621.833.1

Paper No. 185-27

On Tooth Form Accuracy of Roll-Finished Involute Helical Gears*

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In order to improve the quality of roll-finished involute helical gears, a calculation program for roll-finished tooth form has been developed, which can deal with tooth form correction of die-wheel and tooth form error of work-gear before roll-finishing. In this calculation, gradual changing of tooth form of work-gear with progress of plastic deformation is taken in consideration. Some comparisons between calculated and measured tooth forms of helical and spur gears roll-finished by die-wheels with and without die-profile correction under some roll-finishing conditions showed fairly good coincidence. This program can therefore help to decide the optimum tooth form of die-wheel and the optimum working condition.

1. Introduction

Roll-finished cylindrical involute gear has some problems about tooth form quality: the tooth form shows commonly some concave shape from true involute profile at the position near pitch point, and transverse tooth forms at different longitudinal positions of gear are not uniform [1]-[4]. Roll-finishing conditions give large influence on the tooth form, and in order to improve the tooth form quality of roll-finished gear, tooth form correction of die-wheel seems to be necessary. In spite of industrially important demands for helical gears, there are so scarce knowledge and information on the necessary shape of corrected tooth form of die-wheel and on the necessary amount of the correction, and also on the optimum working condition for the roll-finishing of helical gears. The complicated states of tooth contact and load distributions on tooth flanks have made it difficult to perform theoretical investigations on roll-finishing of helical gear. It is also difficult to establish general rules only from experimental investigations, because the tooth form correction of die-wheel, for example, involves possibilities of so many combinations of transverse, longitudinal tooth correcting shapes and their amounts. To find out a better solution against such today's status of roll-finishing of helical gear, it is necessary to develop a computer simulation program which can well reproduce the roll-finishing process of helical gear, and the investigations from both experimental and computer-simulating approaches, that is, by analyzing and feeding back of experimental data to the simulation program and giving effective guides to the experiments by computer simulation to find better roll-finishing conditions and to find a better tooth form correction of die-wheel, are indispensable. In this report, the fundamental structure of the simulation program

using theoretical analysis about load distribution on helical gear tooth and experimental data about plastic deformation between roller and plate is shown, and the prospect of this simulation program is discussed.

2. Plastic deformation between roller and plate

In order to obtain the fundamental data on permanent deformation of tooth flank of work-gear pressed by hard tooth flank of die-wheel, hard rollers of SKS-3 steel were pressed on S45C steel block which has same width with roller length, and the permanent deformations on the surface of S45C block were measured. Fig.1 shows the relation between specific pressing load P (kgf/mm) on roller and the maximum depth D (μ m) of plastic deformation¹, using roller radii R (mm) as a parameter. There seems to be the relation

$$D = k P^2 \quad (1)$$

between D and P , where k is a constant which is decided by steel sort and roller radius. When a definite specific pressing load P_1 and the amount of deformation D_1 under this load are known, the amount of deformation D under arbitrary load P is given from eq.(1) by

$$D = D_1 \left(\frac{P}{P_1} \right)^2 \quad (2)$$

Fig.1 Relation between press. load and plastic indentation

* Received 17th February 1978

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1 The maximum depth D of plastic deformation on worked body given by one contact is called in this report the "amount of deformation".

Fig.2 Relation between geometrical interpenetration and plastic indentation

3. Simulation of roll-finishing

The load distribution along a contact line on a helical gear tooth is given by the solution of

$$\int_{-l}^l K_b(x, \xi) p(\xi) d\xi + K_c(x) p(x) = \{\Delta + e(x)\} \cos \beta_g \quad (4)$$

