539.384.6 : 539.413

Paper No. 185-5

Theoretical and Experimental Researches on Strength of a Rotary Kiln*

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In this paper the strength of a rotary kiln, which is widely used in the field of chemical industries, is discussed from both theoretical and experimental approaches. The analysis is made exactly by regarding the rotary kiln as a cylindrical shell and a full-sized model of kiln is used in the experiment. In the analysis the various factors of loads, such as the weight of kiln and deformation of tire, force of the loading block acting on tire, those due to the contents and driving equipment, etc., are taken into consideration. A detailed discussion on the results of analysis and experiment confirms the importance of an appropriate design for the blocks between kiln and tire. Furthermore the effect of thermal stress on the strength of the rotary kiln is examined.

pertinent design of rotary kiln.

1. Introduction

A rotary kiln is one of the large-scale machines widely used in the field of the chemical industry. There seems, however, to be no prior study which has discussed in detail the strength of the rotary kiln. Especially there may occasionally occur a failure, which is supposed to be attributable to the fatigue of a shell body near the tire, but the cause of the failure has not been fully examined so far. It is understood that this comes from the fact that in order to know the stress state near the tire it is necessary to take into account the relationships among the loading blocks for tire, the shell and the fire-bricks, which makes the theoretical approach very complicated.

In this paper an analysis of the rotary kiln, which is approximately regarded as a cylindrical shell, is carried out exactly in consideration of the thermal stresses as well as the loading blocks, and an experiment is performed with a full-sized model of kiln, whose data are compared with the theoretical predictions.

It is pointed out that the stresses produced in the shell near the tire depend largely upon the structure of the loading blocks, which confirms the importance of a

2. General remarks

In the analysis the following assumptions are made (refer to the schematic drawing shown in Fig.1):

(1) A rotary kiln is considered a cylindrical shell with a constant thickness and radius.

(2) Both ends of the rotary kiln are freely supported.

(3) The brick lining has a constant thickness and is not in contact with shell wall, but with a rise of temperature it comes into contact with shell wall. Hence the curvatures

Fig.1 Schematic drawing of rotary kiln

Fig.2 Shell element: displacement, stress resultants, stress couples and surface forces

* Received 5th April, 1979.

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after the deformation of a cylinder and of bricks are identical with each other. The skill of the lining work is related to the degree of contact, which makes the theoretical analysis much more difficult. In order to avoid this difficulty the method of changing the elastic modulus of the brick is adopted in this analysis.

(4) To be on the safe side, the friction between the bricks and the cylinder is disregarded. Therefore no in-plane strain occurs in the brick. A certain amount of the moment caused by the inclination of the contents in the kiln is transmitted to the cylinder.

(5) As the temperature rises, the clearance between the tire and the loading blocks reduces, resulting in a contact.

(6) A slight inclination of the kiln is neglected.

(7) The pressure distribution acting on the surface of an individual loading block is uniform.

(8) The kiln rotates so slowly that the contents are accumulated uniformly with a slight slope at the bottom of the kiln.

Using these assumptions and referring to Fig. 2, the relations between the displacement components and the internal forces in a shell body and the bricks are expressed as follows ⁽¹⁾:

$$\left. \begin{aligned} T_x &= (12D/h^2) [\partial u / \partial x + (\nu/a) (\partial v / \partial \theta + w)], & T_\theta &= (12D/h^2) [\nu \partial u / \partial x + (\partial v / \partial \theta + w)/a] \\ S &= \{6(1-\nu)D/h^2\} [\partial v / \partial x + \partial u / a \partial \theta], & S' &= S - \mathfrak{M}_b/a \\ M_x + M_{xb} &= (D + D_b) [\partial^2 w / \partial x^2 + (\nu/a^2) (\partial^2 w / \partial \theta^2 - \partial v / \partial \theta)] \\ M_\theta + M_{\theta b} &= (D + D_b) [\nu \partial^2 w / \partial x^2 + (\partial^2 w / \partial \theta^2 - \partial v / \partial \theta)/a^2] \\ \mathfrak{M}_x + \mathfrak{M}_b &= \{(1-\nu)(D + D_b)/a\} [\partial^2 w / \partial x \partial \theta - \partial v / \partial x] \\ D &= Eh^3/12(1-\nu^2), & D_b &= E_b h_b^3/12(1-\nu^2) \end{aligned} \right\} \dots\dots(1)$$

where suffix *b* represents the quantities for the bricks, and the displacements and the radius of the brick correspond to the quantities which are moved to the position equivalent to the shell. The reason is that because of the uncertainty of the mutual behavior between the bricks and the shell a simple approximation is introduced for the bricks.

