A Study on Facilities Location Using the Properties of Cut Set Matrix

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This paper deals with a problem of facilities location. Fundamental suggestion is making an individual location algorithm for combination of "Presupposition" and "Objective function" in each location problem. The presupposition of this paper is "that facilities locate straight", and the objective function is "to minimize Backward Distance x Intensity". This new facilities location algorithm to select "minimum Backward Distance x Intensity location" is developed by using a property on Cut Set matrix that is a part of graph theory. The feature of this algorithm is ability to select the optimal solution by only using easy matrices manipulation. Trial of this algorithm in regard to an actual 6 facilities location is described here in.

1. Introduction

The facilities used to located by a skillful layout planner. But now, a new algorithm which makes satisfactory facilities location is demanded. And some effective "Mathematical Methods" have been developed.

One of the properties of mathematical method is clear objective function. It is disadvantage to replace the problem of facilities location involving many complex factors with a simple problem handling only one factor to be minimized or maximized. When we reflect on the complexity of facilities location, development of an allround algorithm to solve every kind of layout problem is very difficult. Therefore, we think that we should make an individual algorithm for combination of "Presupposition" and "Objective function".

This paper develops a new facilities location algorithm to minimize "Backward Distance x Intensity" without tryanderor, supposing that facilities locate straight, by use of property of Cut Set matrix that is a part of graph theory. The presupposition of this algorithm is "that facilities locate straight", and the objective function is "to minimize Backward Distance x Intensity".

2. Outline of algorithm

Given input information is a "from-to chart" that describes materials flows among facilities, and "Width of facilities". Nodes are defined as facilities and arrows are defined as materials flows, the information of materials movement is described by use of oriented graph. We can make Cut Set matrix from materials flow information by using the property of Cut Set, that is a set of minimal branches needed for open elimination in dividing graph into two parts. Adding an information about existence and direction of arrow, that is coming from Cut Set matrix, to an information about materials flow and width of facilities (distance between centers of facilities i to j), that is coming from input information, we can calculate "Backward Distance x Intensity (Backward DXI)". Then, select the optimal solution according to the heuristic method. To avoid an increase in number of trial runs, that is incidental to heuristic way, we make "minimum Backward Intensity facilities location", that can be obtained by applying Tie Set algorithm to a temporary initial solution. Because "Minimum Backward Intensity" is a part of "Minimum Backward Distance x Intensity", which is objective function of this paper.

3. Fundamental theory

This developed algorithm has two phases, one being [1] Preparative phase, the other [2] Judgement phase. The preparative phase is matrices manipulation of "Backward Distance x Intensity", that is the objective function of this paper. The judgement phase is selection of an optimal solution. The fundamental theory of this algorithm is explained below;

Input information

The information about state of materials flow is described on a from-to chart (Fig.1). Fig.2 is information on width of facilities. The existence of materials...
flow is shown on oriented graph (Fig. 3).

Fig. 1 From-to chart

<table>
<thead>
<tr>
<th>Machine</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

Fig. 2 Width of facilities

Fig. 3 Oriented graph

[1] Preparative phase

(1) Distance between centers of facilities i and j.

When facilities locate straight, we grasp the distance of materials flow and as center-to-center distance of the facilities which is shown in Fig. 4. When the width of facilities is given as in Fig. 2, we can make distance matrix (Fig. 5) by use of $d_{ij}$, that is coming from Fig. 4. Fig. 5 describes the matrix for the distance between adjacent facilities.

Fig. 4 Distance between centers of facilities i and j

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Fig. 5 $d_{ij}$ Matrix

(2) Calculated Cut Set

Fundamental Cut Set is calculated by matrices manipulation. All Cut Set matrices can be obtained as combinations of fundamental Cut Set.

