532.517.6:532.542

Paper No. 186-12

Flow Pattern and Frictional Losses in Pulsating Pipe Flow

Part 3 General Representation of Turbulent Flow Pattern*

By Munekazu OHMI** and Manabu IGUCHI***

The characteristic parameters $\Phi_{t,n}$, $\Phi_{z,n}$, $\alpha_{t,n}$, and $\alpha_{z,n}$ describing the flow pattern in a pulsating turbulent pipe flow are introduced as functions of $\omega'/Re_{ta}^{3/4}$ from quasi-steady analysis, where ω' is the dimensionless frequency, Re_{ta} the time-averaged Reynolds number and the validity of these parameters is confirmed experimentally. In the range of time-averaged Reynolds numbers larger than 1×10^5 , where Blasius' $1/4$ power frictional law is not valid, $\omega'/Re_{ta}^{3/4}$ must be replaced with $\omega'/(\lambda_s Re_{ta})$. Then λ_s is calculated from another friction law. The values of $\omega'/Re_{ta}^{3/4}$ corresponding to the limits between quasi-steady, intermediate, and inertia dominant regions are also proposed.

1. Introduction

The present authors showed in a previous paper⁽¹⁾ that the flow pattern in a pulsating turbulent pipe flow could be described by four characteristic parameters $\Phi_{t,n}$, $\Phi_{z,n}$, $\alpha_{t,n}$, and $\alpha_{z,n}$, and that these parameters were functions of dimensionless frequency ω' at time-averaged Reynolds number Re_{ta} of 5.7×10^4 .

On the basis of these parameters the flow patterns were classified into three types, i.e., quasi-steady, intermediate, and inertia dominant ones with respect to ω' .

By the way, one of the authors has reported that the pulsating velocity distribution for small ω' and small Re_{ta} is qualitatively similar to that for large ω' and large Re_{ta} ⁽²⁾. This indicates that the flow pattern may be affected by Re_{ta} in addition to ω' . Therefore, the limits between the three regions mentioned above also become a function of ω' and Re_{ta} . It is the aim of this investigation to study the effect of time-averaged Reynolds number on the flow pattern.

From the analysis of quasi-steady flow it is expected that the flow pattern may be described as a function of $\omega'/Re_{ta}^{3/4}$. From the result of examina-

tion in intermediate and inertia dominant regions it is evident that the characteristic parameters are functions of $\omega'/Re_{ta}^{3/4}$ in the Reynolds number range over which Blasius' friction law is valid and that they are functions of $\omega'/(\lambda_s Re_{ta})$ in a wider range of the Reynolds numbers.

2. Nomenclature

- A_1 : velocity amplitude ratio
= $|u_{m,os}|/u_{m,ta}$
- D : pipe diameter
- f : pulsation frequency
- K_1 : coefficient defined by Eq.(29)
- l : pipe length of test section
- $M_0(\sqrt{\omega'}), M_1(\sqrt{\omega'}), M_2(\sqrt{\omega'}), \theta_0(\sqrt{\omega'}), \theta_1(\sqrt{\omega'}), \theta_2(\sqrt{\omega'})$
: modulus and phase of Bessel functions of the first order
- p : pressure
- Δp : pressure drop
- R : radius of pipe
- Re : instantaneous Reynolds number = $u_m D / \nu$
- Re_{ta} : time-averaged Reynolds number
= $u_{m,ta} D / \nu$
- St : Strouhal number = $f D / u_{m,ta}$
- t : time
- u_m : cross-sectional mean velocity
- u : axial velocity
- λ_g : quasi-steady friction factor
- λ_s : steady flow friction factor
- ν : kinematic viscosity
- ρ : density
- σ_1 : modification factor = $1 + 21A_1^2/64$
- τ_w : wall shear stress
- $\Phi_{t,n}, \Phi_{z,n}, \alpha_{t,n}, \alpha_{z,n}$: characteristic parameters describing the flow pattern
- ω : angular frequency
- ω' : dimensionless frequency = $R^2 \omega / \nu$
- Subscripts and others
 - q : quasi-steady state
 - n : n th harmonic in the finite Fourier expansion
 - os : oscillating component
 - ta : long-time-averaged value

* Received 28th June, 1978.