Now if we put

$$D + D_b \equiv D_m, \quad D/h^2 = D_m/h_m^2 \dots\dots\dots(2)$$

and denote the sum of the quantities for the shell and the bricks by the subscript *s*, Eqs.(1) may be written

$$\left. \begin{aligned} T_x &= (12D_m/h_m^2) [\partial u / \partial x + (\nu/a) (\partial v / \partial \theta + w)], & T_\theta &= (12D_m/h_m^2) [\nu \partial u / \partial x + (\partial v / \partial \theta + w)/a] \\ S &= \{6(1-\nu)D_m/h_m^2\} [\partial v / \partial x + \partial u / a \partial \theta], & S' &= S - \mathfrak{M}_s/a \\ M_x + M_{xs} &= D_m [\partial^2 w / \partial x^2 + (\nu/a^2) (\partial^2 w / \partial \theta^2 - \partial v / \partial \theta)], & M_\theta + M_{\theta s} &= D_m [\nu \partial^2 w / \partial x^2 + (\partial^2 w / \partial \theta^2 - \partial v / \partial \theta)/a^2] \\ \mathfrak{M}_x + \mathfrak{M}_s &= \{(1-\nu)D_m/a\} [\partial^2 w / \partial x \partial \theta - \partial v / \partial x], & N_{xs} &= \partial M_{xs} / \partial x + \partial \mathfrak{M}_s / a \partial \theta, & N_{\theta s} &= \partial M_{\theta s} / a \partial \theta + \partial \mathfrak{M}_s / \partial x \end{aligned} \right\} \dots\dots\dots(3)$$

The utilization of D_m and h_m makes it possible to employ the general formulas of a cylindrical shell obtained already.

3. External forces acting on shell

3.1 Weight of shell and brick

As for the weight of the brick, as has already been described in Eqs.(1), the equivalent quantities moved to the position of the shell are used.

$$\left. \begin{aligned} p &= -(h_b' \gamma_b + h \gamma) \cos \theta \\ q &= (h_b' \gamma_b + h \gamma) \sin \theta, & f &= 0 \end{aligned} \right\} \dots\dots\dots(4)$$

where

$$h_b' a = h_b [a - (h + h_b)/2]$$

Fig. 3 Position of tire and loading blocks

3.2 Force exerted by loading blocks for tire

There are twelve loading blocks along the periphery of the tires as shown in Fig. 3. Since the forces exerted by the blocks upon the shell are piecemeal, they are expanded into double Fourier series in the *x* and *θ* directions. The series expanded in *x* direction are divided into two cases in consideration of the boundary conditions (19) at both ends, described in the following section, i.e. one is a symmetric function with a period of $2L$ and the other an antisymmetric one with a period of $4L$. Let us denote the unknown pressures acting on the blocks for the symmetric part by p_n ($n=0,1,\dots,11$),

Fig. 4 Diagram of contents

and those for the antisymmetric part by p_n' ($n=0,1,\dots,11$). Therefore the double series contain the unknown quantities p_n and p_n' in their coefficients, respectively.

3.3 Load caused by contents in kiln

The rotation of a kiln causes the inclination of the contents by α with respect to a vertical line. For simplicity, the free surface of the contents is assumed to be a plane as shown in Fig.4. Let us consider a certain point of the contents. Denote the average pressure by p , the shearing stress by τ and the density of the contents material by γ' , and we have

$$\left. \begin{aligned} -\partial p/\partial x + \partial \tau/\partial y + \gamma' \sin \alpha &= 0 \\ -\partial p/\partial y + \partial \tau/\partial x + \gamma' \cos \alpha &= 0 \end{aligned} \right\} \dots\dots\dots (5)$$

Hence

$$-\partial^2 p/\partial x^2 + \partial^2 p/\partial y^2 = 0 \dots\dots\dots (6)$$

The solution of this equation is given by

$$p = F(x+y) + \phi(x-y) \dots\dots\dots (7)$$

The condition that the pressure p is zero at $y=0$ leads to

$$p = F(x+y) - F(x-y) \dots\dots\dots (8)$$

Substituting Eq.(8) into the first and the second of Eqs.(5) and integrating them with respect to y and x , respectively, we get

$$\left. \begin{aligned} \tau &= -\gamma' y \sin \alpha + F(x+y) + F(x-y) + \lambda(x) \\ \tau &= -\gamma' x \cos \alpha + F(x+y) + F(x-y) + \mu(y) \end{aligned} \right\} \dots\dots\dots (9)$$

$$p_n = p \cos^2 \varphi + p \sin^2 \varphi - 2\tau \sin \varphi \cos \varphi = \gamma' \bar{y} \cos \alpha + \gamma' \bar{y} \sin \alpha \sin 2\varphi$$

$$\tau_n = p \sin \varphi \cos \varphi - p \cos \varphi \sin \varphi + \tau (\sin^2 \varphi - \cos^2 \varphi) = \gamma' \bar{y} \sin \alpha \cos 2\varphi$$

$\bar{y}$ represents the value of y at the point P . Substituting the relations (referring to Fig.4)

$$a' \cos \varphi = a' \cos \alpha_0 + \bar{y} \quad \text{and} \quad \theta - \varphi = \pi - \alpha$$

into the above expressions and denoting the new set equivalent values at radius a by p_n and τ_n , we obtain

$$\left. \begin{aligned} p_n &= (a'^2 \gamma' / a) \{ -\cos(\theta + \alpha) - \cos \alpha_0 \} \{ \cos \alpha + \sin \alpha \sin 2(\theta + \alpha) \} \\ \tau_n &= (a'^2 \gamma' / a) \{ -\cos(\theta + \alpha) - \cos \alpha_0 \} \sin \alpha \cos 2(\theta + \alpha) \\ &\quad \pi - (\alpha_0 + \alpha) < \theta < \pi + (\alpha_0 - \alpha) \end{aligned} \right\} \dots\dots\dots (13)$$

zero at the other range of θ

Expressions (13) are again expanded into single Fourier series in the θ direction.

