A One-dimensional Analysis and Performance
Prediction of Subsonic Radial Turbines

By Misao Hamajima

A one-dimensional analysis of subsonic radial turbines in terms of non-dimensional velocity is developed introducing a new incidence loss model. Based on this theory a performance prediction of radial turbine of a tip diameter d_3=228 mm is conducted. The predicted performance is compared with experimental results, and is considered to be of a sufficiently good coincidence. Comparison with incidence loss models, one due to Wallace and the other to NASA, is also presented.

1. Introduction

Many papers are presented already on the theoretical analysis and performance prediction for the subsonic inward flow radial turbines. Wallace(1)(2) has studied the analysis of partial load conditions and effects of design parameters on performance characteristics of inward flow radial turbines, and reported on the performance prediction using a constant-pressure incidence loss model. Putral et al(3) have shown that the incidence loss is equal to the tangential component of kinetic energy destroyed in the performance prediction of off-design point and made comparison between the theoretical calculations and experimental results. Jansen(4) has given a simplified calculation method of off-design point characteristics of inward flow radial turbines. Also, Rohlik(5) has studied the analysis of components for the best design of radial turbines. The studies by Putral, et al are published as the NASA performance prediction programs(6) for a radial-inflow turbine. The parameter which gives the most important influence on the performance prediction of a radial-inflow turbine, is the incidence loss of turbine rotor. Whitfield, et al(7) have compared and examined in detail Wallace’s incidence model and NASA’s one. Furthermore, Wallace, et al(8) have given a one-dimensional analysis and performance prediction of radial and mixed flow turbines and compared the predicted performance by their procedures with the results by NASA’s methods.

According to Wallace incidence loss model, however, it is necessary to introduce an empirical entropy gain multiplier into the static temperature ratio of inlet portion predicted by theoretical calculation, on the other hand, there are some differences from experimental results concerning the maximum efficiency point by NASA’s incidence loss model. And so, author introduced a new incidence loss model and made a theoretical one-dimensional analysis using non-dimensional parameters, and established a performance prediction procedure for the subsonic inward flow radial turbines. Based on the theoretical analysis, a performance prediction is carried out on an actual subsonic inward flow radial turbine and the results are compared with experimental results indicating a good conformity. Further, comparison is made between the above-mentioned works and the former incidence loss models.

2. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>c_0</td>
<td>Critical speed in absolute flow m/s</td>
</tr>
<tr>
<td>c_0^*</td>
<td>Critical speed in relative flow m/s</td>
</tr>
<tr>
<td>c_p</td>
<td>Specific heat at constant pressure J/kgK</td>
</tr>
<tr>
<td>d</td>
<td>Diameter m</td>
</tr>
<tr>
<td>F</td>
<td>Area m^2</td>
</tr>
<tr>
<td>h_3</td>
<td>Blade height at rotor inlet m</td>
</tr>
<tr>
<td>h</td>
<td>Fluid loss J/kg</td>
</tr>
<tr>
<td>m</td>
<td>Constant defined by Eq. (3)</td>
</tr>
<tr>
<td>h</td>
<td>Mass flow rate kg/s</td>
</tr>
<tr>
<td>W</td>
<td>Rotational speed rpm</td>
</tr>
<tr>
<td>F_o</td>
<td>Total pressure in absolute flow N/m^2</td>
</tr>
<tr>
<td>F_Ro</td>
<td>Total pressure in relative flow N/m^2</td>
</tr>
<tr>
<td>q_{(r)}</td>
<td>Non-dimensional mass velocity defined by Eq. (2)</td>
</tr>
<tr>
<td>R</td>
<td>Gas constant J/kgK</td>
</tr>
<tr>
<td>T_o</td>
<td>Total temperature in absolute flow K</td>
</tr>
<tr>
<td>T_Ro</td>
<td>Total temperature in relative flow K</td>
</tr>
<tr>
<td>U</td>
<td>Circumferential velocity m/s</td>
</tr>
<tr>
<td>V</td>
<td>Absolute velocity m/s</td>
</tr>
<tr>
<td>W</td>
<td>Relative velocity m/s</td>
</tr>
<tr>
<td>a</td>
<td>Absolute flow angle</td>
</tr>
<tr>
<td>b</td>
<td>Relative flow angle</td>
</tr>
<tr>
<td>b_s</td>
<td>Blade angle</td>
</tr>
<tr>
<td>d = d_4/d_3</td>
<td>Diameter ratio</td>
</tr>
</tbody>
</table>

