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Basic Characteristics of Cylindrical, Electrostatic Probes for the Measurements of Fluctuating Premixed Flames*

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An experimental study has been conducted to explore the effects of the potential, size, and velocity of a cylindrical, electrostatic probe on the ion current fluctuation recorded when the probe traverses a premixed, two-dimensional nozzle burner flame parallel to the probe axis.

The maximum ion current was found to be proportional to the probe length L_p and to increase with increasing probe diameter d_p , probe velocity V_p , or absolute value $|\psi_p|$ of the probe potential ψ_p . In the ranges of L_p from 1 to 14 mm and V_p from 1 to 22 m/s, the estimated half-value thickness δ_h of the high-ion-density region near the flame decreased and approached about 0.4 mm as $|\psi_p|$ and d_p were decreased. When the probe was operated under practical conditions, e.g., $|\psi_p| \lesssim 40$ V and $d_p \lesssim 0.4$ mm, the value of δ_h would be between 0.4 and 0.8 mm.

1. Introduction

It has been well known that the high ion density due to chemiionization is confined in a narrow region near the reaction zone of a hydrocarbon flame^{(1)~(4)}. Based on this fact, an electrostatic probe or ionization gap has been frequently used for examining the movement of a propagating or turbulent flame^{(5)~(16)}. The ion current fluctuation recorded when a flame passes the electrostatic probe obviously depends on the ion density variation near the probe, which is closely related to the flame velocity, flame configuration, chemical reaction (fuel properties), etc. If the relations between the ion current fluctuation and such characteristics of the flame passing the probe were revealed, the behavior or structure of a fluctuating flame would be elucidated to a greater extent by analyzing the ion current fluctuation measured by using an electrostatic probe. Up to this time, a number of studies have been conducted for exploring such relations as that between the combustion phenomena in a combustion chamber of an internal combustion engine and the ion current fluctuation measured by using an ion gap^{(17)~(19)}, and that between the aspects of a flame propagating in a tube and one measured by using

an electrostatic probe^{(13)~(14)}. Although many valuable facts have been made clear in these studies, most relations between the aspects of fluctuating flames and the ion current fluctuations have remained obscure.

Since the electrostatic probe is of excellent response and spatial resolution, it is very useful for examining the flame front movements of fluctuating flames. However, knowledge of its basic characteristics seems to be insufficient for making best use of its function in the measurements of fluctuating flames.

The ion current to an electrostatic probe immersed in a gas of uniform ion density depends not only on the ion density, but also on the probe potential, probe size, gas velocity, etc.^{(20)~(29)}, so that the fluctuation of the ion current to an electrostatic probe passing through a flame can be expected to depend on such quantities. Furthermore, effects of the ion density distribution on the ion current fluctuation must be taken into account because the ion density changes greatly near a flame.

In the present study, in order to obtain basic knowledge for the measurements of fluctuating premixed flames by using a cylindrical, electrostatic probe, the relations between the ion current fluctuation obtained when a cylindrical, electrostatic probe passed a premixed flame parallel to the probe axis and the probe potential, probe size, and flame velocity with respect to the probe were examined in detail. Based on the results, the basic characteristics of an electrostatic probe for the measurements of fluctuating flames such as propagating and turbulent flames were discussed.

When a propagating or fluctuating flame is measured by using a stationary, electrostatic probe, it is difficult to control the velocity and configuration of the flame traversing the probe. However,

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if a probe was made to move across a stable, stationary flame, not only a flat flame would be obtained easily, but also the flame velocity with respect to the probe could be changed arbitrarily^{(14) (21) (30)}. Therefore, in the present experiments, probes were made to move across a premixed propane-air two-dimensional nozzle-burner flame, and the relative velocities between the probes and the flame were controlled.

