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Bifurcation or Skew of a Rapidly Propagating Crack^{*}

(Case of Mode III Crack)

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In this paper a transient elastodynamic analysis of the stress intensity factors at the propagating tips of both a bifurcating crack and a skew one is presented. In the method of solution, the stress field in the vicinity of the crack tip is analyzed under a peculiar loading which makes the antiplane displacement self-similar, and then the result is used to know the stress intensity factors for the case that a branching or a skew actually occurs. Then, on the assumption that the crack propagates in a direction which maximizes the total energy released in a unit time, the angle and the velocity of the bifurcating or the skew crack are determined. In the present treatment the considerations are restricted to the case of antiplane shear.

1. Introduction

In this paper we consider the phenomena of crack extending, skewing and branching, when an antiplane shear stress wave strikes a semi-infinite crack.

To investigate the behaviour of a crack, it is necessary to incorporate the criterion for tearing of a crack and the stress analysis in the vicinity of a crack tip. A steady-state crack propagation problem was first solved by Yoffe,⁽¹⁾ who revealed the angular variation of a circumferential stress near the tip of a crack moving at a constant speed through a stretched elastic sheet. His result suggests that a running crack may bifurcate at higher velocity. Freund⁽²⁾ found that the angular variation of a stress near a crack tip depended only on the crack propagation velocity and the magnitude of the stress was only affected by the shape of a crack and the loading condition. To estimate the crack propagation velocity at which the crack bifurcates and the angles of branches, Sih⁽³⁾ reviewed the applicability of the maximum hoop stress criterion and the minimum strain energy density criterion. These criteria do not include the information on the stress state after crack bifurcation.

To evaluate the validity of the energy condition after bifurcation, we may have to use a criterion incorporating the stress state after bifurcation. Recently, Achenbach^{(4)~(6)} offered an analytical method to get the stress state after bifurcation

(skew) of a running crack under the antiplane shear load. In the subsidiary problem, he analyzed the skew and the bifurcation of a stationary crack at the instant when the wave front struck it, by assuming the particle velocity to be self-similar. It follows from the results of the above method that a crack may bifurcate under a small stress which can not force the crack to propagate in its plane.

The purpose of this paper is to avoid this undesirable feature. The stress state after crack bifurcation is analyzed by assuming the displacement to be self-similar. We employ the criterion of the balance of energies before and after crack bifurcation. To determine the angles of bifurcation and the velocities of branches, we assume that the crack should behave such as to make the rate of release of energies maximum.

2. Method of solution

The dynamic stress intensity factor k_3 of a stationary semi-infinite plane crack has been solved as Eq.(1) when the step antiplane shear stress τ_0 is applied on the surface of it.⁽³⁾

$$k_3 = \frac{2\sqrt{2}}{\pi} \tau_0 \sqrt{c_2 t'} \dots \dots \dots (1)$$

where $c_2^2 = G/\rho$, G : transverse modulus of elasticity, ρ : density, t' :time. From the principle of superposition, Eq.(1) also holds when the antiplane shear stress wave the front of which is parallel to the crack, impinges on it at time $t'=0$.

A crack will start to propagate, holding the condition of the balance of energies, some time after the instant of the arrival of the wave front at the crack surface. For a running crack with velocity v , k_3 is of the form:⁽⁷⁾

$$k_3 = \frac{2\sqrt{2}}{\pi} \tau_0 \sqrt{c_2 t'} \sqrt{1 - \frac{v}{c_2}} \dots \dots \dots (2)$$

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Employing Freund's idea for Mode I crack,⁽⁸⁾ the intensity factor at the instant when the crack stops suddenly, satisfies Eq.(1).

If a stress wave impinges on a crack plane at time $t'=0$ and the crack bifurcates at time $t'=t_0$, the stress state at time $t'(>t_0)$ can be derived from the superposition of the following cases ① and ②.

- ① Crack stops at time $t'=t_0$.
- ② Bifurcation crack propagates under the load which is applied along each branch to cancel the stress field produced by the condition ①.