$A = A_r, A_1$

$Q_f = [U, (A_r^{-1}A_1)]$

A : Reduced incidence matrix
$A_r$ : Tree of reduced incidence matrix
$A_1$ : Co-tree of reduced incidence matrix
$Q_f$ : Fundamental Cut Set matrix
$U$ : Unity matrix
$A_r^{-1}$ : Inverse matrix

(3) Proposed site and Cut Set

We number the proposed site as in Fig. 6, 1 being the nearest site from IN, 2 the next one and so on. If a facility locate at No. 1 site, any one of B, C, D facilities can be located at No. 2 site (see Fig. 6).

Fig. 6 Proposed site and Cut 1

In this case, arrows (materials flow) between No. 1 site and No. 2 site can be seen on A column of calculated fundamental Cut Set matrix as "(2) Calculate the Cut Set ". Similarly, the existence of arrows for the case of B facility being located at No. 1 site, is described on B column of above-mentioned fundamental Cut Set matrix. "Backward movement of materials" is indicated by a negative sign (-) of Cut Set matrix. We call a set of negative arrows between proposed sites i and i+1 Cut No. 1.

In Fig. 6, a facility locates at No. 4 site, that is the nearest site from OUT side, and B, C, D facilities locate at No. 3 site, the backward materials flows being positive (+) arrows of fundamental Cut Set. And then, we name the set of these backward materials flow Cut No. 3 (see Fig. 6 and 7). The same holds with Cut No. 2 (Fig. 8).

Fig. 7 Cut 1 and Cut 3

Fig. 8 Cut 2
(4) Calculate "the amount of backward materials flows"
Columns of Cut Set matrix, that is calculated in (2), correspond to the arrows. So an amount of backward material flows can be calculated as the sum of negative arrows. Here, Cut Set matrix and the calculated amount of backward material flows of Cuts 1, 2 and 3 are described as Fig.7 and 8.
(5) Calculate "Backward Distance × Intensity"
"Backward Distance × Intensity" is calculated by multiplying "(4) the amount of backward materials flow" by "(1) Distance information". The results are shown in Fig.9-11. Further, pick up the minimum one \( \min (\text{Cut}{.\text{i}}) \) from each matrix for [2] Judgement phase.

Fig.9 Cut 1

\[
\begin{array}{cccc}
A & B & C & D \\
6 & A & 24 & 30 & 12 \\
17 & B & 38 & 51 & 34 \\
3 & C & 15 & 9 & 8 \\
11 & D & 44 & 22 & 33 \\
\end{array}
\]

min(cut1)=9

C-B

C-D

Fig.10 Cut 3

\[
\begin{array}{cccc}
A & B & C & D \\
55 & A & 44 & 55 & 11 \\
55 & B & 12 & 28 & 14 \\
50 & C & 30 & 10 & 5 \\
8 & D & 14 & 6 & 2 \\
\end{array}
\]

min(cut3)=4

B-D

Fig.11 Cut 2

\[
\begin{array}{cccc}
A & B & C & D \\
9 & A & -27 & 18 & - \\
B & - & -45 & 36 & - \\
8 & A & -24 & 24 & - \\
C & - & -32 & 32 & - \\
15 & A & -30 & 45 & - \\
B & C & 55 & - & -33 \\
11 & C & 44 & - & -22 \\
B & D & 80 & - & 60 \\
20 & D & 80 & - & 50 \\
11 & C & 44 & 22 & - \\
D & C & 55 & 33 & - \\
\end{array}
\]

min(cut2)=18

A-B-D

Fig.12 Initial solution

(2) Upper limit of Cut i
This step is an attempt to decrease the number of trials.

Upper limit of Cut 1
\[ \text{Upper limit of Cut 1} = \text{K}_0 - \min(\text{Cut} 2) - \min(\text{Cut} 3) = 58 - 18 - 4 = 36 \]  

Numbers over this upper limit never reduce initial solution (K0), so they are excluded from the object of selection. Other upper limits of Cut Set are as follows;

Upper limit of Cut 2
\[ \text{Upper limit of Cut 2} = 58 - 9 - 4 = 45 \]

Upper limit of Cut 3
\[ \text{Upper limit of Cut 3} = 58 - 9 - 18 = 31 \]

Using there upper limits of Cut i, the matrix which is calculated in "[1] Preparative phase of (5) " is modified as Figs.13 15.