where x indicates the position on tooth flank, at which tooth pair deformation is under consideration. ξ indicates the position on tooth flank, at which the distributed load is under consideration. $K_b(x, \xi)$ is influence function for the bending and shearing deflection of meshing tooth pair. $K_b(x, \xi)$ considers also the influence of frictional force on tooth flank on tooth deflection. $K_c(x)$ is influence function for the contact approach between contacting tooth flanks. $p(\xi)$ is the distributed load. Δ is a rotational delay of driven gear to driving gear, which is reduced to the displacement on the line of action². $e(x)$ is composite error at x , that is the sum of the gear errors of driving and driven gears. β_g is helix angle on base cylinder. l is a half length of contact line, cf. Fig.3. During the process of roll-finishing of gear, both tooth flanks, i.e. driven and followed flanks, mesh and are pressed at the same time. When a state of tooth meshing on three simultaneous lines of contact through points a, b and c shown in Fig.4 is taken into consideration, for example, the following equations are valid:

(i) on a contact line which passes through point a:

$$\int_{-l_a}^{l_a} K_b(x_a, \xi_a) p(\xi_a) d\xi_a + K_c(x_a) p(x_a) + \int_{-l_b}^{l_b} K_b(x_a, \xi_b) p(\xi_b) d\xi_b = \{\Delta_A + e(x_a)\} \cos \beta_g \quad (5)$$

(ii) on a contact line which passes through point b:

$$\int_{-l_a}^{l_a} K_b(x_b, \xi_a) p(\xi_a) d\xi_a + \int_{-l_b}^{l_b} K_b(x_b, \xi_b) p(\xi_b) d\xi_b + K_c(x_b) p(x_b) + \int_{-l_c}^{l_c} K_b(x_b, \xi_c) p(\xi_c) d\xi_c = \{\Delta_B + e(x_b)\} \cos \beta_g \quad (6)$$

(iii) on a contact line which passes through point c:

$$\int_{-l_b}^{l_b} K_b(x_c, \xi_b) p(\xi_b) d\xi_b + \int_{-l_c}^{l_c} K_b(x_c, \xi_c) p(\xi_c) d\xi_c + K_c(x_c) p(x_c) = \{\Delta_C + e(x_c)\} \cos \beta_g \quad (7)$$

where suffixes a, b or c on symbols x , ξ and l indicate that they are quantities on a contact line which passes through points a, b or c respectively. Suffixes A and B indicate that the quantity with this suffix is concerned with plane of action A or with plane of action B. The equilibrium condition that the component of total forces on all meshing tooth flanks in the pressing direction of die-wheel is equal to the roll-finishing load W which is given as a working condition, is given by

$$W = \int_{-l_a}^{l_a} \{\sin \alpha_{ba} \cos \beta_g + S(\xi_a) \mu \cos \alpha_{ba}\} p(\xi_a) d\xi_a + \int_{-l_b}^{l_b} \{\sin \alpha_{bb} \cos \beta_g + S(\xi_b) \mu \cos \alpha_{bb}\} p(\xi_b) d\xi_b + \int_{-l_c}^{l_c} \{\sin \alpha_{bc} \cos \beta_g + S(\xi_c) \mu \cos \alpha_{bc}\} p(\xi_c) d\xi_c \quad (8)$$

Fig.2 shows the relation between amount of deformation and geometrical interpenetration $a^2/(2R)$ (mm) of two contacting bodies, where a is a half of Hertzian contact width and R is the relative radius of curvature. The experimental results are summed up by

$$D = 1330 \{a^2/(2R)\}^{1.227} \quad (3)$$

without receiving any influences of R -value. Same kind of experiments were carried out using rollers of different lengths, but the influence of roller length, i.e. of contact length, on the amount of deformation was found to be negligibly small.