Singularities at the tips of branches are entirely governed by the condition ②. The dominant part of load shown in ② is the singularity term:

$$\tau \approx -\frac{2\tau_0\sqrt{c_2}\sqrt{t_0}}{\pi\sqrt{r}} \cos \frac{\theta}{2} \quad (3)$$

where (r, θ) is a polar coordinates system with the origin at the bifurcation point of crack (Fig.1).

It is not easy to analyze the stress intensity factors $k_{3,x}$ ($x=1,2$) at the points F and D in Fig.1 by using Eq.(3) directly. Let us now suppose the following load which makes the displacement w self-similar.

$$\tau = -\frac{2\tau_0\sqrt{c_2}T}{\pi\sqrt{t}\sqrt{r}} \cos \frac{\theta}{2} \quad (4)$$

where T is a value with magnitude unity and with dimension of time, t is time measured from the instant of the crack bifurcation.

The problem at hand is to obtain the true stress intensity factor $k_{3,x}$ from k_3^x ($x=1,2$) which will be derived later by assuming the load of Eq.(4) acts on the crack surface CDEFG in Fig.1. To make the difference clear between the loading conditions of Eqs.(3) and (4), each surface of branches is divided into n parts and the resultant force on each domain is calculated (Fig.2). For the load of Eq.(4), the stress on the m th domain is:

$$\tau = -\frac{2\tau_0\sqrt{c_2}T}{\pi\sqrt{t}\sqrt{vt(m/n)}} \cos \frac{\theta}{2} \quad (5)$$

Multiplying the width v t/n of the domain, we have the resultant force as:

$$\tau_m = -\frac{2\tau_0\sqrt{c_2}\sqrt{v}T}{\pi\sqrt{mn}} \cos \frac{\theta}{2} \quad (6)$$

Force τ_m of Eq.(6) is independent on time. Similarly, for the load of Eq.(3), τ_m is of the form:

$$\tau_m = -\frac{2\tau_0\sqrt{c_2}\sqrt{v}\sqrt{t_0}\sqrt{t}}{\pi\sqrt{mn}} \cos \frac{\theta}{2} \quad (7)$$

By using Duhamel's integral formula,⁽⁹⁾ $k_{3,x}$ may be obtained from k_3^x as:

$$k_{3,x} \approx \frac{\sqrt{t_0}}{2T} \int_0^t \frac{k_3^x(\xi)}{\sqrt{t-\xi}} d\xi \quad (x=1,2) \quad (8)$$

Right hand side of Eq.(8) will yield a non-negligible error for higher crack propagation velocity, because of the shape change with time of the elastic body.

To investigate the error mentioned above, let us start from the case of the crack propagation in the direction of the crack plane. In case of $\theta=0$ in Eq.(4), k_3 has been solved as:⁽⁷⁾

$$k_3 = \frac{4\sqrt{2}\tau_0\sqrt{c_2}}{\pi^2} \sqrt{1-\frac{v}{c_2}} \frac{T}{\sqrt{t}} F\left(\frac{\pi}{2}, \sqrt{\frac{v}{c_2}}\right) \quad (9)$$

where F is a complete elliptic integral of the first kind. If we estimate k_3 for the case of $\theta=0$ in Eq.(3), it follows from Eqs. (8) and (9) that:

$$k_3 \approx \frac{2\sqrt{2}\tau_0\sqrt{c_2}}{\pi} \sqrt{1-\frac{v}{c_2}} \sqrt{t_0} F\left(\frac{\pi}{2}, \sqrt{\frac{v}{c_2}}\right) \quad (10)$$

An exact value of k_3 for this case has been shown as:⁽⁷⁾

$$k_3 = \frac{2\sqrt{2}\tau_0\sqrt{c_2}}{\pi} \sqrt{1-\frac{v}{c_2}} \sqrt{t_0} \quad (11)$$

By comparing Eqs.(10) and (11), the error in k_3 of Eq.(10) is $F(\pi/2, \sqrt{v/c_2})$. If we assume that the same error occurs for $\theta \neq 0$, a modification of Eq.(8) yields:

$$k_{3,x} \approx \frac{\sqrt{t_0}}{2TF(\pi/2, \sqrt{v/c_2})} \int_0^t \frac{k_3^x(\xi)}{\sqrt{t-\xi}} d\xi \quad (x=1,2) \quad (12)$$

The intensity factor k_3^x in Eq.(12) is produced from the load of Eq.(4) and this load condition makes the displacement self-similar. Now, to analyze k_3^x , we can employ the same method as shown in the paper (6).

