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Analysis of low Reynolds number flow through a pipe

with an annular protrusion*

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In order to examine the influence of an annular protrusion, which is set in a pipe in Poiseuille's flow through the pipe, two kinds of experiments were carried out; that is, head loss of the flow downstream of the protrusion was measured at several positions of the flow, and the aspect of the flow was observed by the aid of a dye streak.

The flow is possibly disturbed by the protrusion even when Reynolds number (Re) is less than its critical one, and then the friction factor becomes greater than $64/Re$. And it also becomes clear that the friction factor is given by such a function of the distance of the flow from the protrusion that the friction factor decreases as the flow goes down, and finally the factor becomes $64/Re$. From the results of a series of experiments the damping-up distance of the local turbulence due to the protrusion is given as a product of power function of internal diameter of the pipe and Reynolds number of the flow.

By observing the dye streak through the pipe the damping-up distance is proved to coincide with the result obtained by the experiments of the pipe friction.

1. Introduction

In the previous paper¹⁾ the results of two kinds of experiments with flow through a circular smooth pipe with a ring or triangular teeth type protrusions are reported; one experiment is carried out to prove the occurrence of a local turbulence under Reynolds number less than the critical one by determining friction factors of the flow by the aid of pressure drop measurements along the pipe, and the other by visualization of the flow by the aid of a dye streak method. Examining the experimental results it becomes clear that the outbreak of a local turbulence is conditioned by the internal diameter of the pipe, dimension and shape of the protrusion and the Reynolds number of the flow, and that the condition of the outbreak can be expressed by a series of simple empirical formulae. From these results it is proposed that the Reynolds number at which the turbulence arises be named the quasi-critical Reynolds number.

In the practical pipe lines in industry there exist many joints and bends, and thus it is apprehended that the flow through such pipe lines may not always be a laminar one, even when its Reynolds number is less than the critical one (i.e. 2300). Therefore in the design of the pipe lines the peculiar property of the local turbulence must be taken into consideration.

For the basic investigation of such a local turbulent flow, friction factors of flow through a pipe with protrusion are determined under a series of the conditions of internal diameter, flow rates, shape and dimension of the protrusion, and the results are expressed by a series of empirical formulae by the aid of multiple regression analysis, and taking the formulae into consideration the damping-up distance of local turbulence is reduced.

2. Friction factor

2.1 Experimental apparatus and measurements

An apparatus, as shown in Fig. 1, is made to feed

water of constant head by the overflow method through an acrylic pipe set horizontally. The water is fed through a rectifier consisting of honeycomb and cloth net into the pipe through a bell mouth attached on the pipe inlet. The flow rate of water is controlled with a valve on the pipe end and measured with a graduated measuring glass and a stop watch.

The pressure difference between two neighbouring static pressure holes of 0.7mm in diameter is measured by a two-liquid U-tube differential manometer of toluene and water, with accuracy of 0.14mm Aq. by a reading microscope, the minimum one division of which is 0.01mm. And the sensitivity can be increased by 3.7 times compared with a combination of water and chloroform in the previous paper.¹⁾

The test pipe consists of an acryl resin pipe of 10mm in internal diameter, having satisfactory smoothness and roundness on its inner surface.

As the measurements are performed on a flow with Reynolds number lower than 2000, the length from pipe inlet to the protrusion is set at 1300mm, according to the formula of inlet length $L=0.065Re d$ to make the flow near the protrusion sufficiently laminar. As seen in the previous paper, a damping-up distance of a local turbulence caused downstream of the protrusion, which is set in a pipe of 10mm in internal diameter for Reynolds number 2000 is about 1100mm, and so the distance from the protrusion to the outlet end is set at 1500mm in consideration of a valve at the end.

The protrusion is a ring-shaped one with two kinds of values of y/d 0.15 and 0.2, y being the protrusion height and d the internal diameter.

As the results on the friction factor of water and oil are similar to those reported in the previous paper, the experiment is performed only on water flow.

y/d : 0.15, 0.2, d : 10mm, L_H : 160, 200, 250, 300, 400, 500, 600, 700mm

Fig. 1 Schematic diagram of experimental apparatus.