** Professor, Faculty of Engineering, Osaka University.

*** Assistant, Faculty of Engineering, Osaka University, Yamada-Kami, Suita, Osaka.

- : short-time-averaged value which is permitted to include the relatively low frequencies for the sinusoidal waves under consideration

3. Basic equations

As a basic equation for investigating the flow pattern in an incompressible pulsating turbulent pipe flow, the following momentum balance equation is employed.

$$\rho_0 \frac{du_m}{dt} + \frac{4}{D} \tau_w = \frac{\Delta p}{l} \quad (1)$$

The terms $\rho_0 du_m/dt$, $4\tau_w/D$, and $\Delta p/l$ in this equation are named inertia, viscous, and pressure gradient terms respectively.

The authors have made experiments on the flow pattern at time-averaged Reynolds number Re_{ta} of about 5.7×10^4 and over wide ranges of ω' and A_1 , and reported the results in a previous paper.⁽¹⁾ Then experimental values of $\Delta p/l$, u_m , and τ_w are fitted by the finite Fourier expansion as

$$\frac{\Delta p}{l} = \frac{\Delta p_{ta}}{l} + \sum_{n=1}^3 \frac{|\Delta p_{os,n}|}{l} \cos(n\omega t + \angle \frac{\Delta p_{os,n}}{l}) \quad (2)$$

$$u_m = u_{m,ta} + \sum_{n=1}^3 |u_{m,os,n}| \cos(n\omega t + \angle u_{m,os,n}) \quad (3)$$

$$\tau_w = \tau_{w,ta} + \sum_{n=1}^3 |\tau_{w,os,n}| \cos(n\omega t + \angle \tau_{w,os,n}) \quad (4)$$

After substitution of these equations into Eq.(1), it is divided into two parts and rewritten as follows:

$$\frac{\Delta p_{ta}}{l} = \frac{4}{D} \tau_{w,ta} \quad (5)$$

$$\begin{aligned} & -\rho_0 n \omega |u_{m,os,n}| \sin(n\omega t + \angle u_{m,os,n}) \\ & + (4/D) |\tau_{w,os,n}| \cos(n\omega t + \angle \tau_{w,os,n}) \\ & = (|\Delta p_{os,n}|/l) \cos[n\omega t + \angle(\Delta p_{os,n}/l)] \\ & \quad (n=1, 2, 3) \end{aligned} \quad (6)$$

On the basis of these equations (5) and (6), then the four characteristic parameters describing the flow pattern are defined as

$$\phi_{1,n} = \frac{\rho_0 n \omega |u_{m,os,n}|}{|\Delta p_{os,n}|/l} \quad (7)$$

$$\phi_{2,n} = \frac{(4/D) |\tau_{w,os,n}|}{|\Delta p_{os,n}|/l} \quad (8)$$

$$\alpha_{1,n} = \angle(\Delta p_{os,n}/l) - \angle u_{m,os,n} \quad (9)$$

$$\alpha_{2,n} = \angle(\Delta p_{os,n}/l) - \angle \tau_{w,os,n} \quad (10)$$

The values of these parameters determined experimentally are plotted in figures against dimensionless frequency ω' . These

figures are termed flow pattern diagrams. From these diagrams it is found that the velocity amplitude ratio has no effect on the flow pattern, but the effect of time-averaged Reynolds number is not discussed. The aim of this investigation is to clarify this effect.