* Received 30th October, 1978
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Tatsuno, Pref. Nagano.
\( \zeta \) Loss coefficient
\( \eta \) Efficiency
\( \theta = \frac{T_{in}}{T_{03}} \) Temperature ratio
\( \vartheta = \frac{T_{in4}}{T_{03}} \) Temperature ratio
\( \varepsilon \) Ratio of specific heats
\( \lambda = \frac{U}{u_0} \) Nondimensional relative velocity
\( \lambda = \frac{V}{u_0} \) Nondimensional circumferential velocity
\( \lambda = \frac{V}{u_0} \) Nondimensional absolute velocity
\( \mu \) Slip factor
\( \nu \) Blade-jet speed ratio
\( \psi \) Velocity coefficient of nozzle
\( \phi \) Velocity coefficient of rotor

Subscripts
0 Turbine inlet
1 Nozzle inlet
2 Nozzle exit
3 Just before flow in rotor
3' After slip (cf. incidence model)
4 After flow in rotor
5 Rotor exit
6 Diffuser exit
7 U Diffuser exit
8 Relative system
U Relative system
d Diffuser
e Rotor exit
s Turbine inlet
sh Incidence
vl Vaneless space

3. Theoretical analysis

A radial turbine consists of scroll, nozzle (stator), vaneless space, rotor, and diffuser as shown in Fig. 1 illustrating subscript numbers at each location.

![Fig. 1 Turbine components and location](image)

In the analysis author assumes that the working fluid is a perfect gas, that the turbine is perfectly isolated with no heat transfer from the surroundings, and further that there is no leak of working fluid through the turbine stage. Based on these assumptions, the condition of continuity is satisfied, and the losses resulting from fluid flow are transformed into heat preserved in working fluid itself, so that the total temperature is kept constant between turbine inlet and rotor inlet as well as between rotor exit and diffuser exit. Further it is assumed that the fluid flows through turbine in steady state and without separation, and that the down flow from the blade passages changes into uniform state within a short
distance. The working process is shown diagrammatically by a temperature-entropy (Ts) diagram in Fig. 2.

To clarify the meaning of nondimensional velocities \( \lambda_\nu \), \( \lambda \nu \), the subscripts of corresponding velocities are used as the subscripts; for example, \( \lambda_\nu \) correspond to absolute velocity \( V_\nu \), and relative velocity \( W_\nu \) respectively.

3.1 Condition of continuity

Provided that the cross-sectional area perpendicular to the velocity \( V \) at a point of flow passage is given by \( F \), and \( T_0, P_0 \) represent respectively total temperature and total pressure of the fluid at that point, the mass flow can be given by the following equation:

\[
\dot{m} = \frac{\sqrt{\gamma}}{\gamma} \frac{A}{T_0} \quad (1)
\]

where

\[
\psi(\lambda_\nu) = \lambda_\nu \left[ 1 - \frac{1}{(\kappa + 1)\frac{\gamma + 1}{2\kappa}} \right] (2)
\]

\[
\psi(\lambda v) = \frac{1}{(\kappa + 1)\frac{\gamma + 1}{2\kappa}} (3)
\]

\( \psi(\lambda) \) is a function of \( \nu \) and \( \nu \nu \) given in the function table of literatures such as (9), (10). \( \alpha \) is flow coefficient.

3.2 Nozzle

The loss in nozzle is given by the velocity coefficient \( \psi \). If \( \nu \) is known, the pressure ratio between nozzle inlet and exit can be calculated from the following equation:

\[
\frac{P_{in}}{P_{out}} = \left[ 1 - \frac{1}{(\kappa + 1)/2\kappa} \right] (4)
\]

Assuming that \( T_0 = T_{in} \), from condition of continuity

\[
\frac{P_{in}}{P_{out}} = \frac{a_1}{a_2} \quad (5)
\]

Then, if \( \psi(\lambda v) \) is known at nozzle inlet, \( \psi(\lambda v) \) can be determined. In actual calculation, \( \nu \) is obtained by iteration so as to satisfy Eq. (4) and Eq. (5).

