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Flow Pattern near the Outlet of a Straight Long Circular Tube*

(2nd Report, Theoretical Calculation)

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A theoretical study was carried out to investigate such a flow that a liquid of low viscosity issues downward from a vertical tube into the atmosphere. Numerical calculations were performed by using both vorticity transfer and momentum theory.

From the study the following results were obtained:

(1) Inside of the tube, velocity profile gradually becomes different from that of the Poiseuille flow as the flow comes close to the tube outlet. For $Re=753$, $F_r=9.75 \times 10^{-2}$, the velocity on the tube axis at the tube end becomes 1.23 times as large as that of the Poiseuille flow. Accordingly, the flow pattern at the tube end is no longer that of the Poiseuille flow. Issuing from the tube outlet, the velocity near the flow axis in the liquid jet increases further downstream, and the flow at a position by one tube-diameter downstream from the tube end settles to uniformity. These tendencies become remarkable with an increase of the ratio of the Reynolds number to the Froude number.

(2) The pressure on the tube wall takes a minimum value at a position by the tube-radius times 0.7 from the tube end. This value is lower than the atmospheric pressure.

1. Introduction

In a previous report⁽¹⁾, the velocity profiles of water flow near the outlet of a vertical tube were studied experimentally. However, flow pattern near the tube wall and near the free surface in the liquid jet was not yet clearly investigated.

Such a laminar jet of Newtonian liquid has long been studied by many researchers. Neglecting the effect of gravity, Goren⁽²⁾, Tillett⁽³⁾ and Miyake & Yamaji⁽⁴⁾ analyzed the jet flow pattern outside of the tube. According to their results, the surface displacement is proportional to a 1/3 power of the distance from the tube outlet. On the other hand, the jet flow outside of the tube at the gravity field was analyzed by Duda & Vrentas⁽⁵⁾, Lienhard⁽⁶⁾ and Miyake⁽⁷⁾. However, since the effect of gravity on these flows is assumed very little, the free surface contracts smoothly. Applying the boundary layer theory to a jet flow outside of the tube and assuming the flow at the tube end to be the Poiseuille flow, Duda & Vrentas, Lienhard numerically calculated the velocity profile in the jet by using the finite difference method. Miyake analytically investigated by using a perturbation method.

However, since a laminar jet of low viscous liquid like water is considerably

affected by gravity, the free surface of a jet contracts rapidly and the curvature of the free surface near the tube outlet becomes fairly sharp. Accordingly, the application of the boundary layer theory to this jet flow was not found appropriate. Furthermore, it was already found from the previous report⁽¹⁾ that the flow at the tube end is no longer the Poiseuille flow. So far as the authors are aware, a theoretical analysis is not yet made because the jet flow of low viscous liquid is considerably affected by gravity. In the present paper, flow patterns near the vertical tube outlet are numerically calculated by using both vorticity transfer equation and momentum theory.

Nomenclature

F_r : Froude number = $U_m^2/(gR)$
 g : acceleration of gravity
 p : dimensionless pressure = $p^*/(\rho U_m^2)$
 $p_h = p - z/F_r$
 R : tube radius
 R_c : radius of curvature on axial plane of the free surface
 Re : Reynolds number = $U_m R/\nu$
 r : dimensionless radial coordinate
 U_m : mean velocity inside of the tube
 u : dimensionless axial velocity component
 v : dimensionless radial velocity component
 We : Weber number = $2\rho U_m^2 R/\sigma$
 z : dimensionless axial coordinate
 ν : kinematic viscosity
 $\xi = \tanh(\alpha_0 z)$
 ρ : density
 σ : surface tension
 ψ : stream function
 ω : vorticity

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ζ : radial displacement of the free surface

Subscripts

0, s and w : variables at the tube outlet, on the free surface and on the tube wall respectively

Abbreviation

cal.1: calculated results from the measured free surface

cal.2: analytically calculated results from Eqs.(9), (12), (13), (14) and (21)

Fig.1 Flow situation and mesh points

2. Basic equations and boundary conditions

We take the reference frame as shown in Fig.1. The flow is assumed to be laminar, incompressible and axisymmetric. The vorticity is expressed as

$$\omega = \partial v / \partial z - \partial u / \partial r \dots \dots \dots (1)$$