(3) Select the optimal solution
The optimal solution is selected from the combinations of modified Cut i (Backward Distance × Intensity). This step remarkably decreases the number of trials, as compared with traditional complete enumeration method. Generally, \( \min(\text{Cut} i) \) is most effective to improve the initial solution, therefore the selection starts with a comparison between \( \min(\text{Cut} i) \) and the initial solution (K0).

Thinking about Cut 1, that is a connection of proposed sites 1 and 2, the smallest number is 9. And the facilities locations in this case are (i) C-B, (ii) C-D (see Fig.9). The facility that can be located at the proposed site 3 is shown as row of CB and CD on modified Cut 2 (Fig.14). There are only 4 cases, as follows (see Fig.14);

In case of (i) : C-B - A, C-B - D
In case of (ii) : C-D - A, C-D - B
Next, think about the proposed site 4. In both (i) and (ii), location of the rest of facilities is described on modified cut 3 matrix (Fig.15). Every column of Fig.15 is then none of combinations used in the min(Cut 1) improve the initial solution. Applying the same procedure to min(Cut 2), we cannot obtain any better locations. For min(Cut 3), the following two locations are selected.

- A = C = B = 0

Backward Distance × Intensity = 52 < K0

- C = A = 0 = 0

Backward Distance × Intensity = 51 < K0

then, the initial solution (K0) is corrected by Eq.(6). Initial solution is renewed to 51.

In the above-mentioned procedure, we finish trial of min(Cut 1). Then, the second smallest value in each Cut is set as a new min(Cut 1).

\[
\begin{align*}
\min(Cut\ 1) &= 15 \\
\min(Cut\ 2) &= 22 \\
\min(Cut\ 3) &= 6
\end{align*}
\]

Upper limit of Cut 1 is calculated by using the renewed initial solution (K0) and min(Cut 1). Eliminate non-improvable com-
binations from Fig.13 15, and so on.

When the initial solution (K0) can be improve or all columns on the matrix of Cut 1 are excluded, the algorithm is stopped. And the very same facilities location is the optimal solution. The selected optimal solution of this example is shown in Fig.16. Fig.17 is a conceptional sketch of Fig.16 using the information on facilities width shown in Fig.2.

![Fig.16 Sketch of the optimal solution](image)

![Fig.17 Conceptional sketch of the optimal location](image)

4. Developed algorithm which minimizes "Backward Distance × Intensity" in facilities location.

The overall procedure of this algorithm is outlined in "3. Fundamental theory ". Here, a detailed procedure is described.

[Step 1]
Grasp the information of materials flow and width of facilities in the shape of From-to chart and matrix for input of algorithm.

[Step 2]
Make the From-to chart into Node to Node matrix and change the information about width of facilities into Distance matrix (distance between centers of facilities i and j).

[Step 3]
Label the existing arrows of Node to Node matrix.

[Step 4]
Make incidence matrix (Ak), that describes materials flow, by using the labeled arrows.

[Step 5]
Make reduced incidence matrix (A). To eliminate an arbitrary row from the incidence matrix (Ak).

[Step 6]
Separate the reduced incidence matrix (A) into Tree part and Co-tree part, and make a new matrix in the shape of A = A, A1

[Step 7]
Calculate the fundamental Cut Set matrix (Qf) by using the result of Step 6. Qf = [ U, (A, A1) ]
[Step 8]
Calculate the whole Cut Set by using the combinations of fundamental Cut Sets.

[Step 9]
Correlate Cut, between facilities, with Cut Set (see Fig.6) and make the matrix of "Backward Distance × Intensity".