3.3 Vaneless space

The flow between nozzle exit and rotor inlet is considered to be a free vortex flow of a compressible fluid subjected to the resistance of skin friction. It can be solved in a similar way to a vaneless diffuser of centrifugal compressor. Assum-
ing the fluid loss of this portion is to be represented by the following equation

$$h_a - h_{a'} = \frac{\epsilon}{2} \frac{k}{2}  \quad (6)$$

the total pressure ratio is given by

$$P_{a} = \left( 1 - \frac{e - 1}{e + 1} \frac{k}{2} \right)^{1/(e-1)} \quad (7)$$

Therefore,

$$\eta = \frac{\eta_a}{\eta_{a'}} = \frac{P_{a} a_{a}}{P_{a} a_{a'}} = \eta_{a} \quad (8)$$

$F_3$ is the cross-sectional area perpendicular to the flow immediately upstream of the rotor, and therefore if the representative area is given by $F_a = \frac{1}{2} \frac{k}{2} \frac{k}{2}$, the value of $F_3$ can be calculated by

$$F_3 = \frac{4}{k} \frac{k}{2} \frac{k}{2} \cos \alpha \quad (9)$$

The absolute flow angle $\alpha$, i.e. the angle between the absolute flow and the inward radial direction, has to be determined to satisfy the condition of continuity given by Eq. (8) and the momentum equation Eq. (10), which is derived from that the time differential of angular momentum is equal to the moment of fluid friction force, simultaneously.

$$\sin \alpha = \frac{\sin \alpha}{\sin \alpha} = \frac{k}{2}(1 - \frac{e - 1}{e + 1}) \quad (10)$$

3. 4 Incidence loss model of rotor inlet

The incidence loss of rotor has a heavy influence on performance characteristics. The flow from vaneless stationary interspace to the inlet of rotating passage restricted by the turbine blades, however, is extremely complicated. Therefore, in performance prediction based on one-dimensional theory, it is routine to estimate the incidence loss by modeling the flow around the inlet portion of rotor.

A number of papers have been published on the slip in the centrifugal compressor, whereas a study of slip in the inward flow radial turbines, which have a reversed relative flow direction, cannot be found. Kohlik (9)(11) however, states that the slip in the radial turbines follows the equation of Stantitz (12).

The mass flow giving the maximum efficiency in radial turbines varies depending on rotational speed, and in most cases of prediction, the performance characteristics at the neighborhood of the maximum efficiency point for every rotational speed is required. For this reason the maximum efficiency point must be estimated as correctly as possible. From this point of view, the author introduces a new one-dimensional flow model around rotor inlet portion mentioned below. That is, as shown in velocity diagram, Fig. 3, the fluid which arrives at the portion just before the rotor inlet with absolute velocity $V_a$, flows into rotor with relative velocity $V_a$, suffering a slip at the periphery of turbine rotor. If there is a difference between the blade inlet angle $\beta_a$ and the direction of $V_a$, an incidence loss occurs. When $\beta_a = 0$, assume that the slip factor $\mu$ is the same as that at the design point of the centrifugal compressor with radial outward blades, and that the amount of slip $(1-\mu)\nu_a$ is invariable irrespective of mass flow so far as rotational speed is equal.

The slip changes the energy transfer through turbine rotor, and the occurrence of slip is assumed to be isentropic, and the incidence loss to flow against rotor is equal to $h_{a} = (\frac{1}{2} \frac{k}{2} \frac{k}{2}) \mu T_{a}$. The process on $\eta$-diagram is shown in Fig. 4.

**Fig. 4** $\eta$-diagram for rotor inlet

The total pressure after the occurrence of slip have the same value as $T_{03}$, $P_{03}$, respectively, but the total temperature on relative system becomes $T_{03}$, and the total pressure does $P_{03}$. After the occurrence of an incidence loss, the relative total temperature is $T_{03}$, but the relative total pressure decreases from $P_{03}$ to $P_{03}$. The fluid, which arrives at the rotor with the conditions indicated by the point $03$ on $\eta$-diagram, flows into the rotor with the conditions indicated by the point $03$ on relative system and flows through the passages between turbine blades.