It is well-known that the quasi-steady state holds at any instant in a cycle when ω' is very small. So the relationship between wall shear stress τ_w and cross-sectional mean velocity u_m in the quasi-steady region is taken up and the effect of Re_{ta} on the flow pattern is investigated. In the quasi-steady region τ_w is represented as a function of u_m as follows:

$$\tau_w = \lambda_t \rho_0 u_m^2 / 8 \quad (11)$$

$$\lambda_t = 0.3164 / Re^{1/4} \quad (Re = u_m D / \nu) \quad (12)$$

The quasi-steady expressions for the characteristic parameters can be obtained by using Eqs.(1), (11), and (12). Since experimental values of $\Delta p/l$, u_m , and τ_w are well fitted by sinusoidal waves, n is taken to be unity in Eq.(3) and zero is substituted for $\angle u_{m,os,n}$. Then Eq.(3) is written as

$$u_m = u_{m,ta} + |u_{m,os,1}| \cos \omega t \quad (13)$$

Substitution of Eq.(13) into Eq.(11) and the usage of Eq.(12) give

$$\tau_w = \frac{\rho_0}{8} \left\{ 0.3164 \left(\frac{\nu}{D} \right)^{1/4} \left(u_{m,ta} + |u_{m,os,1}| \cos \omega t \right)^{7/4} \right\} \quad (14)$$

From assumption that the flow is turbulent over one cycle, the relationship $u_{m,ta} > |u_{m,os,1}|$ is established, i.e., $A_1 < 1$. Under this condition the right-hand side of Eq.(14) can be expanded in a series, resulting in the following equation

$$\begin{aligned} \tau_w = & \frac{0.3164}{8} \left(\frac{\nu}{D} \right)^{1/4} \rho_0 u_{m,ta}^{7/4} \left(1 + \frac{7}{4} A_1 \cos \omega t \right. \\ & \left. + \frac{21}{32} A_1^2 \cos^2 \omega t + \dots \right) \end{aligned} \quad (15)$$

Substitution of Eqs.(13), (15) into Eq.(11) gives

$$\begin{aligned} \frac{\Delta p}{l} = & -\rho_0 \omega |u_{m,os,1}| \sin \omega t + \frac{4}{D} \frac{0.3164}{8} \left(\frac{\nu}{D} \right)^{1/4} \rho_0 u_{m,ta}^{7/4} \left(1 \right. \\ & \left. + \frac{7}{4} A_1 \cos \omega t + \frac{21}{32} A_1^2 \cos^2 \omega t + \dots \right) \end{aligned} \quad (16)$$

Equations (15) and (16) are named the quasi-steady expressions for τ_w and $\Delta p/l$, respectively.

The characteristic parameters are made up of oscillating components, so that it is necessary to take oscillating components out of Eqs.(15) and (16). To do so the time-averaged values of τ_w and $\Delta p/l$

must be calculated beforehand. Values of $\tau_{w,ta}$ can be obtained by integrating Eq.(15) over one cycle with respect to time t (a detailed discussion was made in the previous paper⁽¹⁾) as follows:

$$\tau_{w,ta} = \frac{\rho_0}{8} \lambda_s u_{m,ta}^2 \left(1 + \frac{21}{64} A_1^2 + \frac{105}{16384} A_1^4 + \dots \right) \\ = \frac{\rho_0}{8} \lambda_s u_{m,ta}^2 \sigma_1 \quad \dots\dots\dots (17)$$

$$\sigma_1 = 1 + \frac{21}{64} A_1^2 + \frac{105}{16384} A_1^4 \dots\dots\dots (18)$$

where σ_1 is a modification factor which correlates time-averaged values of τ_w and $\Delta p/l$ with $u_{m,ta}$. Modification factor σ_1 is approximated as

$$\sigma_1 = 1 + (21/64) A_1^2$$

with errors less than 3 percent. In the same way as with $\tau_{w,ta}$, the following equation is obtained:

$$\Delta p_{ta}/l = \lambda_s \rho_0 u_{m,ta}^2 \sigma_1 / 2D \quad \dots\dots\dots (19)$$

Accordingly the oscillating components of τ_w and $\Delta p/l$ are determined by subtracting $\tau_{w,ta}$ and $\Delta p_{ta}/l$ from Eqs.(15) and (16) respectively.