In the physical plane shown in Fig.1, two-dimensional antiplane wave motions are governed by the following equation for the out-of-plane displacement $w(r, \theta, t)$:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial w}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} = \frac{1}{c_2^2} \frac{\partial^2 w}{\partial t^2} \quad (13)$$

Fig.1 Bifurcation of crack

Fig.2 Load on crack surface

Boundary conditions take the form:

$$\left. \begin{aligned} \theta = \pm \pi, \quad 0 < r; \quad \tau_{\theta r} = G \frac{1}{r} \frac{\partial w}{\partial \theta} = 0 \\ \theta_1 = \kappa_1 \pi, \quad 0 < r < r_{1t}; \quad \tau_{\theta r} = G \frac{1}{r} \frac{\partial w}{\partial \theta} = -\frac{2\tau_{01}\sqrt{c_1}T}{\pi\sqrt{r}\sqrt{t}} \cos \frac{\kappa_1 \pi}{2} \\ \theta = -\kappa_1 \pi, \quad 0 < r < r_{1t}; \quad \tau_{\theta r} = G \frac{1}{r} \frac{\partial w}{\partial \theta} = -\frac{2\tau_{01}\sqrt{c_1}T}{\pi\sqrt{r}\sqrt{t}} \cos \frac{\kappa_1 \pi}{2} \\ -\pi < \theta < \pi, \quad r = c_1 t; \quad w = 0 \end{aligned} \right\} \quad (14)$$

In Eq.(14), all the conditions are given by θ and a new variable $s=r/t$. This suggests that the displacement w is self-similar. Chaplygin's transformation $[\beta = \cosh^{-1}(c_2/s)]$ reduces Eq.(13) to Laplace's equation:

$$\frac{\partial^2 w}{\partial \beta^2} + \frac{\partial^2 w}{\partial \theta^2} = 0 \quad (15)$$

The above transformation maps the interior of the circular domain in Fig.1 into a semi-infinite strip in α -plane containing slits as shown in Fig.3. The boundary conditions on w in α -plane are indicated in Fig.3.

The slits CDE, EFG shown in Fig.3 are considered the semi-infinite cracks in the semi-infinite elastic strip. The order of the singularity on $\tau_{\theta r}^x = G(\partial w / \partial p^x)$, $\tau_{\psi z} = (G/p^x)(\partial w / \partial \psi^x)$ are both $1/\sqrt{p^x}$, and the following relation holds:

$$\tau_{\theta r}^x - i\tau_{\psi z}^x \approx \frac{K^x}{\sqrt{2p^x}} \exp\left(\frac{i\phi^x}{2}\right) \quad (x=1,2) \quad (16)$$

The strip in α -plane can be related to the upper half of the ζ -plane by means of a Schwarz-Christoffel transformation (Fig.4):

$$\alpha = \omega(\zeta), \quad \zeta = \xi + i\eta \quad (17)$$

An appropriate transformation is:

$$\omega(\zeta) = C_1 \int_1^\zeta \frac{(u+\xi_D)(u-\xi_F)}{(u+\xi_C)(u-\xi_G)\sqrt{1-u^2}} du + C_2 \quad (18)$$