Fig.3 Load distribution on a line of contact

² The expression "line of action" in this report means a transverse section of plane of action.

where μ is coefficient of friction between tooth flanks of die-wheel and work-gear, α_{bi} is transverse meshing pressure angle and $S(\xi) = -1$, when the component of frictional force on tooth flank in the direction of center distance has the same sign as pressing direction of die-wheel and $S(\xi) = 1$, when the above mentioned component of frictional force has the opposite sign to pressing direction of die-wheel. The equilibrium condition that the torque on work gear owing to the loads on tooth flanks in planes of action A and B is zero, is given by

$$\int_{-l_a}^{l_a} \{r_g \cos \beta_g - S(\xi_a) E(\xi_a) \mu\} p(\xi_a) d\xi_a - \int_{-l_b}^{l_b} \{r_g \cos \beta_g - S(\xi_b) E(\xi_b) \mu\} p(\xi_b) d\xi_b + \int_{-l_c}^{l_c} \{r_g \cos \beta_g - S(\xi_c) E(\xi_c) \mu\} p(\xi_c) d\xi_c = 0 \quad (9)$$

where r_g is radius of base cylinder and $E(\xi)$ is the distance from base cylinder to the contacting point of tooth flanks on the line of action. The distributed loads on lines of contact can be calculated by solving the equations (5) to (9) simultaneously. When the meshing state of die-wheel and work-gear proceeds and the condition of simultaneous contact lines changes from the state shown in Fig.4, the contents of eqs. (5) to (9) change naturally corresponding to the new state of simultaneous contact lines.

The contact approach between tooth flanks of die-wheel and work-gear during roll-finishing process is the sum of elastic deformation D_e and plastic deformation D . The amount of elastic deformation D_e is expressed by

$$D_e = K_e p \quad (10)$$

where K_e is the compliance for elastic contact approach between tooth flanks and p is the distributed loading value at this point. Now if the quantity K_p is incorporated, which fulfills the relation

$$D = K_p p \quad (11)$$

for the amount of plastic deformation D , the nominal compliance K_c for contact approach between tooth flanks of die-wheel and of work-gear during roll-finishing is expressed by

$$K_c = K_e + K_p \quad (12)$$

For the value of K_p , the experimental data of plastic deformation between pressed roller and plate shown in Fig.1 are applicable. Fig.5 shows this relation among K_p , specific pressing load p on roller and radius of roller R . K_p -value changes as a function of specific load p , and the nominal compliance K_c has therefore nonlinear characteristics. In the simulating calculation for roll-finishing of gear, radius of roller used for plastic deformation test is set to correspond to the local relative radius of curvature of contacting tooth flanks of die-wheel and work-gear, and specific pressing load on roller at plastic deformation test is set to correspond to the local value of distributed load on tooth flanks. A linear relation shown in Fig.5 is assumed to hold between K_p , R and p .

Fig.4 Positions of lines of contact (shown in transverse section)

Fig.5 Nominal compliance for contact approach owing to plastic deformation

Fig.6 Flow chart for simulating calculation

The state of work-tooth form and distributed load on a line of contact changes gradually from the beginning of contact between maiden tooth flank of work gear and tooth flank of die-wheel until they are pressed by roll-finishing load given as working condition. This state of changing is treated as follows: at the very beginning of tooth flank contact, tooth deformation is assumed to be full-elastic. Under this assumption, the distributed load p_1 on lines of contact is calculated. Contact stress obtained by this procedure is slightly larger than the threshold value of contact stress corresponding to the beginning of permanent deformation of contacting bodies, which is obtained by plastic deformation test using roller and plate. The geometrical interpenetration between tooth flank profiles of die-wheel and work-gear is then set to correspond to the geometrical interpenetration shown in Fig.2, and the permanent deformation D_1 of work-tooth flank under the distributed load p_1 is calculated. Tooth form error of work-gear is therefore now changed by D_1 . Under this new tooth form condition and giving a little more roll-finishing load, the new distributed load p on the same line of contact is calculated for elastic-plastic state of work-tooth flank. The calculation is possible, because at this second and also in later calculating routines, K_p -value is already obtained from former calculating routine. Work-tooth flank under distributed load p is deformed then plastically by D of eq.(2). That is, the tooth form error of work increases further by D . Under this new tooth form condition of work and giving a little more roll-finishing load, the distributed load for this new condition is calculated. Such routine calculation is repeated until eqs.(8) and (9) are fulfilled, and until tooth form of work-gear will not change after one more routine calculation under constant roll-finishing load. After this calculating process, the composite tooth form error on the lines of contact is also obtained. Then the meshing position of teeth is slightly shifted, and the calculation procedure mentioned above is carried out for new positions of lines of contact. When the calculation is finished for one tooth meshing period, sets of simultaneous contact lines cover the whole plane of action, and the composite tooth form error on plane of action, therefore on whole tooth flank, is obtained. The tooth form error of work-gear, that is, of roll-finished gear, is obtained by subtraction of tooth form error of die-wheel from the composite tooth form error of work-gear and die-wheel, cf. Appendix. Fig.6 shows the flow chart of this calculation procedure.