The relations between the velocity components u , v and the stream function are

$$u = (\partial \psi / \partial r) / r, \quad v = -(\partial \psi / \partial z) / r \dots \dots \dots (2)$$

We introduce a new variable ξ which is defined as

$$\xi = \tanh(\alpha_0 z) \dots \dots \dots (3)$$

where α_0 is a constant coefficient. By using the above described variable, the vorticity transfer equation and the stream function equation are written as follows:

$$\alpha_0(1-\xi^2) \left\{ \frac{1}{r} \frac{\partial \psi}{\partial r} \frac{\partial \omega}{\partial \xi} - \frac{\partial \psi}{\partial \xi} \frac{\partial}{\partial r} \left(\frac{\omega}{r} \right) \right\} = \frac{1}{R_e} \left\{ \frac{\partial}{\partial r} \left[\frac{1}{r} \frac{\partial}{\partial r} (r\omega) \right] + \alpha_0^2(1-\xi^2) \frac{\partial}{\partial \xi} \left[(1-\xi^2) \frac{\partial \omega}{\partial \xi} \right] \right\} \dots \dots \dots (4)$$

$$\alpha_0^2(1-\xi^2) \frac{\partial}{\partial \xi} \left[(1-\xi^2) \frac{\partial \psi}{\partial \xi} \right] + r \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \psi}{\partial r} \right) + r\omega = 0 \dots \dots \dots (5)$$

Applying momentum theory to the control volume as shown in Fig.2, we have

$$\int_a^{r_a} u^2 r dr - \int_0^{r_a} u^2 r dr = \frac{1}{F_r} \int_0^{z_a} \int_0^{r_a} r dr dz + \int_0^{r_a} p r dr - \int_a^{r_a} p r dr \dots \dots \dots (6)$$

where r_a is radius of the jet at any position. The left-hand side represents the momentum change, the first term of the right-hand side does the gravity force and the second and the third term the surface force. Pressure is calculated from the following Navier-Stokes equations.

Fig.2 Control volume

Fig.3 Flow near the free surface

$$\frac{\partial}{\partial r} \left\{ p + \frac{1}{2} (u^2 + v^2) \right\} = -\omega u + \frac{1}{R_c} \alpha_0 (1 - \xi^2) \frac{\partial \omega}{\partial \xi} \quad \dots\dots\dots (7)$$

$$\frac{\partial}{\partial \xi} \left\{ p + \frac{1}{2} (u^2 + v^2) \right\} = \frac{1}{\alpha_0 (1 - \xi^2)} \left\{ \omega v + \frac{1}{F_r} - \frac{1}{R_c} \left(\frac{\partial \omega}{\partial r} + \frac{\omega}{r} \right) \right\} \quad \dots\dots\dots (8)$$

The boundary conditions are taken as follows:
the flow at the infinite upstream inside of the tube is the same as the Poiseuille flow; the vorticities both at the flow axis and at the infinite downstream of the jet are zero. In the case where the effect of gravity is large, the jet contracts rapidly and the curvature of the free surface is so large that the vorticity ω_s at the free surface is regarded as to be finite. We introduce new coordinates (s_c, n_c) with the origin on the free surface as shown in Fig.3. Since the friction force between the liquid surface and the air is very small, the velocity gradient $\partial v_s / \partial n_c$ can be neglected. Accordingly, the vorticity ω_s on the free surface is expressed as

$$\omega_s = v_s / R_c \quad \dots\dots\dots (9)$$

where R_c is radius of curvature on the free surface.