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2.2 Pressure distribution in pipes

To determine positions of static pressure holes in the pipe, pressure distribution downstream of the protrusion for the value of y/d 0.15 is measured. By connecting the two-liquid U-tube differential manometer with static pressure holes which are drilled and reamed every 20mm within 200mm from the protrusion and every 50mm thereafter, head losses of water columns of liquid tube manometer are measured by a reading microscope on every six adjoining intervals.

Fig. 2 shows the empirical results plotted on uniform coordinate i.e., static pressure on axis of ordinate and length from the protrusion on axis of abscissa where $\circ$ -mark indicates static pressure at Reynolds number 1000 and $\times$ -mark at 1500; and a dotted line indicates dynamic pressure drops calculated from the average flow velocity in the pipe.

Pressure distribution in the pipe clearly changes at a border 60mm from the protrusion to downstream; namely up to the distance of $6d$ the pressure drop increases and thereafter it decreases gradually. It is thought that the characteristics of the flow within a distance of $6d$ from the protrusion differs from that at farther distance. In this study, accordingly, the experiment on a flow downstream of $6d$ is performed.

In Fig. 2 the pressure distribution $6d$ downward from the protrusion can be regarded linear at any interval of any static pressure hole less than 200mm, excluding the initial part. Consequently, the distance of adjoining static pressure holes to measure head loss is set at 200mm which is shorter than that in the previous paper.

From the results mentioned above, such representative distances l_m from the protrusion as 8 steps of 160, 200, 250, 300, 400, 500, 600 and 700mm are selected to obtain mean friction factors. And thus eight test pipes are prepared conforming to each distance and

Fig. 2 Pressure distribution down stream of a protrusion.

Table 1 Experimental conditions for measurement of head loss.

internal diameter of the pipe (mm)	10
protrusion coefficient (annual ring type): y/d	0.15, 0.2
flowing distance from the protrusion: l_m (mm)	160, 200, 250, 300, 400, 500, 600, 700
distance between two static pressure holes: l (mm)	200
Reynolds number: Re	800 ~ 2000

in each test pipe two static pressure holes are drilled in each test pipe at 100mm behind and after the representative distance.

Experimental conditions determined in consideration of the above factors are summarised in Table 1.

2.3 Discussion on experimental results

Friction factor λ is obtained by substituting the experimental data of head loss at representative distance l_m as shown in Table 1 into the following equation.

$$\Delta h = \frac{p_i - p_{i+1}}{\gamma} = \lambda \frac{\Delta l}{d} \frac{v^2}{2g} \quad (1)$$

In Eq. (1), Δh : head loss in pipe length Δl ; $p_i - p_{i+1}$: static pressure difference and v : average flow velocity in pipe. The value of λ obtained from Eq. (1) is an average friction factor between adjoining static pressure holes. As the mid-point between two adjoining static pressure holes is assumed to be the representative distance l_m , the value of λ obtained by the pressure drop of the adjoining pressure holes is expressed by λ_m .

(a) Experimental results for the value of 0.2 in y/d of a protrusion

As both Reynolds number Re and the average friction factor λ_m are non-dimensional quantities, l_m/d is used as a parameter instead of l_m so as to make all of the values non-dimensional. Plotting the relation of Re and λ_m on logarithmic coordinate, Fig. 3 is obtained. As clearly seen in this figure, the experimental data of λ_m fall on a straight line downward right on logarithmic coordinate so that the value of λ_m for same Reynolds number increases according as the value of l_m/d decreases, namely the representative position of λ_m comes close to the protrusion. A single-dotted line in the figure shows the relation, $\lambda = 64/Re$, and the plots showing the experimental data of λ_m approaches the single dotted line, according as l_m/d increases. This means that when the protrusion is installed inside a straight smooth pipe the flow therein may become $\lambda_m > 64/Re$ even if Reynolds number is less than the critical one and this value of λ_m gradually approaches the value of friction factor of a laminar flow farther downstream of the protrusion.

The relation of λ_m to Re in Fig. 3 can be expressed by the following empirical equation.

$$\lambda_m = a Re^{-\alpha} \quad (2)$$

As the inclination of $\lambda_m \sim Re$ curve changes according to the parameter l_m/d , a or α in Eq. (2) is expected to be a function of l_m/d .

Fig. 3 $\lambda_m \sim Re$ relation.