Since $T_{03} = 2 U_{a} V_{a} \sin \alpha (1 - 2 \mu) / U_{a}^2$

$$\eta_{03}/\eta_{03}$$ can be found as follows:

$$T_{03} = 1 - \frac{e - 1}{e + 1} \frac{k}{2} \frac{k}{2} \cos \alpha (1 - 2 \mu) / U_{a}^2 \quad (11)$$

Therefore

$$\eta_{03} = (\frac{T_{03}}{T_{03}})^{(e+1)/(e-1)} \quad (12)$$

And, since $EM = (2 \mu) \sin \alpha(1 - 2 \mu) / U_{a}^2$, the incidence loss can be written as follows:

$$h_{a} = \frac{\cos \alpha}{\cos \alpha} = \frac{1}{2} \frac{k}{2} \frac{k}{2} \frac{k}{2} (2 \mu) / U_{a}^2 \quad (13)$$

Accordingly

$$\frac{T_{03}}{T_{03}} = 1 - \frac{e - 1}{e + 1} \frac{k}{2} \frac{k}{2} \cos \alpha (1 - 2 \mu) / U_{a}^2 \quad (14)$$

Thus

$$\frac{P_{03}}{P_{03}} = \eta_{03} \frac{P_{03}}{P_{03}} = \left[ 1 - \frac{e - 1}{e + 1} \frac{k}{2} \frac{k}{2} \cos \alpha (1 - 2 \mu) / U_{a}^2 \right]^{-(e+1)/(e-1)} \quad (15)$$

And
Fig. 5 Flow in turbine rotor duct

\[
\frac{T_{\text{in}}}{T_{c}} = \frac{T_{\text{in}}}{T_{a}} \quad (16)
\]

Therefore

\[
q(\lambda_{av}) = q(\lambda_{av}) + \frac{T_{\text{in}}}{T_{a}} \ln \frac{P_{a}}{P_{\text{in}}} 
\] 
\[
\text{The conditions of fluid at the portion of rotor inlet can be found from Eq. (17).}
\]

3.5 The flow through rotor passages

Figure 5 shows the configuration of a rotor passage and the velocity diagram at the passage inlet and exit. The relative total temperature decreases from \( T_{03} \) to \( T_{04} \) in rotor passage. And the temperature ratio \( \theta_S \), which means the ratio of \( T_{04} \) to \( T_{03} \), is represented by the following equation (confer appendix 1):  
\[
\theta_S = 1 - \frac{2(1-1)(x+1)2x2x \sin \theta_S + \frac{x-1}{x+1} \sin \theta_S}{x-1} 
\] 
\[
\text{Using } \theta_S, \text{ the ratio of relative total temperature of rotor passage inlet to exit can be written as}
\]
\[
\frac{T_{\text{in}}}{T_{\text{ex}}} = \frac{T_{\text{in}}}{T_{a}} \cdot \theta_S 
\]

Also, from velocity coefficient of the rotor, 
\[
P_{\text{m}} = \frac{1 - \left[(x+1)(x+1)x \sin \theta_S \right]}{x+1} 
\] 
\[
\text{can be obtained, so}
\]
\[
P_{\text{m}} = \frac{P_{\text{ex}} - P_{\text{in}}}{P_{\text{in}} - P_{\text{a}}} 
\]

Therefore

\[
q(\lambda_{av}) = q(\lambda_{av}) + \frac{T_{\text{in}}}{T_{a}} \ln \frac{P_{a}}{P_{\text{in}}} 
\] 
\[
\text{Eq. (19) -- (21), } \lambda_{av}, \text{ can be calculated by iteration.}
\]

3.6 Rotor exit

When \( \lambda_{av} \) is calculated, \( \theta = T_{03}/T_{04} \) can be found from the following equation (confer appendix 2),

\[
\theta = \theta_S + \frac{x-1}{x+1} \left(2x \theta_S \sin \theta_S + \frac{x-1}{x+1} \sin \theta_S \right) 
\] 
\[
\text{From } \theta \text{ and } \theta_S
\]
\[
\frac{T_{\text{ex}}}{T_{a}} = \theta_S 
\]