3. Calculation procedure

We try to rewrite Eqs.(4) and (5) by using the central-difference approximation except the first derivatives $\partial \omega / \partial r$, $\partial \omega / \partial \xi$ in Eq.(4). These two terms should be rewritten by using the upstream-difference approximation⁽⁸⁾⁽⁹⁾, otherwise their convergence would be very bad. If we denote mesh points by subscripts (0,1,2,3,4) as shown in Fig.1, these two derivatives will be

$$\alpha = \psi_1 - \psi_3 \quad \left. \begin{array}{l} \alpha \geq 0, \quad (\partial \omega / \partial r)_0 = (\omega_2 - \omega_0) / (r_2 - r_0) \\ \alpha < 0, \quad (\partial \omega / \partial r)_0 = (\omega_0 - \omega_4) / (r_0 - r_4) \end{array} \right\} \quad \dots\dots\dots (10)$$

$$\beta = \psi_2 - \psi_4 \quad \left. \begin{array}{l} \beta \geq 0, \quad (\partial \omega / \partial \xi)_0 = (\omega_0 - \omega_3) / (\xi_0 - \xi_3) \\ \beta < 0, \quad (\partial \omega / \partial \xi)_0 = (\omega_1 - \omega_0) / (\xi_1 - \xi_0) \end{array} \right\} \quad \dots\dots\dots (11)$$

In the case of $\alpha \geq 0$ and $\beta \geq 0$, that is, $u \geq 0$ and $v \leq 0$ respectively, Eq.(4) is expressed as follows:

$$\omega_0 = A_1 \omega_1 + A_2 \omega_2 + A_3 \omega_3 + A_4 \omega_4 \quad \dots\dots\dots (12)$$

where

$$\begin{aligned} A_1 &= \frac{1}{A_s} \left\{ \frac{2r_0 \alpha_0^2 (1 - \xi_0^2)}{(\xi_1 - \xi_3)(\xi_1 - \xi_0)} - \frac{2r_0 \alpha_0^2 \xi_0 (1 - \xi_0^2)(\xi_0 - \xi_3)}{(\xi_1 - \xi_0)(\xi_1 - \xi_3)} \right\} \\ A_2 &= \frac{1}{A_s} \left\{ \frac{2r_0}{(r_2 - r_4)(r_2 - r_0)} + \frac{r_0 - r_4}{(r_2 - r_0)(r_2 - r_4)} + R_c \alpha_0 \frac{1 - \xi_0^2}{r_2 - r_0} \frac{\partial \psi}{\partial \xi} \right\} \\ A_3 &= \frac{1}{A_s} \left\{ \frac{2r_0 \alpha_0^2 (1 - \xi_0^2)^2}{(\xi_1 - \xi_3)(\xi_0 - \xi_3)} + \frac{2r_0 \alpha_0^2 \xi_0 (1 - \xi_0^2)(\xi_1 - \xi_0)}{(\xi_0 - \xi_3)(\xi_1 - \xi_3)} + R_c \alpha_0 \frac{1 - \xi_0^2}{\xi_0 - \xi_3} \frac{\partial \psi}{\partial r} \right\} \\ A_4 &= \frac{1}{A_s} \left\{ \frac{2r_0}{(r_2 - r_4)(r_0 - r_4)} - \frac{r_2 - r_0}{(r_0 - r_4)(r_2 - r_4)} \right\} \\ A_s &= \frac{2r_0 \alpha_0^2 (1 - \xi_0^2)^2}{(\xi_1 - \xi_3)(\xi_1 - \xi_0)} - \frac{2r_0 \alpha_0^2 \xi_0 (1 - \xi_0^2)}{\xi_1 - \xi_0} + \frac{2r_0}{(r_2 - r_4)(r_2 - r_0)} + \frac{1}{r_2 - r_0} \\ &\quad + R_c \alpha_0 \frac{1 - \xi_0^2}{r_2 - r_0} \frac{\partial \psi}{\partial \xi} + \frac{2r_0 \alpha_0^2 (1 - \xi_0^2)^2}{(\xi_1 - \xi_3)(\xi_0 - \xi_3)} + \frac{2r_0 \alpha_0^2 \xi_0 (1 - \xi_0^2)}{\xi_0 - \xi_3} + R_c \alpha_0 \frac{1 - \xi_0^2}{\xi_0 - \xi_3} \frac{\partial \psi}{\partial r} \\ &\quad + \frac{2r_0}{(r_2 - r_4)(r_0 - r_4)} - \frac{1}{r_0 - r_4} + \frac{1}{r_0} + R_c \alpha_0 (1 - \xi_0^2) \frac{\partial \psi}{\partial \xi} \frac{1}{r_0} \end{aligned}$$