Although the oscillating components of τ_w and $\Delta p/l$ obtained as before contain harmonics higher than $n = 2$, they are of the negligible order in comparison with the fundamental wave, so they are neglected here. The coefficient of $\cos \omega t$ in a brace of Eqs.(15) and (16) is rewritten as $(7/4)A_1 - (21/512)A_1^3 + \dots$. This is abbreviated as $(7/4)A_1$ with an error of 2.3 percent. The modulus and phase of τ_w and $\Delta p/l$ are represented respectively after simple treatment as follows:

$$|\tau_{w,os}| = \frac{\rho_0}{8} \lambda_s u_{m,ta}^2 \frac{7}{4} A_1 \quad \dots\dots\dots (20)$$

$$\angle \tau_{w,os,1} = \angle u_{m,os,1} = 0 \quad \dots\dots\dots (21)$$

$$\frac{|\Delta p_{os,1}|}{l} = \rho_0 |u_{m,os,1}| \sqrt{\omega^2 + \left[\frac{7\lambda_s u_{m,ta}}{(8D)} \right]^2} \quad \dots\dots\dots (22)$$

$$\angle \frac{\Delta p_{os,1}}{l} = \tan^{-1} \left\{ \omega / \left(\frac{7}{8} \lambda_s \frac{u_{m,ta}}{D} \right) \right\} \quad \dots\dots\dots (23)$$

Substituting these Eqs.(20)~(23) into Eqs.(7)~(10), the four characteristic parameters $\Phi_{t,1}$, $\Phi_{z,1}$, $\alpha_{t,1}$, and $\alpha_{z,1}$ are obtained readily and written as

$$\Phi_{t,1} = 1 / \sqrt{1 + \left(\frac{7\lambda_s R_{e,ta}}{32 \omega'} \right)^2} \\ = 1 / \sqrt{1 + \left(\frac{1}{14.45} \frac{R_{e,ta}^{3/4}}{\omega'} \right)^2} \quad \dots\dots\dots (24)$$

$$\Phi_{z,1} = \frac{\frac{7\lambda_s R_{e,ta}}{32 \omega'}}{\sqrt{1 + \left(\frac{7\lambda_s R_{e,ta}}{32 \omega'} \right)^2}} = \frac{\frac{1}{14.45} \frac{R_{e,ta}^{3/4}}{\omega'}}{\sqrt{1 + \left(\frac{1}{14.45} \frac{R_{e,ta}^{3/4}}{\omega'} \right)^2}} \quad \dots\dots\dots (25)$$

$$\alpha_{t,1} = \alpha_{z,1} = \tan^{-1} (32 \omega' / 7 \lambda_s R_{e,ta}) \\ = \tan^{-1} (14.45 \omega' / R_{e,ta}^{3/4}) \quad \dots\dots\dots (26)$$

For reference it would be useful to calculate the characteristic parameters for $n \geq 2$. The result is represented as

$$\Phi_{t,n} = 0, \quad \Phi_{z,n} = 1, \quad \alpha_{t,n} = \alpha_{z,n} = 0 \quad (n \geq 2) \quad \dots\dots\dots (27)$$

A summary of the analytical results gives the following explanation in the quasi-steady region that the characteristic parameters are functions of $\omega'/R_{e,ta}^{3/4}$ when λ_s and λ_s obey Blasius' friction law, and that in the Reynolds number range where Blasius' law is not valid $\omega'/(\lambda_s R_{e,ta})$ must be used. Applicability of these expressions in the intermediate and inertia dominant regions has to be judged experimentally.

4. Experimental setups and procedure

Experimental setups are described elsewhere.⁽¹⁾ In the present investigation experiments with water were made in order to obtain data on a relatively high ω' region. Pulsation was given to a steady flow by means of a piston mechanism and its frequencies were chosen to be 0.140, 0.431, 1.03, and 2.03 Hz ($\sqrt{\omega'} = 33.0 \sim 138$). Time-averaged Reynolds number and velocity amplitude ratio ranged from 1×10^4 to 6×10^4 and from 0.05 to 0.9 respectively. Velocity measurements were made mainly with a hot film probe.