Also

\[
P_{\text{m}} = \frac{T_{\text{m}}}{T_{\text{a}}} \left(\frac{T_{\text{m}}}{T_{\text{a}}} \right) 
\]

Next, the loss at rotor exit is considered to be equal to the kinetic energy of circumferential component of absolute exit velocity, therefore

\[
\frac{h_{\text{a}}}{a_{\text{n}}^2} = \left(\frac{\theta_S \sin \beta + \frac{2x2x \sin \theta_S + \frac{x-1}{x+1} \sin \theta_S}{\theta_S} \right)^2 
\]
\[
\text{And so}
\]
\[
P_{\text{m}} = \frac{1 - \frac{x-1}{x+1} \left(T_{\text{m}} \sin \beta + \frac{2x2x \sin \theta_S + \frac{x-1}{x+1} \sin \theta_S}{\theta_S} \right)}{x+1} 
\]
\[
\text{From Eqs. (24) and (26)}
\]
\[
\frac{P_{\text{m}}}{P_{\text{a}}} = \frac{1 - \frac{x-1}{x+1} \left(T_{\text{m}} \sin \beta + \frac{2x2x \sin \theta_S + \frac{x-1}{x+1} \sin \theta_S}{\theta_S} \right)}{x+1} 
\]

3.7 From turbine inlet to nozzle inlet, and diffuser

If the fluid losses in the above-mentioned stationary ducts are defined by the following expressions:

\[
h_{\text{a}}/a_{\text{n}}^2 = \frac{\lambda_{av}}{2} 
\]

\[
h_{\text{a}}/a_{\text{n}}^2 = \frac{\lambda_{av}}{2} 
\]

respectively, \( P_{\text{in}}/P_{\text{a}}, \) \( P_{\text{in}}/P_{\text{a}} \), and \( q(\lambda_{av}), \) \( q(\lambda_{av}) \) can be found without difficulty.

3.8 Performance

All the necessary performance characteristics of inward flow radial turbines can be found based on above analysis. For example,

\[
\text{Pressure ratio: } \frac{P_{\text{m}}}{P_{\text{a}}} \quad \text{Eq. (28)} 
\]

\[
\text{Total-turbine efficiency: } \eta_t = \frac{1}{1 - \left(\frac{P_{\text{in}}}{P_{\text{a}}} \right) \left(\frac{P_{\text{in}}}{P_{\text{a}}} \right)} \quad \text{Eq. (29)} 
\]

\[
\text{Velocity equivalent to isentropic enthalpy drop (non-dimensional):} \quad \lambda_{av} = \frac{2x2x \sin \theta_S + \frac{x-1}{x+1} \sin \theta_S}{\theta_S} 
\]

\[
\text{Velocity ratio: } \nu = \frac{\lambda_{av}}{\lambda_{04}} 
\]

Since the non-dimensional velocities are known, the properties of static condition of fluid at any location as well as

<table>
<thead>
<tr>
<th>Table 1. Specification of calculation turbine</th>
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<tbody>
<tr>
<td><strong>Turbine dimensions</strong></td>
</tr>
<tr>
<td>Rotor tip diameter</td>
</tr>
<tr>
<td>Blade height/tip dia</td>
</tr>
<tr>
<td>RMS exit dia/tip dia</td>
</tr>
<tr>
<td>Nozzle exit dia/tip dia</td>
</tr>
<tr>
<td>Rotor blade exit angle</td>
</tr>
<tr>
<td>Nozzle blade inlet angle</td>
</tr>
<tr>
<td>Nozzle blade exit angle</td>
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<table>
<thead>
<tr>
<th>Table 2. Area ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Area ratio</strong></td>
</tr>
<tr>
<td>( F_0/F_2 )</td>
</tr>
<tr>
<td>( F_1/F_2 )</td>
</tr>
<tr>
<td>( F_3/F_2 )</td>
</tr>
<tr>
<td>( F_4/F_2 )</td>
</tr>
<tr>
<td>( F_5/F_2 )</td>
</tr>
<tr>
<td>( F_6/F_2 )</td>
</tr>
<tr>
<td>( F_8/F_2 )</td>
</tr>
</tbody>
</table>

Note: \( F_0 = \frac{3}{4} d_3^3 \)
the characteristics of turbines based on static condition can also be determined.