3.4 Load from driving equipment

3.4.1 Turning torque

The turning torque is transmitted to the shell through twelve supporting blocks situated along the periphery of the equipment. Assume that the torque is uniformly transmitted and the shearing stress σ produced in each block is constant. Then these quantities may be expanded into double Fourier series referring to Fig.5. In this case only the external force q is considered and it is expanded into a periodic function with a period of $4L$ with respect to y in consideration of the boundary conditions (19) at both ends.

3.4.2 Turning force

Since the turning force is transmitted through thin springs, there also occurs a shearing stress at each block. This force is considered as $\tau \sin \varphi$, in which τ is a constant shearing stress and φ is a subtend angle from the vertical line to the diameter through the driving point. As before, this force may be expanded into double Fourier series with the period of $4L$ with respect to y . Since the turning torque in 3.4.1 is a moment related to the turning force with respect to its center, the equilibrium condition leads to the following relation.

$$\tau = 2\sigma \dots\dots\dots (14)$$

3.5 Weight of driving equipment

The weight of the driving equipment is neglected because it may be considerably small compared with those of the shell, bricks and the contents.

4. Method of solution

The external forces acting on a shell are those for p and q . In this case the differ-

Fig.5 Position of driving equipment and its sketch

$\lambda(x)$ and $\mu(y)$ are the arbitrary functions of x and y , respectively. The condition that two equations in Eqs.(9) are identical to each other requires

$$\lambda(x) = -\gamma' x \cos \alpha, \quad \mu(y) = -\gamma' y \sin \alpha$$

Therefore

$$\tau = -\gamma' x \cos \alpha - \gamma' y \sin \alpha + F(x+y) + F(x-y) \dots\dots\dots (10)$$

$\tau = 0$ at $y = 0$ gives

$$F(x+y) = \gamma' (x+y) \cos \alpha / 2 \dots\dots\dots (11)$$

$$F(x-y) = \gamma' (x-y) \cos \alpha / 2$$

Substitution of this into Eqs.(8) and (10) yields

$$p = \gamma' y \cos \alpha, \quad \tau = -\gamma' y \sin \alpha \dots\dots\dots (12)$$

Denote the pressure in the radial direction by p_n and the shearing stress by τ_n at P , a point at the inner wall of the brick shown in Fig.4. Then by making use of Eqs.(12) we have

ential equation in terms of the radial displacement w is given as follows⁽²⁾:

$$\begin{aligned} A_m \frac{\partial^3 w}{\partial x^3} + \frac{4B_m}{a^2} \frac{\partial^3 w}{\partial x^2 \partial \theta^2} + \frac{6C_m}{a^4} \frac{\partial^3 w}{\partial x^2 \partial \theta^4} + \frac{8-2\nu^2}{a^4} \frac{\partial^3 w}{\partial x^2 \partial \theta^2} + \frac{6L_m}{a^4} \frac{\partial^3 w}{\partial x^2} + \frac{\partial^2}{a^6 \partial \theta^2} \left(4 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{a^2 \partial \theta^2} \right) \left(\frac{\partial^2}{\partial \theta^2} + 1 \right) w \\ = \frac{1}{D_m} \left[A_m \frac{\partial^4 p}{\partial x^4} + \frac{2U_m}{a^2} \frac{\partial^4 p}{\partial x^2 \partial \theta^2} + \frac{B_m}{a^4} \frac{\partial^4 p}{\partial \theta^4} \right] - \frac{1}{D_m} \left[\frac{(2-\nu)}{6(1-\nu)} \left(\frac{h_m}{a} \right)^2 \frac{\partial^3 q}{\partial x^2 \partial \theta} \right. \\ \left. + \frac{(4-3\nu+\nu^2)}{12(1-\nu)a^2} \left(\frac{h_m}{a} \right)^2 \frac{\partial^3 q}{\partial x^2 \partial \theta^3} + \frac{1}{12a^4} \left(\frac{h_m}{a} \right)^2 \frac{\partial^3 q}{\partial \theta^3} - \frac{2+\nu}{a^2} \frac{\partial^3 q}{\partial x^2 \partial \theta} - \frac{\partial^3 q}{a^4 \partial \theta^3} \right] \dots \dots \dots (15) \end{aligned}$$

where

$$\begin{aligned} A_m = 1 + (h_m/a)^2/3, \quad B_m = 1 + (h_m/a)^2/12, \quad C_m = 1 + (1-\nu^2)(h_m/a)^2/72 \\ L_m = 2(1-\nu^2)A_m(a/h_m)^2, \quad U_m = 1 + (2-2\nu+\nu^2)(h_m/a)^2/12(1-\nu) \end{aligned}$$

p and q in Eq.(15) contain all the external forces described in 3.1 through 3.4. A particular solution w found from this expression is also represented by the double Fourier series, whose coefficients include p_n , p_n' and σ , τ . In finding this particular solution we obtain two systems of equations from the condition that the terms with $\sin\theta$ and $\cos\theta$ disappear.

Substituting this particular solution into $w \sim u$ and $w \sim v$ relations⁽³⁾, which do not include p and q , we find u and v for this particular solution, respectively.