The boundary conditions on the line $\eta=0$ are:

$$\left. \begin{aligned} -\infty < \xi < -1; \quad w=0, \quad -1 < \xi < -\xi_C; \quad \frac{\partial w}{\partial \eta} = 0 \\ -\xi_C < \xi < 0; \quad \frac{\partial w}{\partial \eta} = (\kappa_1 + \kappa_2) \frac{\xi_C \xi_G (\xi + \xi_D)(\xi - \xi_F)}{\xi_D \xi_F (\xi + \xi_C)(\xi - \xi_G) \xi \sqrt{1-\xi^2}} \frac{2c_1 \tau_{01} T}{G \pi \sqrt{\cosh \beta_1}} \cos \frac{\kappa_1 \pi}{2} = -\frac{\tau_{\theta r 1}}{G} \\ 0 < \xi < \xi_C; \quad \frac{\partial w}{\partial \eta} = (\kappa_1 + \kappa_2) \frac{\xi_C \xi_G (\xi + \xi_D)(\xi - \xi_F)}{\xi_D \xi_F (\xi + \xi_C)(\xi - \xi_G) \xi \sqrt{1-\xi^2}} \frac{2c_1 \tau_{01} T}{G \pi \sqrt{\cosh \beta_1}} \cos \frac{\kappa_1 \pi}{2} = -\frac{\tau_{\theta r 1}}{G} \\ \xi_C < \xi < 1; \quad \frac{\partial w}{\partial \eta} = 0, \quad 1 < \xi < \infty; \quad w=0 \end{aligned} \right\} \quad (19)$$

where

$$\left. \begin{aligned} \beta_1 = (1-\kappa_2) \ln \left(\frac{1+\xi_C \xi + \sqrt{1-\xi_C^2} \sqrt{1-\xi^2}}{\xi + \xi_C} \right) + (1-\kappa_1) \ln \left(\frac{1-\xi_C \xi + \sqrt{1-\xi_C^2} \sqrt{1-\xi^2}}{\xi_C - \xi} \right) \\ + (\kappa_1 + \kappa_2) \ln \left(\frac{1+\sqrt{1-\xi^2}}{\xi} \right) \\ \beta_2 = (1-\kappa_2) \ln \left(\frac{1+\xi_C \xi + \sqrt{1-\xi_C^2} \sqrt{1-\xi^2}}{\xi + \xi_C} \right) + (1-\kappa_1) \ln \left(\frac{1-\xi_C \xi + \sqrt{1-\xi_C^2} \sqrt{1-\xi^2}}{\xi_C - \xi} \right) \\ + (\kappa_1 + \kappa_2) \ln \left(\frac{1+\sqrt{1-\xi^2}}{-\xi} \right) \end{aligned} \right\} \quad (20)$$

The problem proposed here may be equivalent to that of an infinite body with a crack (crack length is 2) subjected to the antiplane shear load of Eq.(19). The values of $\partial w / \partial \xi$ at the points F,D yield: (11)

$$\left(\frac{\partial w}{\partial \xi} \right)_{\xi=\xi_{F,D}} = \frac{1}{G \pi \sqrt{1-\xi_{F,D}^2}} \left(\int_{-\xi_C}^0 \frac{\tau_{\theta r 1} \sqrt{1-\xi^2}}{\xi - \xi_{F,D}} d\xi + \int_0^{\xi_C} \frac{\tau_{\theta r 1} \sqrt{1-\xi^2}}{\xi - \xi_{F,D}} d\xi \right) \quad (x=1,2, \quad \zeta_1 = \xi_F, \quad \zeta_2 = -\xi_D) \quad (21)$$

The values of K^1, K^2 in Eq.(16) can be obtained from those of $\partial w / \partial \xi$ at F,D and it follows that:

$$K^x = \sqrt{2} G \lim_{\zeta \rightarrow \zeta_x} \sqrt{\omega(\zeta) - \omega(\zeta_x)} \frac{1}{\omega'(\zeta)} \frac{\partial w}{\partial \xi} \quad (x=1,2) \quad (22)$$

Fig.3 Boundary conditions in α -plane ($\alpha = \beta + i0$)