[Step 10]
Calculate min(Cut1) by use of each Cut matrix, and extract the facilities location of this case.

[Step 11]
Using the algorithm on "Facilities location with Tie Set" to make the minimum backward materials flow facilities location. Add the information on distance between centers of facilities i and j, that is shown as Dij matrix (Step 2) to above-mentioned location, and calculate Backward Distance × Intensity of this case and put it as the initial solution (K0).

[Step 12]
Calculate upper limit of each Cut by use of the initial solution (K0) and min(Cut1).

[Step 13]
Using the upper limit of Cut1 (see Step 12), eliminate the combination of facilities, which never correct the initial solution (K0) and make a modified Cut matrix (Fig.13 ~ 15). The initial solution (K0) after all combinations eliminated, is the optimal solution.

[Step 14]
Enumerate the possible facilities locations in the case of min(Cut1), that is most effective to improve the initial solution (K0) by using the modified Cut1.

[Step 15]
Calculate the "Backward Distance × Intensity" for possible facilities locations that is selected by Step 14.

[Step 16]
Compare the minimum value that is selected by Step 15 with the initial solution (K0), minimum value < initial solution go to Step 17
minimum value ≧ initial solution the facilities location of the initial solution is optimum.

[Step 17]
Make the minimum value of Step 15 a new initial solution.

[Step 18]
After checking min(Cut1), take the second smallest value in each Cut as a new min Cut1.

[Step 19]
Go back to Step 12, and continue this algorithm until the optimal solution is reached by using the renewed initial solution (K0) and the renewed min(Cut1).

Above-mentioned procedure is a developed algorithm which minimizes "Backward Distance × Intensity" in facilities location.

When in Step 13, every column of modified Cut1 is eliminated or in Step 16 the minimum value, that is selected in Step 15, is greater than the initial solution (K0), we stop this algorithm. Then this facilities location is the optimum solution.

5. Conclusions

As a fundamental suggestion on the facilities location an individual algorithm for combination of "Presupposition" and "Objective function" is proposed. This paper describes a new facilities location algorithm to minimize "Backward Distance × Intensity" by use of property on Cut Set matrix that is a part of graph theory. The developed algorithm is shown in Chapter 4. The special features of the developed algorithm are as follows;

(1) Each Cut Set is calculated by easy matrices manipulation.
(2) The existence and direction of arrows are checked by Cut Set.
(3) Use of the initial solution (K0) and the upper limit of Cut1 to eliminate impossible combinations of facilities.

6. Supplementary note

Here is described an investigation about the efficiency of this developed algorithm. Examined is a 6 facilities problem and input information is described in Fig.18 and 19. The oriented graph of this example is shown in Fig.20.

Fig. 18 From-to chart

Fig.19 Width of facilities

Fig.20 Oriented graph

The traditional complete enumeration method needs 6! = 720 trials to select the minimum Backward Distance × Intensity facilities location". The application of the developed algorithm to this 6 facilities problem is made as follows;
The result of Step 9, that is a final part of "Preparative phase", is shown in Fig.21.

Fig.21 "Backward Distance x Intensity" Matrix (Step 9)

The result of Step 11, that is reduction of trial number by using the initial solution (K₀) and the upper limit, is shown in Fig.22.

Fig.22 Initial solution (K₀) and reduction of upper limit of Cut 1

In Fig.22, the optimal solution (K₀ = 275) is selected through three times repetition of Steps 12 to 19. A sketch of the optimal solution is given in Fig.23.

In the course of applying the developed algorithm to several actual examples, we could gain a remarkable reduction of trial number and always select the optimal solution.

Fig.23 Selected optimal solution

In the future, we will develop other algorithm and other "Objective functions".

Dr. HITOMI.K and FUJII.S pointed out a mistake in the calculation of the initial solution (K₀) in Fig.12. We express our thanks to them for good advice.

References