4. Performance prediction and comparison with experimental results

For the purpose of comparing the theoretical analysis mentioned above with the results of experiments of actual inward flow radial turbine, the performance of a radial turbine with rotor tip diameter \( d_3 = 228 \text{ mm} \), is calculated on the basis of theoretical analysis. The specifications, installations, test methods, and test results of the experimental turbine are already reported (13), (14). The theory of this paper is intended to analyze the problem applying the non-dimensional parameters, and so the calculation is conducted by using only the data given in Tables 1 and 2, and by using the coefficients indicated in Table 3. The flow coefficients

<table>
<thead>
<tr>
<th>Assumed coefficients</th>
<th>( \varphi )</th>
<th>0.97</th>
</tr>
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<tbody>
<tr>
<td>( \mu )</td>
<td>0.88</td>
<td></td>
</tr>
<tr>
<td>( \varphi' )</td>
<td>0.8947</td>
<td></td>
</tr>
<tr>
<td>( \varphi'' )</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>( \varphi''' )</td>
<td>0.115</td>
<td></td>
</tr>
<tr>
<td>( \gamma' )</td>
<td>0.40</td>
<td></td>
</tr>
</tbody>
</table>

may differ for each sectional area, but the calculation is carried out by assuming that the ratio of flow coefficients of two sectional areas is approximately equal to unity. Assume a non-dimensional flow rate \( m_{\text{in}} \sqrt{T_{\text{in}}} \), and a non-dimensional rotational speed \( \omega_{\text{in}} = N \sqrt{T_{\text{in}}} \), \( q(\lambda_{\text{in}}) \) can be determined from area ratio, and from

\[
\lambda_{\text{in}} = \frac{m_{\text{in}} \sqrt{T_{\text{in}}}}{\rho_0 \sqrt{T_{\text{in}}}} \frac{1}{\sqrt{\frac{2\pi G}{\gamma'}}} \frac{N}{\sqrt{T_{\text{in}}}}
\]

(33)

\( \lambda_{\text{in}} \) can be calculated, so the calculation can be done successively. To make easy the comparison of the theoretical results with the experimental ones, the calculation is conducted for each non-dimensional rotational speed \( \lambda_{\text{in}} = 500, 760, 1005, \) and 1258 respectively.

The results of calculation are shown in Fig. 6~9. The results of theoretical calculation show a sufficiently good coincidence with the experimental results for both non-dimensional flow rates and total-total efficiencies. Because the number of experimental points are somewhat small, the blade-jet speed ratio giving the maximum efficiency point is not obtained accurately, but it is clearly illustrated that the experimental results agree well with the calculation, showing the appropriateness of above theoretical analysis. That is, the blade-jet speed ratio giving the maximum efficiency is equal to 0.70~0.74 at \( N = 1005 \), being smaller for the lower rotational speed, and vice versa, the maximum efficiency for each rotational speed becomes maximum in \( N = 1005 \), being smaller for higher and lower rotational speeds than this range. The flow rate of choke which is shown by the point where the non-dimensional flow rate curve becomes horizontal on the figure, is nearly coincident with the results of experiment.

5. Consideration

The theoretical analysis of subsonic inward flow radial turbines by non-dimen-
in the velocity coefficient.

Hitherto, there are two incidence loss models, one being a constant pressure incidence loss model proposed and developed by Wallace and his group, and the other, NASA's which assumes that the loss is given by the tangential component of relative kinetic energy destroyed. Fig. 10 shows a comparison of the incidence losses, which is originally carried out by Wallace, et al. (8) on the turbine rotor $d_2=104$ mm showing the predicted entropy gains due to incidence losses. Author's data from the performance prediction above-mentioned are also given in Fig. 10. Wallace introduces a multiplier $K$ and raises the static temperature ratio calculated from his theory to the power $K$, so as to make coincidence with experimental results. As this figure shows, the value of multiplier $K$ significantly affects the amount of incidence loss, but it is not reported how to decide the value of $K$ for performance prediction. When $K=1.0$, there is very little difference between Wallace's and NASA's (Futral and Wasserbauer) methods. The incidence losses predicted based on author's theory come approximately between the curve $K=1.0$ and $K=0.1$, and the losses are different for the same value of blade-jet speed ratio depending on $\theta$.