Fig.4 ζ -plane

Then, we have:

$$K^1 = \frac{1}{\pi} \sqrt{\frac{\xi_D \xi_F (\xi_F + \xi_C) (\xi_C - \xi_F)}{(\kappa_1 + \kappa_2) \xi_C \xi_D (\xi_F + \xi_D)}} \frac{1}{(1 - \xi_F^2)^{1/4}} \left(\int_{-\xi_C}^0 \frac{\tau_{yzt} \sqrt{1 - \xi^2}}{\xi - \xi_F} d\xi + \int_0^{\xi_D} \frac{\tau_{yzt} \sqrt{1 - \xi^2}}{\xi - \xi_F} d\xi \right) \dots (23)$$

$$K^2 = \frac{1}{\pi} \sqrt{\frac{\xi_D \xi_F (\xi_C - \xi_D) (\xi_D + \xi_C)}{(\kappa_1 + \kappa_2) \xi_C \xi_D (\xi_F + \xi_D)}} \frac{1}{(1 - \xi_D^2)^{1/4}} \left(\int_{-\xi_C}^0 \frac{\tau_{yzt} \sqrt{1 - \xi^2}}{\xi + \xi_D} d\xi + \int_0^{\xi_D} \frac{\tau_{yzt} \sqrt{1 - \xi^2}}{\xi + \xi_D} d\xi \right) \dots (23)$$

In the physical plane, the stress in the vicinity of the tip of the crack becomes:

$$\tau_{yz} \approx \frac{k_3^*}{\sqrt{2R^*}} f(\varphi^*) \quad (x=1, 2) \dots (24)$$

where $f(\phi^X)$ is determined from the crack propagation velocity as follows: (6)

$$f(\varphi^*) = \frac{\sin \varphi^*}{\sqrt{2} \sqrt{1 - (v_x/c_1)^2} \sqrt{1 - (v_x/c_2)^2} \sin^2 \varphi^*} \sqrt{\sqrt{1 - (v_x/c_1)^2} \sin^2 \varphi^* - \cos \varphi^*} + \frac{\cos \varphi^*}{\sqrt{2} \sqrt{1 - (v_x/c_1)^2} \sin^2 \varphi^*} \sqrt{\sqrt{1 - (v_x/c_2)^2} \sin^2 \varphi^* + \cos \varphi^*} \quad (x=1, 2) \dots (25)$$

The intensity factor k_3^X can be expressed as:

$$k_3^* = \frac{K^*}{\sqrt{v_x} \sqrt{t}} \left\{ 1 - \left(\frac{v_x}{c_1} \right)^2 \right\}^{1/4} \quad (x=1, 2) \dots (26)$$

Let us suppose a crack which bifurcates in two directions under angles $\chi_1\pi$, $-\chi_2\pi$ and with velocities v_1, v_2 at time t_0 after the out-of-plane shear wave strikes it. The stress intensity factors $k_{3,x}$ at tips of branches follow from Eq.(12) and Eq.(26) as:

$$k_{3,x} = \frac{\pi \sqrt{t_0}}{2 \sqrt{v_x} F(\pi/2, \sqrt{v_x/c_1}) T} \left\{ 1 - \left(\frac{v_x}{c_1} \right)^2 \right\}^{1/4} K^* \quad (x=1, 2) \dots (27)$$

Similarly, the intensity factor k_3 at the tip of a skewing crack can be determined:

$$k_3 = \frac{\pi \sqrt{t_0}}{2 \sqrt{v} F(\pi/2, \sqrt{v/c_1}) T} \left\{ 1 - \left(\frac{v}{c_1} \right)^2 \right\}^{1/4} K \dots (28)$$

where

$$K = -\frac{2c_1\tau_0 T}{\pi^2} \sqrt{1 - \kappa} \cos \frac{\kappa\pi}{2} \sqrt{\xi_C + \xi_E} (1 - \xi_E^2)^{1/4} \sqrt{\xi_E} \int_{-\xi_C}^{\xi_E} \frac{1}{(\xi + \xi_C)(\xi - \xi_E) \sqrt{\cosh \beta}} d\xi \dots (29)$$

$$\beta = (1 + \kappa) \ln \left(\frac{\sqrt{1 - \xi_C^2} \sqrt{1 - \xi_E^2} + \xi_C \xi_E + 1}{\xi + \xi_C} \right) + (1 - \kappa) \ln \left(\frac{\sqrt{1 - \xi_E^2} \sqrt{1 - \xi_C^2} - \xi_E \xi_C + 1}{\xi_E - \xi} \right) \dots (30)$$

In Fig.5, the points A ~ E in ζ -plane correspond to those in the physical plane, respectively.