Fig. 4 $R_e \sim l_m/d$ relation. Fig. 5 $\lambda_m \sim R_e (l_m/d)^{1/2}$ relation.

Fig. 4 shows the result obtained by plotting the relation of Reynolds number and l_m/d on logarithmic coordinate using λ_m as a parameter. The relation of $R_e \sim l_m/d$ becomes a group of parallel lines downward right on logarithmic coordinate, having a directional coefficient of about $-1/2$. Thus, the relation of $R_e \sim l_m/d$ can be expressed by the following empirical equation.

$$R_e = b (l_m/d)^{-1/2} \quad (3)$$

As b is a function of λ_m here, Eq. (3) can be reformed to the following equation using b as $f(\lambda_m)$.

$$R_e (l_m/d)^{1/2} = f(\lambda_m) \quad (4)$$

By finding the relation of $R_e (l_m/d)^{1/2}$ and λ_m from the experimental data of λ_m , R_e and l_m/d and thus plotting them on logarithmic coordinate, Fig. 5 is obtained. In this figure, all dots fall on a straight line with a directional coefficient of about $-2/3$. Correlation coefficient of r between $\log \lambda_m$ and $\log R_e (l_m/d)^{1/2}$ is given by $r = -0.9904$. In the test of a significance of correlation using the r -table, $\log \lambda_m$ and $\log R_e (l_m/d)^{1/2}$ are highly significant for the degree of freedom 42 at the level of 1%.

Therefore, the relation in Eq. (4) is expressed taking c as a constant by

$$\lambda_m = c R_e^{-2/3} (l_m/d)^{-1/3} \quad (5)$$

By generalizing now the indices of R_e and l_m/d as

in

$$\lambda_m = c R_e^{-\alpha} (l_m/d)^{-\beta} \quad (6)$$

and transforming both sides into logarithmic expression, the following equation can be obtained,

$$\log \lambda_m = \log c - \alpha \log R_e - \beta \log (l_m/d) \quad (7)$$

and performing a multiple linear regression analysis to obtain the partial regression coefficient, the values of c , α and β are obtained. First, the multiple correlation coefficient is obtained and $F_0 = F_r/F_v = 1144$. Considering $F(2, 40, 0.01) = 5.18$, $\log R_e$ and $\log (l_m/d)$ are highly correlated with $\log \lambda_m$ at the level of 1%. Then, to obtain c , α and β as partial regression coefficients, Eq. (6) becomes,

$$\lambda_m = 26 R_e^{-0.66} (l_m/d)^{-0.36} \quad (8)$$

Comparing the index of R_e $|\alpha| = 0.66$ with the index of l_m/d $|\beta| = 0.36$, $0.66 \times 1/2 \approx 0.36$ is obtained, i.e., $|\beta|$ becomes nearly $1/2$ of $|\alpha|$.

Eq. (8) which is a function of Reynolds number and l_m/d , gives the average friction factor at downstream when a straight smooth pipe contains a ring-shaped protrusion of 0.2 in y/d . Fig. 6 shows the result on λ_m for l_m/d for flows with Reynolds numbers 800~2000 from Eq. (8) where the relation of $\lambda_m \sim l_m/d$ causes parallel lines downward right taking Reynolds number as a parameter on logarithmic coordinate.

Assuming that λ_m of Eq. (8) is equal to $64/R_e$ of the laminar flow, the following equation is obtained;

$$64/R_e = 26 R_e^{-0.66} (l_m/d)^{-0.36} \quad (8')$$

The value of l_m obtained by the above equation expresses a flowing distance where the average friction factor exceeding $64/R_e$ is returned to $64/R_e$ again, namely the position where the local turbulence downstream of the protrusion is damped up. Expressing this l_m by l_{mc} , the following equation is given;

$$l_{mc} = 0.082 R_e^{0.93} d \quad (9)$$

(b) Experimental results for the value of 0.15 in y/d

Obtaining the relation of λ_m and $R_e (l_m/d)^{1/2}$ from the experimental data of λ_m , R_e and l_m/d at 0.15 in y/d by changing dimensions of the protrusion and plotting them on logarithmic coordinate, Fig. 7 is obtained where the dots fall on a straight line with a

Fig. 6 $\lambda_m \sim l_m/d$ relation.