The optimum incidence condition, which means the incidence loss is the smallest, is one according to NASA incidence loss model as follows:

$$\theta_{\text{opt}} \sin \alpha = \frac{\sqrt{\rho}}{\sqrt{M}}$$

It is known from Eq. (13) in the case of author's model, the optimum incidence condition is:

$$\theta_{\text{opt}} \sin \alpha = 2 - \rho$$

Accordingly, the optimum incidence condition is $\beta_0<0$ by NASA's model, and $\beta_0>0$ by author's. As the slip of rotor occurs in the direction of the counter rotational speed, the relative flow angle must be $\beta_0>0$ in a one-dimensional flow model, if, after the occurrence of slip, the fluid flows into the radial bladed rotor without incidence angle. It is known from Fig. 10 that the blade-jet speed ratio of the smallest incidence loss by author's theory is smaller than the others. This means that the flow rate of the maximum efficiency may become larger.
6. Conclusions

A one-dimensional theoretical analysis of subsonic radial turbines in terms of non-dimensional velocity is developed. It is shown that the performance prediction by non-dimensional parameters is possible, and on the basis of this theory a performance prediction of radial turbine of a tip diameter 228 mm is conducted for four rotational speeds. The predicted performance is compared with experimental results and a sufficiently good coincidence is obtained. Author introduced a new incidence loss model in theoretical analysis and, owing to this model, the effects of rotational speed on characteristics can be included in performance prediction. Comparison of the author's incidence loss model with Wallace's and NASA's is also presented.

Appendix

[Appendix 1]

Total enthalpy drop in the turbine rotor is as follows:

\[ c_s T_4 - c_s T_3 = \frac{1}{2} (F_1 - V_1) + (U'_1 - U_1) + (W'_1 - W_1) \]

therefore

\[ c_s T_3 = c_s T_4 - U'_1 V_1 \sin \alpha + \frac{1}{2} U'_1^2 \]

from Fig. 3

\[ V_1 \sin \alpha_1 - V_4 \sin \alpha_4 = (1 - \beta) U_4 \]

consequently

\[ c_s T_4 = c_s T_3 + V'_1 \sin \alpha_4 + \left[ 1 - \beta + \frac{1}{2} \beta^2 \right] U'_4 \]

Equation (18) will be found by rewriting this equation in non-dimensional form using \( c_{s, s} \).

[Appendix 2]

From Fig. 5

\[ V'_1 = W'_1 + U'_1 + 2W'_1 U'_1 \sin \beta_1 \quad (\beta_1 < 0) \]

therefore

\[ T_4 = T_3 + \frac{1}{2} \beta_1 (W'_1 + U'_1 + 2W'_1 U'_1 \sin \beta_1) \]

then

\[ \frac{T_4}{\beta_1} = \frac{T_3}{\beta_1} \left( W'_1 + U'_1 + 2W'_1 U'_1 \sin \beta_4 \right) \]

Using \( k = \frac{W_{s, s}}{a_{s, s}} \times \sin \beta_4 \sqrt{\beta_4} \), above equation can be rewritten in non-dimensional form as follows:

\[ \frac{\beta_4}{T_4} = \frac{1}{T_4} + \frac{1}{T_4} \left( \frac{2 \beta}{\sqrt{\beta_4}} \frac{W_{s, s}}{a_{s, s}} \sin \beta_4 + \frac{1}{\beta_4} \frac{W_{s, s}}{a_{s, s}} \right) \]

Now using

\[ \frac{T_4}{T_3} = \frac{T_4}{T_3} \quad \frac{T_4}{T_3} = \frac{T_4}{T_3} \quad \frac{T_4}{T_3} = \frac{T_4}{T_3} \]

Eq. (22) can be found.

References