Similar finite difference equation can be obtained when $\alpha < 0$ and $\beta < 0$. Equation (5) is also rewritten as

$$\phi_0 = B_1 \phi_1 + B_2 \phi_2 + B_3 \phi_3 + B_4 \phi_4 + B_5 \omega_0 \quad \dots\dots\dots (13)$$

where

$$\begin{aligned} B_1 &= \frac{1}{B_s} \left\{ \frac{2\alpha_0^2 (1 - \xi_0^2)^2}{r_0 (\xi_1 - \xi_3)(\xi_1 - \xi_0)} - \frac{2\alpha_0^2 \xi_0 (1 - \xi_0^2)(\xi_0 - \xi_3)}{r_0 (\xi_1 - \xi_3)(\xi_1 - \xi_0)} \right\} \\ B_2 &= \frac{1}{B_s} \left\{ \frac{2}{r_0 (r_2 - r_4)(r_2 - r_0)} - \frac{r_0 - r_4}{r_0^2 (r_2 - r_0)(r_2 - r_4)} \right\} \end{aligned}$$

$$\begin{aligned}
B_3 &= \frac{1}{B_1} \left\{ \frac{2\alpha_0^2(1-\xi_0^2)^2}{r_0(\xi_1-\xi_3)(\xi_0-\xi_3)} + \frac{2\alpha_0^2\xi_0(1-\xi_0^2)(\xi_1-\xi_0)}{r_0(\xi_1-\xi_3)(\xi_0-\xi_3)} \right\} \\
B_4 &= \frac{1}{B_1} \left\{ \frac{2}{r_0(r_2-r_4)(r_0-r_4)} + \frac{r_2-r_0}{r_0^2(r_0-r_4)(r_2-r_4)} \right\}, \quad B_5 = \frac{1}{B_1} \\
B_6 &= \frac{2\alpha_0^2(1-\xi_0^2)^2}{r_0(\xi_1-\xi_3)(\xi_0-\xi_3)} - \frac{2\alpha_0^2\xi_0(1-\xi_0^2)}{r_0(\xi_1-\xi_0)} + \frac{2}{r_0(r_2-r_4)(r_2-r_0)} - \frac{1}{r_0^2(r_2-r_0)} \\
&\quad + \frac{2\alpha_0^2(1-\xi_0^2)^2}{r_0(\xi_1-\xi_3)(\xi_0-\xi_3)} + \frac{2\alpha_0^2\xi_0(1-\xi_0^2)}{r_0(\xi_0-\xi_3)} + \frac{2}{r_0(r_2-r_4)(r_2-r_0)} + \frac{1}{r_0^2(r_0-r_4)}
\end{aligned}$$

Equations (12) and (13) should be used at the interior mesh points in the flow field. The vorticity on the tube wall is

$$\omega_w = -3(\psi_{w-1} - \psi_w)/(\Delta r^2 r_{w-1}) - \frac{1}{2} r_w \omega_{w-1}/r_{w-1} \dots\dots\dots (14)$$

where the subscripts w and $w-1$ denote the values at a point on the tube wall and at the nearest point away from the wall respectively as shown in Fig.1.

We assume that radial displacement ζ of the liquid surface is expressed by the following polynomial equation.

$$\zeta = \sum_{k=1}^5 a_k (z^{1/6})^k \dots\dots\dots (15)$$

Two kinds of numerical methods are available. In one method the coefficient a_k in Eq.(15) and the radius R_c of curvature are determined from the measured surface shape. The initial guess all over the flow field except on the boundaries is zero. Then, the stream function and the vorticity are iterated by using Eqs.(12), (13), (14) and (9). The results are denoted by the abbreviation "cal.1". The other method is as follows: First, the coefficient a_k , the radius R_c of curvature are determined from the measured surface shape. Secondly, the stream function and the vorticity are iterated by using Eqs.(12), (13), (14) and (9). Thirdly, when these values converge, they are substituted into Eq.(6). If Eq.(6) is not satisfied by these results, the surface shape is converted to another shape, and the stream function and the vorticity are iterated over again. These calculations are repeated until Eq.(6) is satisfied. The final values of the stream function, the vorticity and the surface shape are denoted by the abbreviation "cal.2".