3. Numerical calculation and results

The following nondimensional quantities are employed:

$$\left. \begin{aligned} v^* &= \frac{v}{c_1}, \quad \gamma^* = \frac{\gamma(v^*)}{\gamma(0)}, \quad g^* = \frac{g}{2\gamma(0)} \\ k_3^* &= \frac{\sqrt{\pi}}{2 \sqrt{c_1 \gamma(0)} \sqrt{G\rho}} k_3, \quad t^* = \frac{2\tau_0^2}{\pi \gamma(0) \sqrt{G\rho}} t \end{aligned} \right\} \dots (31)$$

where v : crack propagation velocity, k_3 : stress intensity factor, g : rate of release of energies, t : time, $\gamma(v^*)$: specific surface energy.

When a crack starts to propagate, the values k_3^* , g^* and t^* are equal to unity. It is well known that the stress intensity factor and the rate of release of energies must satisfy the following relation: (12)

$$g^* = k_3^{*2} / \sqrt{1 - v^{*2}} \dots (32)$$

If we suppose that a crack behaves satisfying the condition of the balance of energies, we obtain:

$$g^* = \gamma^* \dots (33)$$

If a stress wave impinges on a crack at time $t^*=0$ and it propagates at time $t^*=t_0^*$, the stress intensity factor is obtained from Eq.(2) as:

$$k_3^* = \sqrt{1 - v^{*2}} \sqrt{t_0^*} \dots (34)$$

The condition of the balance of energies is from Eqs.(32) ~ (34):

$$\sqrt{(1 + v^*)/(1 - v^*)} \gamma^* = t_0^* \dots (35)$$

For a simple case of $\gamma^* \equiv 1$, we obtain:

$$v^* = (t_0^{*2} - 1)/(t_0^{*2} + 1) \dots (36)$$

Fig.5 Skew of crack

The rate of release of energies per unit time follows:

$$S = \gamma^* \times v^* = (t_0^{*2} - 1) / (t_0^{*2} + 1) \quad \dots\dots\dots(37)$$

When the crack skews at time $t'^* = t_0^*$ after the stress wave strikes it at time $t'^* = 0$, the stress intensity factor is of the form, from Eq.(28),:

$$k_{s,*} = \frac{\pi^2(1-v^{*2})^{1/4}K}{4\sqrt{2}c_1\tau_0\sqrt{v^*}F(\pi/2, \sqrt{v^*})T}\sqrt{t_0^*} \quad \dots\dots\dots(38)$$

In the case that the crack bifurcates at time $t'^* = t_0^*$, it follows that:

$$k_{s,*} = \frac{\pi^2(1-v^{*2})^{1/4}K^*}{4\sqrt{2}c_1\tau_0\sqrt{v_s^*}F(\pi/2, \sqrt{v_s^*})T}\sqrt{t_0^*} \quad (x=1,2) \dots\dots\dots(39)$$

In Fig.6, the rate of release of energies for the skew crack has been plotted versus an angle K . From Fig.6, we note that the rate of release of energies takes the maximum at $K=0$. This means that a crack can not skew under the antiplane load.