Experiments with air were made in a relatively small ω' region. Pulsation was generated by means of a butterfly valve mechanism. In this case $R_{e,ta}$ was $(1.9 \pm 0.2) \times 10^4$, $(6.8 \pm 0.1) \times 10^4$, and $(9.6 \pm 0.4) \times 10^4$. Frequency and velocity amplitude ratio ranged from 0.0432 to 2.06 Hz ($\sqrt{\omega'} = 3.35 \sim 25.0$) and 0.1~0.85 respectively.

5. Experimental results and discussion

Short-time-averaged value of axial velocity $\bar{u}$, cross-sectional mean velocity u_m , and pressure gradient $\Delta p/l$ are shown for three different values of $\omega'/R_{e,ta}^{3/4}$ in Figs.1~3. Data in Figs.1 and 2 were obtained with air and those in Fig.3 with water. It is evident from these figures that $\Delta p/l$ and u_m in each run are well fitted by sinusoidal waves.

Experimental values of characteristic parameters $\Phi_{t,1}$, $\Phi_{z,1}$, $\alpha_{t,1}$, and $\alpha_{z,1}$ are plotted against ω' in Figs.4~7 for $R_{e,ta}$ as a parameter. A curve (solid line) in each figure is drawn through the mean of the data points following the method described in the previous paper.⁽¹⁾ Quasi-steady expressions for characteristic parameters are shown by the broken lines. The experimental values of characteristic parameters for each Reynolds number are well fitted by the quasi-steady expression in the region where ω' is relatively small and approaches the dot-dash-line for laminar

flow as the time-averaged Reynolds number decreases. Laminar characteristic parameters are represented as follows:⁽³⁾

$$\left. \begin{aligned} \Phi_{t,1} &= M_2(\sqrt{\omega'})/M_0(\sqrt{\omega'}) \\ \Phi_{t,1} &= (2/\sqrt{\omega'})M_1(\sqrt{\omega'})/M_0(\sqrt{\omega'}) \\ \alpha_{t,1} &= \theta_0(\sqrt{\omega'}) - \theta_2(\sqrt{\omega'}) + 3\pi/2 \\ \alpha_{t,1} &= \theta_0(\sqrt{\omega'}) - \theta_1(\sqrt{\omega'}) + 3\pi/4 \end{aligned} \right\} \dots\dots\dots(28)$$

Characteristic parameters are replotted against $\omega'/Re_{ta}^{3/4}$ in Figs. 8 ~ 11. The solid lines are calculated from those in Figs. 4 ~ 7. Experimental $\Phi_{t,1}$ and $\alpha_{t,1}$ obtained by using a hot film anemometer in the region near the limit between intermediate and inertia dominant regions

are shown in Figs. 12 and 13 respectively. The solid lines are the same as those in Figs. 8 and 10. Experimental $\Phi_{t,1}$ and $\alpha_{t,1}$ are not given, because their experimental uncertainties in the region under consideration are large. In Figs. 8 ~ 13 all of the experimental values in each figure are fitted by a solid line with sufficient accuracy, so that it is concluded that the flow pattern in a pulsating turbulent flow can be described schematically as a function of $\omega'/Re_{ta}^{3/4}$.

On the basis of characteristic parameters $\Phi_{t,1}$ and $\Phi_{t,1}$, a method for determining limits between quasi-steady, intermediate, and inertia dominant regions was proposed in the previous paper.⁽²⁾ Here again, a more rational method is to be proposed. The limits between the quasi-steady and the intermediate regions and between the intermediate and inertia dominant regions are indicated by $(\omega'/Re_{ta}^{3/4})_q$ and $(\omega'/Re_{ta}^{3/4})_t$ respectively. The former can

(a) $\bar{u}$

(b) $u_m, \Delta p/l$

Fig.1 Short-time-averaged value of axial velocity component $\bar{u}$, cross-sectional mean velocity u_m , and pressure gradient $\Delta p/l$ in the quasi-steady region

(a) $\bar{u}$

(b) $u_m, \Delta p/l$

Fig.2 Short-time-averaged value of axial velocity component $\bar{u}$, cross-sectional mean velocity u_m , and pressure gradient $\Delta p/l$ in the intermediate region

be read off Figs.8 and 9. Then, $\sqrt{(\omega'/Re_{ta}^{3/4})_s}$ = 0.06 when $\Phi_{t,1}$ = 0.05 or 0.145 when $\Phi_{z,1}$ = 0.95, and the latter is represented by $\sqrt{(\omega'/Re_{ta}^{3/4})_t}$ = 1.5 from Fig.9 when $\Phi_{t,1}$ = 0.95.