Fig. 7 $\lambda_m \sim Re (l_m/d)^{1/2}$ relation.

directional coefficient $-2/3$. This phenomenon coincides with the relation of λ_m and $Re (l_m/d)^{1/2}$ regarding the protrusion of 0.2 in y/d in Fig. 5. Therefore, applying Eq. (6) also to experimental data for the protrusion of 0.15 in y/d , and obtaining the coefficient c and indices α and β by means of the partial regression analysis, the average friction factor λ_m is given by the following equation,

$$\lambda_m = 17Re^{-0.65} (l_m/d)^{-0.28} \quad (10)$$

Assuming that λ_m is equal to $64/Re$ in Eq. (10), the distance l_{mc} where a local turbulence is damped up is given by

$$l_{mc} = 0.009Re^{1.23}d \quad (11)$$

3. Observation of damping-up phenomena of local turbulence by the aid of flow visualization

3.1 Object of experiment

In the experiment in the previous chapter, the average friction factor λ_m is an average value between two static pressure holes on a straight pipe. As the value of λ_m decreases with an increase of flowing distance, the value of damping-up distance of local turbulence l_{mc} may possibly contain a biased deviation. In order to examine the biased deviation, damping-up distance of local turbulence is observed by visualizing a flow by the aid of dye streak method.

3.2 Measurements of damping-up distance

Under the conditions shown in Table 2 the relation between damping-up distance of disturbed streak line l_c downstream of the protrusion and Reynolds number is investigated.

In the experimental apparatus in the previous

Table 2 Experimental conditions for visualization of flow

internal diameter of the pipe (mm)	10, 15, 20
protrusion coefficient (annular ring type): y/d	0.15, 0.2
distance between protrusion and intermediate nozzle (mm)	300, 520, 680, 830, 980
Reynolds number: Re	300 ~ 1350

Fig. 8 Schematic illustration of a transparent pipe for visualization of flow by the aid of two sources of dye streaks.

chapter a pipe is replaced by a visualizing pipe as shown in Fig. 8, in which the first nozzle is attached at the centre of the inlet bell mouth and the intermediate nozzle downstream of the protrusion, and thus a tracer of aqueous solution of methylene blue can be injected through both of the nozzles. For the intermediate nozzle, five positions i.e. 300, 520, 680, 830 and 980mm, are selected downstream of the protrusion as shown in Fig. 8. The nozzles are injection needles of 0.7mm in outer diameter and 0.4mm in internal diameter.

Water is fed into the pipe and the flow rate is gradually increased by operating an outlet valve. While the flow rate is sufficiently small, that is, the Reynolds number is far lower than the critical one, both the tracers injected through the nozzles make a straight line. When the flow rate is increased and thus the Reynolds number reaches a certain limit, the tracer from the first nozzle begins to be disturbed between protrusion and intermediate nozzle but the tracer from the intermediate nozzle remains straight. This means that a turbulence is caused near the protrusion but it has already damped up at a position of the intermediate nozzle. Moreover, if the flow rate increases up to a certain value, the tracer from the intermediate nozzle shows a turbulence.

When a turbulence of the tracer from the intermediate nozzle just takes place, the distance between the protrusion and the intermediate nozzle is a value to indicate a damping-up under the condition of the flow at the time.

Fig. 9 is a photograph of a visualizing experiment with a pipe of 20mm in internal diameter. The figure shows an aspect of a streak line where flow rate is gradually increased from zero up to 400 in Reynolds number. The intermediate nozzle is set at 520mm downstream of the protrusion. While the streak line is straight behind the protrusion and is turbulent after the protrusion, the streak line is straight after the intermediate nozzle. From the photograph the turbulence is recognized between the protrusion and the intermediate nozzle only.

Setting the distance of l_c by selecting the position of the intermediate nozzle, and finding the flow rate when the streak line from the intermediate nozzle begins to be disturbed, the relation of flow rate and damping-up distance l_c can be obtained.