4. Comparison between calculated and measured tooth forms of roll-finished gear

Experiments of roll-finishing and the corresponding computer simulation were carried out on a pair of work-gear and die-wheel shown in Table 1. Fig.8 shows the calculated transverse tooth forms at three different longitudinal positions of roll-finished gear, cf. Fig.7. Straight line corresponds to the true involute curve. The calculation was made for a case of using an error free die-wheel and a roll-finishing load $W=1450$ kgf, and for two cases of coefficients of friction $\mu=0$ and $\mu=0.2$ between tooth flank of work-gear and that of die-wheel. The calculated transverse tooth form inspection

Fig.7 Position of transverse and longitudinal tooth form measurement and calculation (shown on the plane of action)

Table 1 Specifications of die-wheel and work-gear (helical gears)

	DIE	WORK
tooth form	normal	
module	2 mm	
press. angle	20°	
tooth number	80	23
prof. mod. factor	0	0
helix angle	23° LH	23° RH
face width	20 mm	15 mm
material	SKS3	S45C
flank hardness	$H_{ac}=60$	$H_B=150$
quality	JIS M1	(hobbed)

Fig.8 Calculated transverse tooth form roll-finished with error free die-wheel under $W=1450$ kgf

Fig.9 Measured transverse tooth form roll-finished with almost error free die-wheel under $W=1450$ kgf

curves of work-gear roll-finished by the error free die-wheel show concave shape near pitch point. In a case of $\mu = 0$, the tooth forms of driven and followed tooth flanks of work-gear become the same. But when $\mu \neq 0$, tooth forms of both flanks are different: driven tooth flank has a concave part at a position a little nearer to tooth tip from pitch point, and followed tooth flank has a concave part at a position near pitch point. Fig.9 shows the transverse tooth forms of four different teeth located each at 90 deg. position of a work-gear which was roll-finished by an almost error free die-wheel (JIS Quality M1) under roll-finishing load $W=1450$ kgf, roll-finishing time 5 sec. and rotational speed of die-wheel 25 rpm. Tooth form was measured at the middle of tooth width and it corresponds to the calculated curve (S2) in Fig.8. The shapes of measured and calculated tooth forms coincide well at the following points: Measured tooth form inspection curves of driven flank show a concave part at a position a little nearer to tooth tip from pitch point and those of followed flank show a concave part at a position near pitch point. Fig.10 shows the calculated transverse tooth forms of a work gear roll-finished by an error free die-wheel under $W=2000$ kgf loading, and Fig.11 shows the measured tooth form of a work-gear whose roll-finishing conditions correspond to those of Fig. 10. In this case the measured and the calculated tooth forms show fairly good qualitative agreement. The calculated tooth form error after roll-finishing of $W=2000$ kgf loading, Fig. 10, is larger than that of $W=1450$ kgf loading, Fig.8, and these calculated results confirm the fact that there is an optimum roll-finishing loading value and excess loading makes the roll-finished tooth form of work-gear worse as known from recent experimental investigation [1].