The consideration of the moment equilibrium about an axis of the shell gives

$24\sigma\delta t + a'^2(L/a)\gamma'\sin\alpha(\sin\alpha_0 + \sin 3\alpha_0/3 - \cos\alpha_0\sin 2\alpha_0) = 0$ (16)

From this σ is found and then from (14) τ is obtained. Hence the unknown constants reduce to p_n and p_n' ($n=0,1,\dots,11$) only.

In addition to the above particular solution, the following solutions due to the uniform bending are taken into account.

$$w_C = M_C x^2 \cos\theta, \quad u_C = -2aM_C x \cos\theta, \quad v_C = -M_C(x^2 - 2\nu a^2) \sin\theta \dots \dots \dots (17)$$

and

$$w_S = M_S x^2 \sin\theta, \quad u_S = -2aM_S x \sin\theta, \quad v_S = M_S(x^2 - 2\nu a^2) \cos\theta \dots \dots \dots (18)$$

where M_C and M_S are unknown constants.

5. Boundary conditions at both ends of kiln

The end condition of the kiln may be considered freely supported as a whole, but $T_x=0$ does not always hold along all the circumference of the edges because of the attached frame made of steel and the resistance of the bricks. Therefore the following boundary conditions at the ends are adopted⁽⁴⁾.

$$\left. \begin{aligned} \int_{-\pi}^{\pi} T_x a d\theta = 0, \quad \int_{-\pi}^{\pi} (T_x a - M_{xx}) \cos\theta a d\theta = 0, \quad \int_{-\pi}^{\pi} (T_x a - M_{xx}) \sin\theta a d\theta = 0, \quad S'_s - \mathfrak{M}_s/a = 0 \\ N_{xs} + \partial \mathfrak{M}_s / a \partial \theta = 0, \quad u = a\zeta_c \cos\theta + a\zeta_s \sin\theta + \xi, \quad \partial w / \partial x = -\zeta_c \cos\theta - \zeta_s \sin\theta \end{aligned} \right\} \dots \dots \dots (19)$$

where ζ_C , ζ_S and ξ are unknown constants. By considering the principle of energy we can prove that the above expressions are the sufficient conditions.

Substituting each displacement found in the chapter 4 into Eqs.(3), T_x , T_θ , ..., $N_{\theta s}$ corresponding to the displacements may be obtained.

Judging from the expansion form of the loads caused by the loading blocks and the driving equipment and the form of solutions of Eqs.(17) and (18), it is understood that the sixth and the seventh conditions in expressions (19) are automatically satisfied. The first condition can be satisfied by the selection of the absolute terms of the expansion formula. The fourth and the fifth conditions are also automatically satisfied in a similar manner to the expansion form of the loads and the forms of Eqs.(17) and (18) as in the case of the sixth and the seventh conditions. Finally we arrive at the conclusion that we have only to consider the second and the third conditions. Substitution of T_x and M_{xx} obtained into the second and the third conditions in Eqs.(19) leads to four sets of equations for $x=\pm L$.

6. Stresses in tire

The tire is subjected to the action of the loading blocks and the outside rollers. Here the tire is treated as a two-dimensional problem because the effect of the intrusion produced by these forces is considerably large and the method by the curved beam would give erroneous results. The cross section of the tire is of the form $\square$ shown in Fig.6, but since this section makes the

Fig.6 Diagram of tire

analysis more complicated, a rectangular cross section with a width of $2t$ and the height of b having the equivalent bending rigidity as a beam is considered. Denote the moment of inertia of area for the original cross section by I and that for the rectangular cross section by I' , and we have $IE_t = I'E_t'$ where $I' = tb^3/6$ (20)

The stress function ϕ satisfies the following differential equation⁽⁵⁾.

$$(\partial^2/\partial r^2 + \partial/r\partial r + \partial^2/r^2\partial\theta^2)\phi = 0 \text{(21)}$$

The solution of Eq.(21) is represented by the sum of the Fourier expansion with respect to θ and the terms of $\theta\sin\theta$ and $\theta\cos\theta$. The stresses in the radial and the circumferential directions and the shearing stress are, respectively, given as

$$\partial\phi/r\partial r + \partial^2\phi/r^2\partial\theta^2, \partial^2\phi/\partial r^2, -\partial(\partial\phi/r\partial\theta)/\partial r$$

The radial stress at $r=r_1$ is equal to the pressure of loading block, and the one at $r=r_2$ to the pressure of the supporting roller. The shearing stresses at $r=r_1$ and $r=r_2$ are both equal to zero. Using these conditions, all the unknown constants in the solutions of Eq.(21) can be represented in terms of p_n and p_n' ($n=0,1,\dots,11$).

7. Determination of unknowns p_n and p_n'

In order to determine the unknowns it is necessary to consider the deformation of the tire.

