_{\theta} / 2) / (h_2 + L_{\theta})$. From Boussinesq's theory for $Re_f \leq 2320$, the coefficient of pipe friction in the right side term of $\tau_0 = (\lambda/4) \rho v_z^2 / 2$ is given by

$$\lambda = 64 / Re_f \quad (6)$$

The coefficient of Boussinesq B is listed in Table 1. If $2320 < Re_f < 1.5 \times 10^5$, Blasius' equation (Eq.(7)) is used.

$$\lambda = 0.3164 Re_f^{-1/4} \quad (7)$$

3.3 Numerical calculations of Eq.(4)

Average liquid-film thickness h_{lm} , listed in Table 2, is used in numerical calculations of Eq.(4). Its thickness is linearly interpolated in the range of $127 \leq Re_l \leq 1.53 \times 10^3$.

The height of liquid-fillet h_2 is given by the hydraulic jump theory

$$\frac{h_2}{h_{lm}} = \frac{1}{2} (\sqrt{1 + 8 Fr} - 1) \quad (8)$$

, where the Froude number Fr is

$$Fr = \sqrt{R/h_{lm}} \quad (9)$$

When the centrifugal acceleration $R(k\omega)^2$ acts on the liquid-film at the outlet, the critical axial velocity of liquid-film at the outlet v_c is given by $v_c = \sqrt{R(k\omega)^2 h_e}$.

The critical axial velocity v_c mentioned above is assumed to equal to the traveling velocity of long wave. k^2 is the ratio of the centrifugal acceleration, which acts on the liquid-film close to outlet of evaporator, to that which acts on the blade.

The volumetric flow rate in the liquid-film Q_e gives

$$Q_e = \pi D h_e \sqrt{R(k\omega)^2 h_e} \quad (10)$$

, when the liquid-film of uniform thickness h_e is formed circumferentially at the outlet. From Eq.(10), we obtain $h_e = \{Q_e / (\pi D \sqrt{R(k\omega)^2})\}^{2/3}$. When $h_e = h_{lm}$, Q_e is equal to the critical volumetric flow rate Q_c . Q_c increases with an increasing ω and h_{lm} . The disappearance point of liquid-fillet z_e is predicted by Oki's equation⁽⁵⁾ for back water curve

$$L - z_e = \frac{(\pi D)^2 R(k\omega)^2 (h_{lm}^4 - h_e^4)}{(\lambda/2) Q^2} - \frac{2(h_{lm} - h_e)}{\lambda/4} \quad (11)$$

, when $Q < Q_c$.

In the outlet region, where the liquid-fillet does not exist, the Reynolds number for axial flow in liquid-film is equal to the liquid-film Reynolds number $Re_l = (4Q/\pi D)/\nu$. Then, $\lambda = 1.5 \times 64 / Re_l$ and the Blasius' equation $\lambda = 0.3164 Re_l^{-1/4}$ are used to calculate the coefficient of pipe friction λ in Eq.(11), when $Re_l \leq 2320$ and $2320 < Re_l < 1.5 \times 10^5$, respectively. Using Eq.(4) and the formula of Euler-Richardson⁽⁶⁾, we calculate dL_{θ}/dz and the correlation between L_{θ} and z . Thereby the condition of $(dL_{\theta}/dz)dz = 0.5\text{mm}$ is set for the assumption of $L_{\theta} \gg (dL_{\theta}/dz)dz$ to hold. Because the de-

Table 1 Coefficient of Boussinesq⁽⁴⁾

h_2/L_{θ} OR L_{θ}/h_2	1	2	3	4	5	...	∞
B	0.895	0.973	1.07	1.14	1.19	...	1.50

Table 2 Average liquid-film thickness h_{lm}/δ

	Re _l	127	1.53×10^3
Re _ω	1.26×10^4	0.88	0.98
	1.68×10^4	0.85	0.96
	2.09×10^4	0.81	0.94
	2.51×10^4	0.78	0.92

nominator of Eq.(4) has to be minus, a critical value of L_θ exist. This critical value $L_{\theta c}$ is obtained as

$$L_{\theta c} = \left\{ \frac{12Q^2}{n^2(D/2 - h_2/3)h_2^3\omega^2} \right\}^{1/2} \dots\dots\dots(12)$$

, When $dL_\theta/dz \rightarrow -\infty$.

Only when $L_\theta > L_{\theta c}$, Eq.(4) is available. In the numerical calculation, $1.05L_{\theta c}$ is chosen as the minimum of L_θ . From Eq.(12), we know that $L_{\theta c}$ increases with both a decreasing ω and an increasing Q . When $Q \geq Q_c$, $L_\theta > L_{\theta c}$ in the evaporator and $L_\theta = L_{\theta c}$ at the outlet. That is, the shear force acts on the liquid-fillet at every axial position in the evaporator except at the outlet.

When $Q < Q_c$, the circumferential length of liquid-fillet L_θ is equal to $L_{\theta c}$ at the position z_e given by Eq.(11). The working liquid in liquid-fillet flows to the liquid-film through clearance δ in the outlet region. The correlation between L_θ and z is determined by calculating the variation of L_θ in the axial direction. Next, the liquid holdup W and the average residence time $T=W/Q$ are calculated.

These numerical calculations based on the flow chart shown in Fig.2 are performed by using computer FACOM-230-28S.