In Fig.7, an example of a bifurcation crack is given. The rates of release of energies with respect to t_0^* have been plotted versus the ratio of the crack tip velocities of the bifurcation branches. The necessary condition of the balance of energies is:

$$g_1^*/t_0^* : g_2^*/t_0^* = \gamma^*(v_1^*) : \gamma^*(v_2^*)$$

We have $g_1^*/t_0^* = g_2^*/t_0^*$, if $\gamma^* \equiv 1$. Thus, in Fig.7, points of intersection of solid lines and dotted lines have special meaning. For an example of $K_1 = K_2 = 0.5$, $v_2^* = 0.1$, we obtain $g^*/t_0^* = 0.45$. The balance of energies $g^* = \gamma^*(v^*) = 1$ yields $t_0^* = 1/0.45 \approx 2.22$. The rate of release of energies S is given by $S = v_1^* + v_2^*$ for $\gamma^* \equiv 1$. Keeping $v_1^* + v_2^* = \text{constant}$, we can draw figures like Fig.7 for various combinations of K_1 and K_2 . If $v_1^* + v_2^* = 0.672$, for instance, points of intersection yield curves like Fig.8. In Fig.8, g^*/t_0^* takes the maximum value of 0.443 with $K_1 = K_2 (=0.22)$. This means that the crack may bifurcate at time $t_0^* = 2.26$. Similarly, we can obtain the result that g^*/t_0^* takes the maximum when the crack bifurcates symmetrically, if $\gamma^* \equiv 1$.

In Fig.9, the maximum rates of release of energies are plotted versus t_0^* . This figure shows that the rate of release of energies for bifurcation crack is faster than that for straight running crack if $t_0^* = 2.26$. This means that when a crack propagation velocity reaches the value of 0.672, the crack bifurcates with $K_1 = K_2 = 0.22$ and $v_1^* = v_2^* = 0.336$.

The above results are obtained assuming that a stress wave the front of which is parallel to a crack strikes it. It should be noted that these are also applicable in case of a more general loading condition where t_0^* means a parameter of the magnitude of the stress field. For the load of Eq.(3), if t_0^* becomes 2.26, the crack propagation velocity reaches $v^* = 0.672$. Conversely, when a crack with a speed $v^* = 0.672$ stops abruptly under any loading condition, a stress of Eq.(3) corresponding to $t_0^* = 2.26$ occurs at the tip of the crack.

Fig.6 Rate of release of energies (skew crack)

Fig.7 Rate of release of energies (bifurcation crack)

Fig.8 Rate of release of energies ($g_1^* = g_2^*$)

Fig.9 Crack propagation velocity and rate of release of energies ($\gamma^* \equiv 1$)

If, at this moment, the crack bifurcates with angles $\chi_1 = \chi_2 = 0.22$, the propagation velocities of bifurcation crack become $v_1^* = v_2^* = 0.336$. In this case, S becomes larger than that for a straight running crack.

In case of Mode III crack, after all, if $\gamma^* = 1$, when the crack propagation velocity reaches $0.672c_2$, it bifurcates at angles $\pm 0.22\pi$ and each tip propagates with the same speed of $0.336c_2$.

In general, γ^* may increase with v^* . In this case, if we assume that a crack bifurcates symmetrically, we can obtain the behaviour of the crack by modifying Fig.9. In Fig.9, for example, the conditions of $v_1^* = v_2^* = 0.25$ ($s = 0.5$) are satisfied at $t_0^* = 2.05$ which means $g^* = 1$. The condition of the balance of energies $g^* = \gamma^*(0.25)$ holds at $t_0^* = 0.25 \times \gamma^*(0.25)$. In Fig.10, chain lines are obtained by multiplying γ^* to t_0^* , and dotted lines by multiplying γ^* to S from Fig.9. In Fig.10, curves are obtained by employing the data of γ^* for PMMA from the paper (13). In Fig.10, a crack in PMMA may bifurcate at $t_0^* = 3.55$ with velocity $v^* = 0.28$ and the velocities of branches will be $v_1^* = v_2^* = 0.20$. In Fig.11, $g^*/\gamma^*t_0^*$ is plotted versus χ_2/χ_1 for $S = 0.74$. From Fig.11, we can confirm that a crack in PMMA bifurcates symmetrically.

Finally, we can reach the conclusion that the crack in PMMA bifurcates at the velocity $v^* = 0.28$ at angles $\pm 0.22\pi$ and with velocities $v_1^* = v_2^* = 0.20$.

Fig.10 Crack propagation velocity and rate of release of energies (PMMA)

Fig.11 Rate of release of energies (PMMA, $S = 0.74$)

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