In regard to the limit between the intermediate and the inertia dominant regions, it is possible to adopt the value of $\omega'/Re_{ta}^{3/4}$ when $\Phi_{z,1}$ = 0.05. However, experimental uncertainty of $\Phi_{z,1}$ in the region near this limit is noticeable as before, so that it would be adequate to adopt the limit when $\Phi_{t,1}$ = 0.95.

In Fig.10 the solid line drawn between $\sqrt{\omega'/Re_{ta}^{3/4}}$ = 0.5 and 2 was obtained experimentally by Nunobe, et. al.⁽⁴⁾ but $\Phi_{t,1}$, $\Phi_{z,1}$, and $\alpha_{z,1}$ are not considered. By assuming that the wall shear stress τ_w in Eq.(1) obeys the quasi-steady relation, i.e., $\tau_w = \lambda_g \rho_0 u_m^2/8$, they rewrote Eq.(1) as

$$\frac{dp}{l} = \lambda_g \frac{\rho_0}{2D} u_m^2 + K_1 \rho_0 \frac{du_m}{dt} \quad \dots\dots\dots(29)$$

(a) $\bar{u}$

(b) $u_m, dp/l$

Fig.3 Short-time-averaged value of axial velocity component $\bar{u}$, cross-sectional mean velocity u_m , and pressure gradient dp/l , in the inertia dominant region

Fig.4 Flow pattern diagram for $\Phi_{t,1}$ against ω'

Fig.5 Flow pattern diagram for $\Phi_{z,1}$ against ω'

Fig.6 Flow pattern diagram for $\alpha_{t,1}$ against ω'

Fig.7 Flow pattern diagram for $\alpha_{z,1}$ against ω'

where K_1 is an experimental constant, λ_s the quasi-steady friction factor represented by Nikuradse's friction law as

$$\lambda_s = 0.0032 + 0.221 R_e^{-0.287} \quad (30)$$

By using experimental values of $\Delta P/l$ and U_m , K_1 can be evaluated readily from Eq.(29) and after some rearrangement the characteristic parameter $\alpha_{t,1}$ is written as follows:

$$\alpha_{t,1} = \tan^{-1} \left(\frac{2\pi K_1}{0.905} \frac{St}{\lambda_s} \right) \quad (31)$$

where $St (= fD/U_m, \tau_a)$ is the Strohal number, $K_1 = 0.907$, λ_s the steady flow friction factor represented by Nikuradse's friction law. Since in their time-averaged Reynolds number range of 7.4×10^4 to 2×10^5 , the difference between Nikuradse's and Blasius' friction laws remains less than 3 percent, then Eq.(31) after replacing λ_s in Eq.(31) with Blasius' law is rewritten as

$$\alpha_{t,1} = \tan^{-1} (13.55 \omega' / R_e \tau_a^{3/4}) \quad (32)$$

This equation compares favourably with Eq.(26) and the present experimental values.

Notwithstanding the relatively large difference between experimental values and quasi-steady expressions of $\alpha_{z,1}$ in the intermediate and inertia dominant regions, good agreements between them for $\Phi_{t,1}$, $\Phi_{z,1}$, and $\alpha_{t,1}$ are seen in Figs. 8~10. This does not come from the fact that the quasi-steady state holds in these regions, but from the fact that the effect of viscosity becomes smaller with an increasing $\omega' / R_e \tau_a^{3/4}$.

Characteristic parameters $\Phi_{t,n}$, $\Phi_{z,n}$, $\alpha_{t,n}$, and $\alpha_{z,n}$ for $n = 2, 3$ are shown in Figs. 14~17. Since wave forms of $\Delta P/l$ and U_m obtained in the present experiments are well fitted by sinusoidal waves as before, the experimental uncertainties of the characteristic parameters for $n = 2, 3$ become inevitably large. However, it is possible in this case to obtain flow pattern diagrams which are very similar to those for $n = 1$.