Fig.2 Flow chart

4. Numerical results and discussion

4.1 Froude number Fr and height of liquid-fillet h_2

In this section, comparisons between numerical and experimental results are made. In the range of $127 \leq Re_l \leq 1.53 \times 10^3$ and $1.26 \times 10^4 \leq Re_N \leq 2.51 \times 10^4$, the numerical results are calculated. Fr , calculated from the average liquid-film thickness h_{1m} and Eq.(9), and h_2/δ , calculated from Fr and Eq.(8), are shown in Fig.3. It can be said that Fr increases with an increasing Re_l and with a decreasing Re_N , and h_2/δ decreases with an increasing Re_l and with a decreasing Re_N . h_{1m}/δ approaches 1.0 and Fr approaches 7.0 with an increasing Re_l . When $Re_N = 1.38 \times 10^4$, the numerical results of h_2/δ are in good accord with the experimental results.

4.2 Effects of Re_l and Re_N on the variation of the circumferential length of liquid-fillet $L_\theta/(\pi D/4)$ in the axial direction

The following results are obtained under $k=0.4$. The effects of Re_l on the variation of $L_\theta/(\pi D/4)$ in the axial direction are shown in Fig.4. It can be said that $L_\theta/(\pi D/4)$ decreases with an increasing z/L , and the liquid-fillet disappears in the evaporator when $Re_l \leq 764$.

$L_\theta/(\pi D/4)$ increases with an increasing Re_l . When Re_l is low, the theoretical results (full line) are in good accord with the experimental results (broken line). When Re_l is high, the theoretical values of $|dL_\theta/dz|$ are lower than the experimental ones.

Under the condition of $Re_l = 764$, the effects of Re_N on the variation of $L_\theta/(\pi D/4)$ in the axial direction are shown in Fig.5. It can be said that $L_\theta/(\pi D/4)$ increases with a decreasing Re_N . When $Re_N \geq 2.09 \times 10^4$, the liquid-fillet disappears in the evaporator. When z/L is small, the theoretical values (full line) are in good accord with the experimental values (broken line).

Fig.3 Froude number Fr and height of liquid-fillet close to blade h_2/δ

Fig. 4 Effect of Re_l on the variation of the circumferential length of liquid-fillet $L_\theta/(\pi D/4)$ in the axial direction

Fig. 5 Effects of Re_N on the variation of the circumferential length of liquid-fillet $L_\theta/(\pi D/4)$ in the axial direction

But the axial distance from the inlet to the theoretical liquid-fillet disappearance point is larger than the experimental one. The experimental values shown in Fig. 4 and Fig. 5 are obtained using the electrode method⁽²⁾.

4.3 Effects of Re_l and Re_N on the liquid-fillet disappearance point z_e/L

z_e in Eq. (11) is shown in Fig. 6. It can be said that z_e/L increases with an increasing Re_l and with a decreasing Re_N . In other words, the liquid-fillet is not likely to disappear in the evaporator either with an increasing Re_l or with a decreasing Re_N .

From the discussions, it can be said that the effects of Re_N on the variation of $L_\theta/(\pi D/4)$ in the axial direction are at least analyzed qualitatively, although the quantitative differences exist between the theoretical liquid-fillet disappearance point and the experimental one.

4.4 Non-dimensional liquid holdup W^* and non-dimensional holdup of liquid-film h_m^*

W^* increases with an increasing Re_l and with a decreasing Re_N as shown in Fig. 7. h_m^* increases with an increasing Re_l and with a decreasing Re_N . However, h_m^* is independent of Re_l and Re_N when $Re_l \geq 10^3$.

h_m is defined as the average liquid-film thickness over the total length of evaporator. In Fig. 7, the liquid holdup in the case of $h_m = \delta$ is shown by a three-dot chain line. From the definition, $W^* - h_m^*$ shows the non-dimensional holdup of liquid-fillet. Fig. 7 shows that the ratio of the holdup of liquid-fillet to the total holdup increases with an increasing Re_l .

Fig. 6 Effects of Re_l and Re_N on the disappearance point of liquid-fillet z_e/L

Fig. 7 Non-dimensional liquid holdup W^* and non-dimensional holdup of liquid-film h_m^*

4.5 Non-dimensional residence time T^*

The predicted values of T^* calculated from the theoretical values of W^* in Fig. 7 are compared with the experimental values⁽⁴⁾ in Fig. 8. Fig. 8 shows that the theoretical values are in good accord with the experimental values. As we have mentioned above, the purpose of this report is to check the

adaptability of our theory by comparing the theoretical values with the experimental values⁽¹⁰⁾.

From Eq.(4) we can also deduce the effects of many parameters on T^* , such as n , D , ν , L and k . For example, this theory indicates that T^* increases with an increasing n , D and L and with a decreasing k , if h_{lm} is independent of these parameters.

5. Conclusions

We theoretically analyzed the flow patterns in an agitated thin film horizontal evaporator neglecting thermal effects when the viscosity of working liquid was low.

We have the following conclusions:

- (1) When the height of liquid-fillet agrees with that given by the hydraulic jump theory (Eq.(8)), the variation of L_θ in the axial direction is determined from Eq.(4) which is derived when the liquid-fillet cross-section is a right-angled triangle.
- (2) Equation (4) is available when L_θ is larger than $L_{\theta c}$ given by Eq.(12). The axial position at which $L_\theta = L_{\theta c}$ is given by Eq.(11).
- (3) The theoretical values of residence time, which is the most important factor in the agitated thin film evaporator, are in good accord with the experimental values. However, the effects of Re_l and Re_w on the variation of L_θ in the axial direction are qualitatively clarified.
- (4) This theory indicates that T^* increases with an increasing n , D , ν and L and with a decreasing k , if h_{lm} is independent of these parameters.

Fig.8 Comparison between non-dimensional residence time T^* and the experimental equation

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