The iterate procedure for stream functions and vorticities is carried out using the following expressions:

$$\phi^{(l+1)} = \phi^{(l)} + \gamma_\phi \{\phi^{(l+1)} - \phi^{(l)}\} \dots\dots\dots (16)$$

$$\omega^{(l+1)} = \omega^{(l)} + \gamma_\omega \{\omega^{(l+1)} - \omega^{(l)}\} \dots\dots\dots (17)$$

where γ_ϕ , γ_ω are the relaxation factors for the stream function and the vorticity respectively. The criteria of convergence are

$$|\{\phi^{(l+1)} - \phi^{(l)}\}/\phi^{(l+1)}| < 2 \times 10^{-5} \dots\dots\dots (18)$$

$$|\{\omega^{(l+1)} - \omega^{(l)}\}/\omega^{(l+1)}| < 5 \times 10^{-5} \dots\dots\dots (19)$$

The relaxation factors of the stream function and the vorticity were chosen as $\gamma_\phi=1.0$ and $\gamma_\omega=0.8$ except on the tube wall where $\gamma_\omega=0.4$ was used. For the surface shape, using the once converged stream function and vorticity, an estimation of the following equation is carried out for each cross section of the jet. Error index e_a and mean error index E_r were calculated from the following equation.

$$e_a = \left\{ \int_a u^2 r dr - \int_a u^2 r dr + \int_a p r dr - \int_0 p r dr - \frac{1}{F_r} \int_0^{z_a} \int_0^{r_a} r dr dz \right\} / \int_a u^2 r dr \dots\dots\dots (20)$$

$$E_r = \int |e_a| dz / \int dz \dots\dots\dots (21)$$

When E_r was less than 7.0×10^{-2} , we considered that the momentum theory had nearly been satisfied.

4. Numerical results and discussion

In this present study, numerical calculation was carried out for a constant mesh size of $\Delta\xi=0.05$ and $\Delta r=0.025$. We have 1681 mesh points for the half region. The coefficient α_0 is determined as follows: According to the previous report⁽¹⁾, the

flow at a position by one tube-radius upstream from the tube end is nearly the same as the Poiseuille flow and the flow at a position by one tube-diameter downstream from the tube end has a nearly uniform velocity profile. From these facts, the coefficient α_0 is set at 0.5 so that the axial distance of the mesh point next to the downstream boundary may be

longer than one tube-diameter. It was found that increasing mesh points does not contribute to better precision of the calculation.

The comparison of the calculated surface shape "cal.2" with the measured surface shape "exp" is shown in Fig.4. The surface shapes were calculated by taking the rate of momentum, the surface force and the body force into account. It is supposed, however, in physical phenomena that the jet flow is also affected by surface tension. The effect⁽⁶⁾ of surface tension is estimated at any cross section aa' of the jet (cf. Fig.2).

$$F_s = \frac{1}{W_s} r_a^2 \left(\frac{1}{r_a} - \frac{1}{R_c} \right) \dots\dots\dots (22)$$

Generally speaking, radius r_a is smaller than radius R_c at the cross section aa', that is, F_s is positive. This implies that the calculated surface shape contracts more remarkably than the measured surface shape. The effect of surface tension agrees very well with the report⁽⁵⁾ in which the effects of both gravity force and surface tension on the jet were assumed to be very little. When the calculated surface shape is compared with the measured surface shape, it is found near the tube outlet that the calculated surface shape of the jet contracts more remarkably than the measured result. It seems that this results from the effects of surface tension, of a sudden change of the wall friction at the tube end and of the adhesive force on the section of the tube end. However, from this study it was not possible to clarify the reason.

Fig.4 Shape of the free surface

Fig.5 Distribution of the error indices e_a ($R_e=753$, $F_r=9.75 \times 10^{-2}$)

The distribution of the error index e_a of cal.2 for $R_e=753$, $F_r=9.75 \times 10^{-2}$ is shown in Fig.5, and the mean error index E_n is therefore 4.90×10^{-2} . Although e_a is -0.177 at the position near the tube outlet, $|e_a|$ decreases further downstream. It is also supposed that the reason why $|e_a|$ is comparatively large at a position near the tube outlet is in the discontinuity at the tube end as described above. The distribution of e_a in the other cases in Fig.4 is similar to that of Fig.5.