Denote the radial and the circumferential displacements by u and v , respectively, and we have

$$\left. \begin{aligned} \partial u/\partial r &= (\sigma_r - \nu\sigma_\theta)/E_t' \\ \partial v/r\partial\theta + u/r &= (\sigma_\theta - \nu\sigma_r)/E_t' \\ \partial u/r\partial\theta + \partial v/\partial r - v/r &= \tau/G_t' \end{aligned} \right\} \text{(22)}$$

$$G_t' = E_t'/2(1+\nu)$$

Substituting σ_r and σ_θ obtained above into the first and the second equations of Eqs. (22) and integrating these equations, we obtain u and v with arbitrary integral functions, which can be determined from the following conditions: the condition that the substitution of u , v into the third equation of Eqs.(22) makes both sides of this equation identically equal to each other, and the condition that the displacement u of the tire is zero at the contact point of the tire and the roller. On the other hand, the value of w consists of the sum of the particular solution described in the chapter 4, the values of Eqs.(17) and (18), and those of the rigid translation and rotation as given by

$$\left. \begin{aligned} \omega_t &= A_c \cos\theta, & s\omega_t &= A_s \sin\theta \\ \omega_r &= B_c x \cos\theta, & s\omega_r &= B_s x \sin\theta \end{aligned} \right\} \text{(23)}$$

The summation of these w should be equal to u at $r=r_1$ of the tire.

Since each tire has 12 loading blocks, we have 24 equations. In addition to this, there are 4 conditions at both ends described in the chapter 5 and 2 equations found in the derivation of the particular solution of w . Hence we finally get 30 equations. On the other hand, the number of unknowns is also 30, that is to say, 24 for p_n ($n=0,1,\dots,11$) and p_n' ($n=0,1,\dots,11$), M_c

and M_s in Eqs.(17) and (18), and A_c , B_c , A_s , B_s in Eqs.(23). Therefore all the unknown constants can be determined by solving these thirty simultaneous equations. In this case it should be noted that $p_n + p_n'$ and $p_n - p_n'$ caused by the loading blocks must be always compressive or zero in view of the structure of the blocks and the tire. This is done in the following way: if we have some solutions with positive sign among the solutions of the above simultaneous equations, we put some of them zero and iterate this procedure until all the solutions become negative and/or zero.

8. Dimensions of kiln used in experiment

The analytical results developed so far are applied to a kiln being in use and the validity of the design of the kiln is examined. For this purpose a full-sized model of kiln is made in order to compare the analytical results with the experimental ones. The dimensions of the kiln are given, referring to Fig.1, as

$$2a=1212\text{mm}, 2L=10100\text{mm}, h=12\text{mm}$$

$$l=3050\text{mm}, h_b=150\text{mm}$$

$$E=2.1 \times 10^5 \text{kg/cm}^2, E_b=1.0 \times 10^5 \text{kg/cm}^2 \sim 0$$

$$\nu=0.3, \gamma=7.85 \times 10^{-3} \text{kg/cm}^3$$

$$\gamma_b=2.0 \times 10^{-3} \text{kg/cm}^3$$

the base area of a loading block=160×100mm²

the dimensions of tire $b=82.5\text{mm}$, $t=30\text{mm}$

$c=100\text{mm}$, $r_2=1468\text{mm}$, the position of

supporting roller=position making an angle

30° with a vertical line (refer to Fig.6),

$l_d=2050\text{mm}$, the base area of a supporting

block of driving equipment=250×60mm², the

setting angle of the gear of motor=the

position making an angle 30° with a vertical

line, $\alpha=37^\circ$ (actual measurement, referring

to Fig.4), $\gamma'=1.45 \times 10^{-3} \text{kg/cm}^3$, $2a_0=115^\circ$

$\alpha'=450\text{mm}$.

9. Method of experiment

It is expected that the largest stress occurs in the vicinity of both tires and next in the place near the driving equipment. The measurement of the stresses is mainly made in these parts. The strain gauges which can measure the strains in the axial, circumferential directions and at an angle of 45° with these directions are pasted on the shell along both sides of the tire at the position nearly 50mm apart from the tire. In the neighborhood of the driving equipment the same type of the strain gauges are pasted at 70mm distance from the edge of the equipment. The measurements are made for every 30° rotation of an empty kiln based on the values of the uppermost position.

10. Numerical results and discussion

The dimensionless values of the displacement, T_x , T_θ , M_x and M_θ are illustrated in Figs.7 through 9, in which the angle from the uppermost position to 360° of one turn is taken on the abscissa. The figures indicate the values near the tire and near the gear, and at the center of the kiln

Fig.7 Radial displacement and stresses near tire

Fig.8 Radial displacement at the center of kiln

whose stresses are so small that only the displacement is depicted in the figure. The figures show that M_x and M_θ near the tire oscillate with a large amplitude, which suggests that the effect of the loading blocks on the stresses is so large.

Figures 10 and 11 illustrate the numerical results and the experimental data on an empty kiln. The numerical values based on the uppermost position are selected so as to correspond with the experimental data. The values for $\bigcirc$ and $\bigotimes$ or $\bullet$ and $\circ$ would coincide with each other if the kiln were manufactured in a geometrically perfect form. It should be noted that the discrepancy between the numerical results and the experimental ones is as a whole so much wider than the irregularity of the experimental data. The experimental values at the center of the kiln are in good agreement with the numerical ones except for some irregularity in the measurement, but they are omitted here because they are too small.

The reason why both results in the experiment and calculation near the tire do not coincide with each other is considered

Fig.9 Radial displacement and stresses in the neighbourhood of gear

in the following way.