As shown in Figs.8,9 and 10, the transverse tooth form of a work-gear roll-finished by an error free die-wheel gives a tooth form

Fig.10 Calculated transverse tooth form roll-finished with error free die-wheel under $W=2000$ kgf

Fig.13 Calculated transverse tooth form roll-finished with form-corrected die-wheel under $W=1450$ kgf

Fig.11 Measured transverse tooth form roll-finished with almost error free die-wheel under $W=2000$ kgf

Fig.12 Corrected transverse tooth form of die-wheel (Driving and following tooth form is same)

Fig.14 Measured transverse tooth form roll-finished with form-corrected die-wheel under $W=1450$ kgf

Fig. 15 Calculated longitudinal tooth form roll-finished with error free die-wheel under $W=2000$ kgf

inspection curve which has a concave part near pitch point. To improve the roll-finished tooth form quality, tooth form correction of die-wheel was tried: tooth forms of both driving and following flanks of die-wheel are corrected in concave shape, whose tooth form inspection curves are shown in Fig. 12. Fig. 13 shows the calculated transverse tooth forms of a work-gear roll-finished by this corrected die-wheel, and Fig. 14 shows the measured tooth forms. The calculated and measured tooth forms show that the tooth form accuracy of a work-gear roll-finished by the corrected die-wheel is improved on followed tooth flank, but on driven tooth flank a clear improvement of tooth form accuracy cannot be recognized: the tooth form inspection curve of driven flank shows higher heap near tooth root of roll-finished work-gear. The comparison between Fig. 8 and Fig. 13 shows that the calculated improvement of tooth form error by using the corrected die-wheel is about $3\text{ }\mu\text{m}$ even on followed tooth flank. This amount of improvement is far more smaller than the amount of tooth form correction, ca. $12\text{ }\mu\text{m}$, of die-wheel. Such difference between amount of tooth form correction of die-wheel and improvement amount of tooth form error of roll-finished work-gear has been always observed in experiments, which is also recognized by the

Fig. 17 Calculated transverse tooth form of profile shifted spur gear roll-finished with error free die-wheel under $W=800$ kgf

comparison between Fig. 9 and Fig. 14. Fig. 15 shows the calculated longitudinal tooth forms (in tooth width direction) of a work-gear roll-finished by an error free die-wheel under $W=2000$ kgf loading. See Fig. 7 for the position of tooth form indicated. Fig. 16 shows a measured longitudinal tooth form. The measured and calculated tooth forms show a dull-rounding down at both tooth flank sides, and even at the middle part of tooth width the longitudinal tooth form is not flat. The necessity for some kind of longitudinal tooth form correction of die-wheel is recognized.

In order to inspect the general applicability of this simulation program for roll-fining of cylindrical involute gear, the calculated and measured results with spur gear must also be examined, in which the load distribution on tooth flank and the load sharing by each simultaneously meshing tooth pairs are very different from those of helical gears. Fig. 17 shows the calculated transverse tooth form of a spur work-gear roll-finished by an error free spur die-wheel whose dimensions are listed in Table 2. Fig. 18 shows the measured transverse tooth form corresponding to the case of Fig. 17 [2]. The quantitative amount of calculated and measured tooth form errors do not coincide, but both tooth forms show the same

Table 2 Specifications of die-wheel and work-gear (spur gears)

	DIE	WORK
tooth form	normal	
module	4 mm	
press. angle	20°	
tooth number	36	28
prof. mod. factor	0.2	0.488
helix angle	0	0
tooth width	14.4 mm	10.0 mm
material	SKH9	S43C
flank hardness	$H_{RC}=61$	$H_V=240$

Fig. 18 Measured transverse tooth form of profile shifted spur gear roll-finished with almost error free die-wheel under $W=800$ kgf

trend, that the concave part of tooth form inspection curve occurs at a position a little nearer to tooth tip from the pitch point and the depth of this concave part is larger on followed tooth flank than on driving tooth flank. From the results shown in Figs.8 to 18, it is concluded that the calculated transverse and longitudinal tooth forms of spur and helical roll-finished gears are qualitatively the same figure as those of actually roll-finished gears.