The streamlines (upper half) and the equi-vorticity lines (lower half) for $R_e=753$, $F_r=9.75 \times 10^{-2}$ are shown in Fig.6. It is found that the calculated surface shape contracts more remarkably than the measured surface shape, so that the change of the flow pattern in Fig.6(b) becomes more remarkable than that in Fig.6(a).

(a) cal.1

(b) cal.2

Fig.6 Streamlines and equi-vorticity lines ($R_e=753$, $F_r=9.75 \times 10^{-2}$)

Whereas the magnitudes of Re/F_r for laminar liquid jet investigated by Duda & Vrentas⁽⁵⁾, Lienhard⁽⁶⁾ and Miyake⁽⁷⁾ are several tens, the magnitude in this study is $Re/F_r = 7.73 \times 10^3$ for $Re = 753$. Accordingly, it is assumed in this study that the flow is considerably affected by gravity. Velocity profiles near the tube outlet are shown in Fig.7. The change of the velocity u_a near the tube axis and the surface velocity u_s are shown in Fig.8. It can be seen from Fig.7 that when Re/F_r becomes large the velocity near the tube axis increases as the flow comes close to the tube outlet, while the velocity near the tube wall decreases. This tendency agrees very well with the result of the previous report⁽¹⁾. Particularly, it can be seen that the flow at the tube end is fairly different from the Poiseuille flow. For example, the ratio of the velocity on the tube axis at the tube end to that of the Poiseuille flow is $u_a/2 = 1.12$ in cal.1 and $u_a/2 = 1.23$ in cal.2 for $Re = 753$, $F_r = 9.75 \times 10^{-2}$. Furthermore, it is seen that there is no eddy region near the tube wall. This fact is also deduced from the stream-lines in Fig.6. When the liquid issues from the tube outlet, the velocity near the jet axis increases like that inside of the tube. However, the acceleration near the axis is smaller than that near the free surface and the jet rapidly contracts. Finally, the flow at a position by one tube-radius downstream from the tube end shows a uniform velocity profile. This tendency agrees very well with the result of the previous report⁽¹⁾. Table 1 shows the ratio of the surface velocity u_s to the velocity u_a on the jet axis at a position by one tube-diameter downstream from the tube end. Although the velocity gradient $\partial u / \partial r$ has a finite value near the tube outlet, it approaches zero further downstream. Accordingly, it is understood that the vorticity ω_s near the tube outlet can not be neglected.

The comparison of the velocity u_a from cal.1 with the experimental results of the previous report⁽¹⁾ is shown in Fig.9. In

Fig.8 Velocity u_a near the tube axis and surface velocity u_s ($Re = 753$, $F_r = 9.75 \times 10^{-2}$)

(a) $Re = 551$, $F_r = 5.33 \times 10^{-2}$

(b) $Re = 753$, $F_r = 9.75 \times 10^{-2}$

(c) $Re = 920$, $F_r = 14.7 \times 10^{-2}$

Fig.7 velocity profile

Fig.9(a) the velocity u_a on the tube axis is compared with the experimental results inside of the tube for $Re \approx 350$, $D=15.88\text{mm}$. In Fig.9(b) the velocity u_a on the jet axis is compared with the experimental results outside of the tube for $Re \approx 600$, $D=15.88\text{mm}$. It can be seen from Figs.9(a) and 9(b) that these calculated results agree very well with the experimental ones.