Under a uniform distribution of pressures at the base of the block as assumed in the calculations, the shell would generally deform as shown in Fig.12. But in reality the blocks of the test kiln are welded to the shell at four edges with hard steel plates and further a hard splice metal is welded on them. Each block is also connected with a band steel by welding. Therefore the deformation as shown in Fig.12 is constrained, and the pressure distribution will no longer be uniform. Consequently

Fig.10 Surface stresses at the inside of tire (Comparison with experimental data)

Fig.11 Surface stresses in the neighbourhood of gear (Comparison with experimental data)

we have to consider the state of shell subjected to a line load at each point of A and B. It might be possible to get the solutions for this case, but an analysis would be extremely complicated and not practical. The deformation of the test kiln is nearly axisymmetric on account of the band steel. Hence let us examine the axisymmetric case in order to anticipate the general behavior of the deformation. Consider a shell under a line load P per unit length at A and B, as shown in Fig.13, then the bending moment M_{xA} at A and the displacements w_A , w_C at A and C are given, respectively, as follows⁽⁶⁾:

$$M_{xA} = (P/4\beta) [1 + e^{-2\beta s} (\cos 2\beta s - \sin 2\beta s)] \quad \dots\dots\dots (24)$$

$$\left. \begin{aligned} w_A &= (P/8\beta^3 D) [1 + e^{-2\beta s} (\cos 2\beta s + \sin 2\beta s)] \\ w_C &= (P/4\beta^3 D) e^{-\beta s} (\cos \beta s + \sin \beta s) \end{aligned} \right\} \quad \dots\dots\dots (25)$$

where

$$\beta^4 = 3(1 - \nu^2)/a^2 h^2$$

Substituting the values of a , h and $2s = 16\text{cm}$ into Eqs.(24) and (25), we have

$$\left. \begin{aligned} M_{xA} &= 0.87338(P/4\beta) \\ w_A &= 0.99298(P/8\beta^3 D) \\ w_C &= 0.77322(P/8\beta^3 D) \end{aligned} \right\} \quad \dots\dots\dots (26)$$

The corresponding values $\bar{M}_{xA}$, $\bar{w}_A$, $\bar{w}_C$ under the uniformly distributed load p_0 along A to B may be obtained by integrating the values for the line load, that is,

$$\left. \begin{aligned} \bar{M}_{xA} &= (P/4\beta^2 s) e^{-2\beta s} \sin 2\beta s \\ \bar{w}_A &= (P/8\beta^4 s D) [1 - e^{-2\beta s} \cos 2\beta s] \\ \bar{w}_C &= (P/4\beta^4 s D) [1 - e^{-\beta s} \cos \beta s] \end{aligned} \right\} \quad \dots\dots\dots (27)$$

where

$$p_0 s = P$$

Substitution of a , h and $2s$ into the above equations gives

$$\left. \begin{aligned} \bar{M}_{xA} &= 0.04959(P/4\beta) \\ \bar{w}_A &= 0.88473(P/8\beta^3 D) \\ \bar{w}_C &= 1.485(P/8\beta^3 D) \end{aligned} \right\} \quad \dots\dots\dots (28)$$

Fig.12 Shell under uniform load

Fig.13 Shell under line loads

Next let us consider a shell under the line loads of ξP at A and B and under a uniform load with the magnitude of $2\eta P$ along AB, as shown in Fig.14. Denote the deformation at the points A and B by w_A' and the deformation at the center of the shell by w_C' , and they are found from Eqs.(26) and (28). From $w_A' = w_C'$ we have

$$\xi = 0.73202, \quad \eta = 0.26798 \quad \dots\dots(29)$$

The bending moment M_{xA}' at A in Fig.14 is thus determined from Eqs.(26) and (28) as follows:

$$M_{xA}' = (0.87338\xi + 0.04959\eta)(P/4\beta) \\ = 0.65262(P/4\beta) \quad \dots\dots\dots(30)$$

The ratio of this value to that for the uniform load becomes

$$M_{xA}'/\bar{M}_{xA} = 0.65262/0.04959 = 13.16 \quad \dots\dots(31)$$

which means that M_{xA}' , the bending moment in the case of being firmly welded, is about 13.16 times larger than that for the uniform load. Moreover in this case $\bar{M}_x$ is far larger than $\bar{M}_\theta$ because of $\bar{M}_\theta = \nu \bar{M}_x$.

Although the above discussion is confined to the axisymmetric case, the actual state of the test kiln is fairly similar to this case, because the blocks are connected with a band steel by welding. This may be the main reason why the numerical results do not coincide with the experimental ones. This can also be recognized from the fact that (a) and (b) in Figs.10 show that σ_θ is in considerably good agreement with the numerical values, while σ_x in the experiment is larger than σ_θ .

Therefore we can conclude that the loading blocks should be designed so as not to be rigid, but to be flexible. This kind of phenomenon is essentially the same as in the case of a rigid saddle of a horizontal reservoir⁽⁷⁾. In fact a kiln larger than the present test kiln manufactured based on the concept of flexible design exhibits a test measurement that σ_θ is larger than σ_x and its ratio is close to our numerical results, showing nearly uniform pressure distribution in the block.