5. Conclusions

A simulation program for roll-finishing of involute spur and helical gears has been developed, which has the following features:

- 1) The nonlinear changing states of load distribution on contact lines, tooth stiffness, tooth form error etc. in the progress of plastic deformation of tooth flank material during roll-finishing are followed as exactly as possible.
- 2) The three dimensional tooth meshing of helical gears is exactly considered.
- 3) Because of the difficulty to consider, the plastic flow of material in the direction tangential to tooth flank is not considered in the calculation. The roll-finishing in this simulation is assumed to proceed only by the indentating plastic deformation between work- and die-tooth flanks.

The comparison between measured and calculated tooth forms of roll-finished gears showed qualitatively good coincidence. It is proved therefore, that the here developed simulation program for roll-finishing of involute spur and helical gears becomes a good help in designing of the tooth form of die-wheel and in investigation of the optimum working condition for roll-finishing.

Acknowledgement

Sincere gratitude is extended to Mr. Y. Toko-o, engineer of the Kyoto Institute of Technology, for his help in the experimental work of this investigation. A part of this research was carried out with a scientific research subsidy of the Cultural Ministry of Japanese Government for 1974 and 1975. The numerical calculation was performed by using the computer FACOM M-190 of the Data Processing Center of Kyoto University.

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Appendix

Fig.19 shows the procedure to obtain the composite tooth form error on the plane of action. The contacting state of die- and work-tooth flank is expressed by

$$\frac{1}{\cos \beta_g} \left[\int_{-l}^l K_b(x, \xi) p(\xi) d\xi + \{K_p(x) + K_s(x)\} p(x) \right] = \Delta - \{e_d(x) + e_w(x)\} \quad (13)$$

where $e_d(x)$ and $e_w(x)$ are tooth form errors of die- and work-gear respectively. By the operation (ii) in Fig.19, an indefinite amount of $e_d(x)$ and $e_w(x)$ owing to the definition of the zero-level of $e_d(x)$ and $e_w(x)$ is included in the value of Δ . And the sum of the deformations of die- and work tooth $\Delta - \{e_d(x) + e_w(x)\}$ is obtained with no influence of above mentioned indefinite amount of composite error. The sole solution for distributed load $p(x)$ is therefore obtained, cf. Fig.20. $e_w(x)$ changes gradually during the calculation with progress of plastic deformation of work tooth flank, fulfilling the equilibrium condition that the instantaneous deformed tooth form of work-gear is the distributed load on the contact line. During this calculating process the constant deformation of work-tooth on the contact line is included in Δ , and not in $e_w(x)$, as the approach between zero-levels of work- and die-tooth form error. The state (vi), at which the deformed tooth form of work gear is defined on the contact line, corresponds to the equilibrium state of roller pressed on steel block. And the approach between zero levels of work- and die-tooth form error at this stage corresponds to the amount of indentation D shown in Fig.2. The composite tooth form after roll-finishing is the sum of the distance between unloaded tooth flank position and the zero level for tooth form measurement of die-wheel and that of work-gear after stage (vi), and this sum is expressed by $D + e_d(x) + e_w(x)$. In the value of $e_d(x) + e_w(x)$ there still remains an indefinite amount caused by the selection of

Fig.19 Calculating procedure of composite tooth form error on plane of action

zero level for the definition of work-tooth form before roll-finishing. But this indefinite amount is constant on the plane of action and this means a changing of calculated tooth form by constant amount on the plane of action, which is of no hindrance to this analysis. The calculating procedure mentioned above represents a simulation of one contact between die- and work tooth flanks. In order to simulate multiple contacts between die- and work tooth flanks, the routine calculation must be repeated considering the work hardening effect of tooth flank material until the amount of plastic deformation D_A becomes constant all over the A-plane of action and D_B all over the B-plane of action (it is not necessary that $D_A = D_B$). But former experimental investigations show [11]-[13] that the plastic deformation of work-tooth is almost completed after one contact of die- and work-tooth flanks. Therefore the simulation of only one contact of die- and work-tooth flanks is sufficient for the qualitative investigation of roll-finishing.

Fig.20 Deformation of die-and work-tooth