The pressure distributions for $Re=753$, $F_r=9.75 \times 10^{-2}$ are shown in Figs.10(a) and 10(b), where the pressure inside of the tube is indicated as P_h . The pressure on the jet axis has a minimum value near the

Table 1 Ratio of the surface velocity u_s to the velocity u_a on the jet axis ($z=1.95$)

Re	Re/F_r	u_s/u_a	
		cal.1	cal.2
551	10.3×10^3	0.978	0.986
753	7.73	0.959	0.972
920	6.25	0.935	0.952

Fig.9(a) Comparison of the velocity u_a (cal.1) of the tube axis with the experimental results ⁽¹⁾ ($Re \approx 350$, $D=15.88\text{mm}$)

Fig.9(b) Comparison of the velocity u_a (cal.1) of the jet axis with the experimental results ⁽¹⁾ ($Re \approx 600$, $D=15.88\text{mm}$)

tube end which is lower than the atmospheric pressure, then the pressure gradually increases further downstream, and at $z=0.67$ in Fig.10(a) and $z=0.40$ in Fig.10(b) the pressure becomes equal to the atmospheric one. Further downstream, the pressure on the jet axis still increases, and at $z=1.50$ in Fig.10(a) and $z=1.0$ in Fig.10(b) the pressure takes a maximum value which is higher than the atmospheric pressure. Still further downstream, the pressure on jet axis decreases and finally approaches the atmospheric pressure asymptotically. At a cross section near the outlet inside of the tube, the pressure near the tube axis is lower than that near the tube wall. This implies that as the flow comes close to the tube outlet, the velocity near the tube axis becomes larger than that of the Poiseuille flow, whereas the velocity near the tube wall becomes smaller. Such variation in pressure near the flow axis described above is more remarkable in cal.2 than in cal.1.

The pressure P_h on the tube wall is shown in Fig.11. The pressure gradient far upstream from the tube outlet is coincident with that of the Poiseuille flow. As the flow comes close to the tube outlet, the pressure decreases and becomes lower than the atmospheric pressure near the tube outlet. In cal.2, a minimum pressure that is lower than the atmospheric pressure occurs at a position by about tube-radius times 0.7 upstream from the tube end. This becomes remarkable as the value of Re/F_r increases. Furthermore, these tendencies are more remarkable in cal.2 than in cal.1.

(a) cal.1

(b) cal.2

Fig.10 Pressure distribution ($Re=753$, $F_r=9.75 \times 10^{-2}$)

The pressure P_h on the tube wall for $Re=753$ is shown in Fig.12. It can be seen that the minimum pressure on the tube wall decreases as the Froude number decreases. Table 2 shows the ratio of the surface velocity u_s to the velocity u_a on the jet axis for $Re=753$ at a position by one tube-diameter downstream from the tube end. It can be seen that the flow becomes remarkably uniform as the Froude number decreases.

Table 2 Ratio of the surface velocity u_s to the velocity u_a on the jet axis ($Re=753$, $z=1.95$)

Fr	Re/Fr	u_s/u_a	
		cal.1	cal.2
12.8×10^{-2}	5.89×10^3	—	0.962
9.75	7.73	0.959	0.972
7.29	10.3	—	0.980

Fig.11 Distribution of the pressures P_h on the wall

Fig.12 Effect of the Froude number on the Pressure P_h on the tube wall

5. Conclusions

The flow patterns were numerically calculated in the case when a liquid of small viscosity issued downward from a circular tube into the atmosphere. Consequently, the following results were obtained:

(1) Inside of the tube, the velocity near the tube axis becomes larger than that of the Poiseuille flow as Re/F_r increases and as the flow comes close to the tube outlet, the velocity near the tube wall becomes smaller. This tendency agrees very well with the result in the previous report ⁽¹⁾. Particularly, the velocity profile at the tube end is fairly different from that of the Poiseuille flow.

(2) Outside of the tube, although the velocity near the jet axis increases as Re/F_r increases and further downstream, the acceleration near the jet axis is smaller than that near the free surface. The jet surface rapidly contracts and the flow becomes nearly uniform. This tendency agrees very well with the result in the previous report ⁽¹⁾.

(3) At a sufficiently upstream region, the pressure gradient $\partial P_h / \partial z$ on the tube wall is coincident with that of the Poiseuille flow. As the flow comes close to the tube outlet, the pressure becomes lower than the atmospheric pressure and the minimum pressure occurs at a position by about tube-radius times 0.7 from the tube end. At a cross section inside of the tube near the outlet, the pressure near the tube axis is lower than that near the tube wall.

(4) The flow patterns near the tube outlet change rapidly, and it is found that there is no eddy region near the tube wall.

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