Next let us consider the strength of a shell near the loading block. Since the kiln rotates, the fatigue strength should be taken into account. For this purpose it is necessary to find the peak and the bottom of stresses σ_x , σ_θ experienced during one turn of the kiln. This may be done by making the construction line from Figs.10 (a) and (b), an example of which is shown in Fig.15. Figures 10 indicate that the peak and the

Fig.14 Shell under a uniform load and line loads

bottom of the maximum σ_x almost coincide with those of the maximum σ_θ . Therefore when an appropriate design of the block based on the uniform pressure distribution is made, the amplitudes of σ_θ and σ_x are given from this construction line in the case when the contents are neglected and $E_b = 0$, respectively, as follows:

$$\left. \begin{aligned} (7\gamma a^2/h + 15\gamma a^2/h)/2 &= 11.0\gamma a^2/h \\ (3.4\gamma a^2/h + 3.8\gamma a^2/h)/2 &= 3.6\gamma a^2/h \end{aligned} \right\} \dots\dots(32)$$

The shearing strain energy theory⁽⁸⁾ states that the fatigue begins when

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2k^2 \quad (33)$$

If the effect of the shearing stress is considered to be slight, the stress state near the blocks is

$$\sigma_1 = \sigma_\theta, \quad \sigma_2 = \sigma_x, \quad \sigma_3 = 0 \quad \dots\dots\dots(34)$$

Substitute the relations (32) into these expressions, and the root of the left-hand side of Eq.(33) becomes $13.7375\gamma a^2/h$ and the substitution of $\gamma a^2/h = 24.0234\text{kg/cm}^2$ leads to

$$330.021\text{kg/cm}^2 \quad \dots\dots\dots(35)$$

If we select the value of k as 2200kg/cm^2 ⁽⁹⁾ though it may be different depending on the material and the safety factor of 4 (the effect of the contents is included in this factor), the root of right-hand side of Eq.(33) becomes 777.817kg/cm^2 , which suggests that the shell is for the present safe from the fatigue, because the value of Eq.(35) is smaller than this one. $E_b \neq 0$ is more on the safe side.

When the loading block is made as rigid as the test kiln, that is, when the experimental data (the values for Φ , though with considerable irregularities) are taken, the amplitudes of σ_θ and σ_x are, respectively, $(0.5+12.9)\gamma a^2/2h = 6.7\gamma a^2/h$, $(3.4+21.0)\gamma a^2/2h = 12.2\gamma a^2/h$. With these values the root of left-hand side of Eq.(33) becomes

$$359.537\text{kg/cm}^2 \quad \dots\dots\dots(36)$$

which is also a safe value because this is smaller than 777.817kg/cm^2 . In this case, however, the loading blocks and the band steel being welded together causes a stress concentration because of an abrupt change of the cross section at this position. Take the strength reduction factor of 2.5⁽¹⁰⁾, and the stress produced is $359.537 \times 2.5 = 898.843\text{kg/cm}^2$, which is beyond 777.817kg/cm^2 , and it may cause a crack, requiring the adoption of a design based on the flexible character.

Fig.15 Example of construction (for σ_θ in Fig.10 (b))

11. Thermal stresses in kiln

A general differential equation with regard to the temperature distribution of a cylindrical shell has been already introduced by Mizoguchi⁽¹¹⁾, but this is not directly applicable to the present problem, because in a rotary kiln employed the shell and the bricks are in contact with each other. Hence the following transformation is introduced. Referring to Eqs.(20) and (21) in reference (11) and Fig.16 we obtain the following equation

$$h\Delta T_m - (k_0/\lambda)(T_0 - \theta_0) - (\partial T/\partial z)_{-h/2} = (c\gamma h/\lambda)(\partial T_m/\partial t) \quad (37)$$

In a similar manner, we have from Eq.(24) in reference (11)

$$(h^2/6)\Delta G + (k_0/\lambda)(T_0 - \theta_0) - (\partial T/\partial z)_{-h/2} - 2G = (c\gamma h^2/6\lambda)(\partial G/\partial t) \quad (38)$$

The elimination of $(\partial T/\partial z)_{-h/2}$ from these two equations gives

$$h\Delta T_m - \frac{h^2}{6}\Delta G - \frac{2k_0}{\lambda}(T_0 - \theta_0) + 2G = \frac{c\gamma h}{\lambda}\left(\frac{\partial T_m}{\partial t} - \frac{h}{6}\frac{\partial G}{\partial t}\right) \quad (39)$$

The same procedure carried out for the brick leads to

$$h_b\Delta T_{mb} + \frac{h_b^2}{6}\Delta G_b - \frac{2k_{ib}}{\lambda_b}(T_{ib} - \theta_{ib}) - 2G_b = \frac{c_b\gamma_b h_b}{\lambda_b}\left(\frac{\partial T_{mb}}{\partial t} + \frac{h_b}{6}\frac{\partial G_b}{\partial t}\right) \quad (40)$$

Since the shell and the bricks are in contact with each other, we obtain

$$\lambda(\partial T/\partial z)_{-h/2} = \lambda_b(\partial T_b/\partial z)_{h_b/2} \quad (41)$$

Multiply Eq.(37) by λ and the equation for the brick equivalent to this equation by λ_b . Then the addition of them gives

$$\lambda h\Delta T_m + \lambda_b h_b\Delta T_{mb} - k_0(T_0 - \theta_0) - k_{ib}(T_{ib} - \theta_{ib}) = c\gamma h\partial T_m/\partial t + c_b\gamma_b h_b\partial T_{mb}/\partial t \quad (42)$$

and

$$T_i \approx T_{0b} = T_m + hG/2 = T_{mb} - h_b G_b/2, \quad T_0 = T_m - hG/2, \quad T_{0b} = T_{mb} + h_b G_b/2 \quad (43)$$

If θ_{ib} is given as a function of x , θ and t by Eqs.(39) through (43), the unknown quantities T_m , T_{mb} , G and G_{mb} can be determined, and substituting them into the fundamental differential equation of general temperature distribution⁽¹¹⁾, each displacement and then the stresses will give.