3.3 Discussion on experimental results

Fig. 10 shows the results obtained by finding the positions where turbulence of streak line is damped up downstream of the protrusion of 0.2 in y/d from the experimental data of l_c/d plotted on axis of ordinate and Reynolds number plotted on axis of abscissa on logarithmic coordinate. In the figure, the value of l_c/d increases in proportion to an increase of Reynolds number. When the value of y/d is fixed, the value of l_c/d with respect to same Reynolds number is independent of pipe internal diameter and the dots on graph showing their experimental data fall on a straight line. Consequently, the relation of $l_c/d \sim Re$ can be expressed by the following empirical equation.

Fig. 9 Photographic illustration of streak line.

Fig. 10 Influence of R_e number on damping-up distance of local turbulence ($y/d; 0.2$).

$$l_c/d = kR_e^\alpha \quad (12)$$

Transforming both sides of the equation into logarithmic expression yields

$$\log l_c/d = \log k + \alpha \log R_e \quad (13)$$

Obtaining a regression coefficient by performing a linear regression analysis, k and α are made available. A correlative coefficient is obtained as $r = 0.9757$ where $r(15, 0.01) = 0.4124$ according to the r -table, and thus, correlation is significant at the level of 1%, because $r = 0.9757 \gg 0.4124$. Obtaining now k and α as linear regression coefficients, Eq. (12) is converted to

$$l_c/d = 0.052R_e^{1.00} \quad (14)$$

and the damping-up distance of turbulence of streak line is expressed by the following equation.

$$l_c = 0.052R_e^{1.00}d \quad (15)$$

By plotting the relation of l_c/d and R_e of Eq. (14), a full line is obtained as shown in Fig. 10 where the dots of the experimental data fall on the line.

Obtaining a damping-up distance of turbulence of streak line downstream of the protrusion of 0.15 in y/d , the following equation is given

$$l_c = 0.0018R_e^{1.45}d \quad (16)$$

It is made clear that a distance where turbulence of streak line downstream of the protrusion is damped up in the flow with low Reynolds number through a straight smooth pipe can be expressed as a function of pipe internal diameter and Reynolds number.

When plotting the relation of $l_{mc}/d \sim R_e$ from Eq. (9) of the previous chapter onto Fig. 10, a broken line upward right is made, which is overlapped on a

Fig. 11 Influence of R_e number on damping-up distance of local turbulence ($y/d; 0.15$).

straight line indicating Eq. (15).

Fig. 11 shows the relation of $l_c/d \sim R_e$ according to Eq. (16) regarding a protrusion of 0.15 in y/d by a full line and $l_{mc}/d \sim R_e$ according to Eq. (11) by a broken line, both of which are straight lines upward right being overlapped on each other.

The damping-up distance l_c of turbulence of streak line downstream of the protrusion is similar to the damping-up distance l_{mc} for local turbulence introduced from the result of the measurement of head loss downstream of the protrusion.

It can be concluded that Eqs. (9) and (11) do not contain systematic deviation according to the fixing manner of experimental points and Eqs. (8) and (10) are satisfactory to express an average friction factor of local turbulence.

4. Conclusions

In order to investigate the aspect of local turbulent in a flow of low Reynolds number through a straight smooth pipe of an acrylic pipe, with an annular protrusion, the relation between flowing distance from the protrusion and head loss is studied. In addition, aspects of streak line in the flow under a series of Reynolds number are observed by visualizing the state of the flow by the aid of the dye streak method. The following conclusion are obtained.

(1) l_m/d obtained by dividing the flowing distance l_m by the pipe internal diameter d and Reynolds number R_e is highly correlative with the average friction factor λ_m downstream of the protrusion, which can be expressed as a function of l_m/d and R_e .

(2) As to the experimental data of λ_m , l_m/d and R_e a multiple correlation analysis, test of significance of correlation and estimation of partial regression coefficient are performed, and thus the quantitative relations among quantities of the three items are avail-

able.

(3) Streak line downstream of the protrusion is observed to make the damping-up distance of turbulence clear. The distance thus obtained corresponds to the damping-up distance of a local turbulence introduced from the relation mentioned in (2) above.

(4) Damping-up distance of local turbulence caused downstream of the protrusion can be expressed as a function of Reynolds number and internal diameter of the pipe.

(5) By visualization of flow, propriety of the results obtained by the head loss measurement is confirmed.

Reference

- 1) Taniuchi, K., Hayashi, K., Bulletin of the JSME, Vol. 20, No. 143 (1977-5), p. 607.
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