By the way, even if the cross-sectional mean velocity changes sinusoidally like Eq.(13), $\Delta P/l$ and U_m contain terms of harmonics higher than $n = 2$ in the quasi-steady region as is obvious from Eq.(27). Consideration is extended to the quasi-steady expression for the case that U_m contain harmonics up to $n = 2$.

For the sake of simplicity, it is assumed that cross-sectional mean velocity U_m is represented as follows:

$$U_m = U_{m,ta} + |U_{m,os,1}| \cos \omega t + |U_{m,os,2}| \cos 2\omega t$$

Then, τ_w and $\Delta P/l$ can be obtained in much the same as described in chapter 3, namely

$$\tau_w = \frac{0.3164}{8} \left(\frac{\nu}{D} \right)^{1/4} \rho_0 U_{m,ta}^{7/4} \left[1 + \frac{21}{64} A_1^2 + \frac{21}{64} A_2^2 - \frac{21}{512} A_1^2 A_2 + \dots \right]$$

$$\begin{aligned} & + \left(\frac{7}{4} A_1 + \frac{21}{32} A_1 A_2 - \frac{21}{512} A_1^3 - \frac{21}{256} A_1 A_2^2 + \dots \right) \cos \omega t \\ & + \left(\frac{7}{4} A_2 + \frac{21}{64} A_1^2 - \frac{21}{512} A_2^3 + \dots \right) \cos 2\omega t + \dots \end{aligned} \quad (33)$$

$$\Delta P/l = -\rho_0 \omega |U_{m,os,1}| \sin \omega t - 2\rho_0 \omega |U_{m,os,2}| \sin 2\omega t + 4\tau_w/D \quad (34)$$

where $A_2 (= |U_{m,os,2}|/U_{m,ta} < 1)$ is the velocity amplitude ratio. These two equations converge when $(A_1 + A_2) < 1$.

As is understood from the above Eq.(33), each coefficient of the fundamental wave of τ_w and its harmonics is a function of A_1 and A_2 from the nonlinear relation between U_m and τ_w . Therefore, Eqs.(24)~(26) for fundamental wave do not hold in a strict sense. In the present experiments, however, $A_1 \gg A_2$, so that Eqs.(24)~(26) hold with sufficient accuracy.

Since the second term of coefficient of $\cos 2\omega t$ in Eq.(33) cannot be neglected in comparison with the first term, expressions for $\Phi_{t,n}$, $\Phi_{z,n}$, $\alpha_{t,n}$, and $\alpha_{z,n}$ are different from those for $n = 1$ and they are represented after considerable rearrangement as follows:

Fig.8 Flow pattern diagram for $\Phi_{t,1}$ against $\omega'/R_e \tau_a^{3/4}$

Fig.9 Flow pattern diagram for $\Phi_{z,1}$ against $\omega'/R_e \tau_a^{3/4}$

Fig.10 Flow pattern diagram for $\alpha_{t,1}$ against $\omega'/Re_{ta}^{3/4}$

Fig.14 Flow pattern diagram for $\Phi_{t,n}$ ($n = 2, 3$)

Fig.11 Flow pattern diagram for $\alpha_{z,1}$ against $\omega'/Re_{ta}^{3/4}$

Fig.15 Flow pattern diagram for $\Phi_{z,n}$ ($n = 2, 3$)

Fig.12 Values of $\Phi_{t,1}$ near the limit between intermediate and inertia dominant regions

Fig.16 Flow pattern diagram for $\alpha_{t,n}$ ($n = 2, 3$)

Fig.13 Values of $\alpha_{t,1}$ near the limit between intermediate and inertia dominant regions