The description above-mentioned is a general case, but the thermal stresses caused by the temperature distribution over the test kiln is not so important, because the temperature is almost constant along the circumference and its variation is also small along the generator. In this case it may be rather important to consider the contact pressure between the shell and the bricks by the temperature rise. Under the assumption that the temperature distribution is constant along both the circumference and the generator, and the shell and the bricks are in contact with each other at the initial state, we try to find the thermal stresses. Equations (39), (40) and (42) become in this case as follows:

$$k_0(T_0 - \theta_0) - \lambda G = 0, \quad k_{ib}(T_{ib} - \theta_{ib}) + \lambda_b G_b = 0, \quad k_0(T_0 - \theta_0) + k_{ib}(T_{ib} - \theta_{ib}) = 0 \quad (44)$$

With θ_{ib} and θ_0 given, T_{ib} , T_0 , G and G_b are found from Eqs.(44) and (43).

Next, denote the components of the displacement in the axial and the circumferential directions with the temperature rise by u and w , and assume the axial stress $T_x = 0$. Then we get

$$du/dx + \nu w/a - (1+\nu)\alpha(T_m - \theta_0) = 0^{(11)}$$

Substitution of this into the expression of T_θ ⁽¹¹⁾ yields

$$T_\theta = [12(1-\nu^2)D/h^2][w/a - \alpha(T_m - \theta_0)] \quad (45)$$

In a similar manner we have for the brick

$$T_{\theta b} = [12(1-\nu^2)D_b/h_b^2][w/a_b - \alpha_b(T_{mb} - \theta_{0b})] \quad (46)$$

Denoting the contact pressure between the shell and the brick by p and referring to Fig.17, the following equations are obtained from Eqs.(45) and (46).

Table 1

θ_{ib} °C	1 500	1 300	1 100	1 000
T_0 °C	143.4	126.6	109.9	101.6
σ_θ kg/cm ²	1 497.5	1 296.9	1 092.8	991.6

Fig.16 Temperature state of rotary kiln

Fig.17 Sectional diagram of shell and brick

$$\left. \begin{aligned} [12(1-\nu^2)D/h^2][w/a-\alpha(T_m-\theta_0)] &= a_c p, \\ [12(1-\nu^2)D_b/h_b^2][w/a_b-\alpha_b(T_{mb}-\theta_0)] & \\ &= -a_c p \end{aligned} \right\} \dots\dots\dots(47)$$

Elimination of p from Eqs.(47) gives

$$w = \frac{(\alpha D/h^2)(T_m-\theta_0) + (\alpha_b D_b/h_b^2)(T_{mb}-\theta_0)}{(D/ah^2) + (D_b/a_b h_b^2)} \dots\dots\dots(48)$$

T_θ produced in the shell can be obtained by substituting this into Eq.(45).

Since the kiln rotates, the gases on both inner and outer surfaces are supposed to be flowing. The heat transfer coefficients k_{ib} and k_o on both surfaces are assumed to be $50 \text{ Kcal/m}^2\text{h}^\circ\text{C}$ ⁽¹²⁾. And the coefficients of thermal conductivity of the steel and the brick are taken as $\lambda = 50 \text{ Kcal/mh}^\circ\text{C}$ and $\lambda_b = 0.75 \text{ Kcal/mh}^\circ\text{C}$, and the linear coefficients of expansion as $\alpha = 11.76 \times 10^{-6}/^\circ\text{C}$ and $\alpha_b = 5 \times 10^{-6}/^\circ\text{C}$, respectively. When the temperature of ambient air is $\theta_0 = 20^\circ\text{C}$ and the temperature in the kiln $\theta_{ib} = 1500^\circ \sim 1000^\circ\text{C}$, the temperature on the shell surface T_θ and the circumferential stress produced in shell σ_θ can be calculated from the formulas above-mentioned, whose results are given in Table 1. The values of T_θ measured actually, being somewhat different depending on the positions, lie as a whole in the range of 110° to 130° . Contrasting with Table 1 it can be said that the temperature of the gas on the inner surface is 1300° to 1100° and the corresponding stress σ_θ is about 1200 kg/cm^2 , which affects the mean value of the repeated load and reduces the fatigue strength. Now that the kiln is operated under the circumstance of high temperature, this sort of thermal stress necessarily occurs and an appropriate design of the loading block is still more required.

12. Conclusions

(1) A method of analysis of a rotary kiln is developed in consideration of the effects of the brick lining, the loading blocks for tire, the driving equipment, the contents, the deformation of tire, the thermal stresses, etc., and it is described.

(2) This method is applied to a full-sized model of kiln and thereby the validity of the design is examined. The result shows that the loading block requires a design based on the concept of flexibility, because a rigid design is likely to cause a crack near the block and the thermal stress reduces the fatigue strength.

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