Fig.17 Flow pattern diagram for $\alpha_{z,n}$ ($n = 2, 3$)

$$\left. \begin{aligned} \phi_{1,2} &= 1 / \sqrt{1 + \left\{ \frac{f(A_1, A_2)}{14.45} \frac{Re_{ta}^{3/4}}{2\omega'} \right\}^2} \\ \phi_{1,2} &= \left\{ \frac{f(A_1, A_2)}{14.45} \frac{Re_{ta}^{3/4}}{2\omega'} \right\} / \sqrt{1 + \left\{ \frac{f(A_1, A_2)}{14.45} \frac{Re_{ta}^{3/4}}{2\omega'} \right\}^2} \\ \alpha_{1,2} &= \alpha_{1,2} = \tan^{-1} \left\{ \frac{14.45}{f(A_1, A_2)} \frac{2\omega'}{Re_{ta}^{3/4}} \right\} \end{aligned} \right\} \quad (35)$$

$$f(A_1, A_2) = 1 + \frac{3}{16} \frac{A_1^2}{A_2} - \frac{3}{128} A_2^2 + \dots \quad (36)$$

That is, characteristic parameters for $n = 2$ can be obtained easily in the flow pattern diagrams by shifting the curves for $n = 1$ in the lateral direction by a factor $f(A_1, A_2)$.

The scattering of experimental values near the quasi-steady region in Figs. 14~17 is partly attributable to the fact described above. For the third harmonic, $f(A_1, A_2, A_3)$ can be obtained in a similar manner to $f(A_1, A_2)$.

From the preceding discussion, it follows that if the quasi-steady state holds for $n = 2$ in the whole region of dimensionless frequency experimental values of characteristic parameters for $n = 2$ must tend to fall within the right side of curves for $n = 1$. However, any clear tendency is not seen and the experimental values are scattered around each curve for $n = 1$. This means that for $n=2$ the relation between u_m and τ_w , $\Delta P/l$ may become linear with an increasing ω' , but nothing definite can be said because of such a scattering of experimental results.

6. Conclusions

The results obtained in this investigation can be summarized as follows:

(1) The flow pattern in a pulsating turbulent pipe flow can be described as a function of $\omega'/Re_{ta}^{3/4}$ when steady flow friction factor obeys Blasius' friction law and, in general, as a function of $\omega'/(\lambda_s Re_{ta})$.

(2) The flow patterns are classified into three types, i.e., quasi-steady, intermediate, and inertia dominant ones with respect to $\omega'/Re_{ta}^{3/4}$. If the limits between the quasi-steady and the intermediate regions and between the intermediate and the inertia dominant regions are indicated by $(\omega'/Re_{ta}^{3/4})_g$ and $(\omega'/Re_{ta}^{3/4})_t$ respectively, $\sqrt{(\omega'/Re_{ta}^{3/4})_g}$ is 0.06 when $\Phi_{t,1} = 0.05$ or 0.145 when $\Phi_{z,1} = 0.95$ and $\sqrt{(\omega'/Re_{ta}^{3/4})_t}$ is 1.50 when $\Phi_{t,1} = 0.95$. The adequate limiting value of $\sqrt{(\omega'/Re_{ta}^{3/4})_g}$ must be judged after more detailed experiments and discussion.

(3) The curves for characteristic parameters $\Phi_{t,1}$, $\Phi_{z,1}$, $\alpha_{t,1}$, and $\alpha_{z,1}$ plotted against ω' move towards the respective laminar curves with a decreasing dimensionless frequency ω' , but the process of overlapping is not clear because of the ambiguity of transition from laminar to turbulent in the pulsating pipe flow.

References

- (1) Ohmi, M., et. al., Trans. Japan Soc. Mech. Engrs. (in Japanese), Vol.46, No. 404, B (1980 - 4), p. 630.
- (2) Ohmi, M., et. al., Trans. Japan Soc. Mech. Engrs. (in Japanese), Vol.41, No. 349 (1975-9), p. 2632.
- (3) Ohmi, M., et. al., Trans. Japan Soc. Mech. Engrs. (in Japanese), Vol.46, No. 405, B (1980 - 5), p. 846.
- (4) Nunobe, M., et. al., Preprint of Japan Soc. Mech. Engrs. (in Japanese), No. 764 - 9 (1976 - 6), p. 45.