2. Experimental apparatus and procedure

A system for supplying propane and air to a rectangular nozzle burner is shown in Fig. 1. Air was supplied from a reciprocating compressor through a purifier, a pressure regulator, a control valve, and an orifice to the burner. Propane(99.8% pure) was fed from a cylinder through a pressure regulator, a control valve, and an orifice to the air stream. In order to obtain a low turbulence stream of the mixture at the burner port, the burner consisted of a diffuser, a settling chamber and a nozzle, and seven screens(100 mesh) were placed in the diffuser and settling chamber. The velocity of the stream at the burner port, the cross section of which was rectangular of 8 mm x 50 mm, was uniform in the region except for the vicinities of the burner walls. Stable, two-dimensional premixed

Fig. 1 Schematic diagram of supply system.

Fig. 2 Experimental apparatus

flames could be established on the burner port. In the present experiments, a stoichiometric mixture was supplied to the burner, and the mixture velocity at the burner port was kept at 1.4 m/s.

Figure 2 shows the structure of the cylindrical electrostatic probe system, the rotating system for moving probes, and the electric circuit for detecting ion currents. The probe wires were of platinum, which are usually used in flames or combustion gases because platinum is electrically conductive and hardly oxidized in high temperature gases. Except for the detecting part, the wire was covered with a finely drawn quartz tube projecting from a porcelain tube. Further, the porcelain tube was settled in a stainless steel tube to increase the stiffness of the system. The diameters of the probe wires used in the present experiments were 0.05, 0.07, 0.1, 0.15, 0.2, 0.3, 0.4, 0.5, 0.7, and 1.0 mm, and the probe length(the length of the exposed detecting part of the probe wire) was varied from 1 to 14 mm.

The probe was fixed on a rotating circular disk so that the probe axis would be parallel to the flame and pass it at 6 mm from the burner port. The probe velocity was varied by changing the rotation frequency of the disk. The rotation frequency was measured by using a digital-counter connected to a frequency detecting system which consisted of a photoelectric cell and a rotating disk with rectangular cuts. When the probe velocity was very high, the flame would be so distorted at its passing that reliable results could not be expected. On the other hand, when the probe velocity was extremely low, the probe would be so heated that the insulation degradation of the quartz tube, the oxidization of the fine probe wire, the thermal emission of electrons from the probe surface, etc. would disturb the ion current fluctuation. Accordingly, the present experiments were performed at the probe velocities from 1 to 22 m/s. The temperature of the probe at the probe velocity of 1 m/s was confirmed to be kept below 300 °C during its passage across the flame, so that the effect of the thermal emission of electrons could be neglected^{(23) (31)}. Furthermore, the quartz tube covering the probe wire could remain as an insulator.

The ion current to the probe was collected through mercury in a vessel and recorded by an oscilloscope. Although the ion current changed rapidly, its fluctuation could be recorded accurately, because the time constant (about 10^{-6} s) of the recording system of the ion current was small enough compared to the period(longer than 2×10^{-4} s) during which the probe passed the high-ion-density region. In order to prevent the electric noise sealed wires were used for the electric circuit, and the motor and support of the rotating circular disk were sealed with screens. A synchronizing system was used such that the oscilloscope would start sweeping just before the probe passed the flame. In the present experiments, the probe potential with

respect to the burner port, which was used as the reference electrode, was varied from -1 V to -120 V.

3. Experimental results and discussions

3.1 Ion-current fluctuations

A typical record of ion current fluctuation when a cylindrical, electrostatic probe (the detecting part) passes the flame parallel to the probe axis from the burned gas side to the unburned gas side is shown in Fig. 3. When the probe passes the flame, the ion current to it increases very rapidly to a sharp maximum and then decreases. The rate of the ion-current change at the unburned gas side is found to be larger than that at the burned gas side.

To qualitatively express the variations of the ion-current fluctuation, its maximum j_m and half-value period t_h defined in Fig. 3 were adopted as representative quantities for describing it, and were examined in detail by changing such variables as the probe potential ψ_p , probe length L_p , probe

Fig. 3 A typical ion-current fluctuation and definitions of maximum ion current j_m and half-value period t_h . $L_p = 2$ mm, $d_p = 0.1$ mm, $\psi_p = -10$ V, $V_p = 5$ m/s, Sweep: 0.5 ms/div, Gain: 0.98×10^{-5} A/div.

Fig. 4 Variation of maximum ion current with absolute value of probe potential. $L_p = 2$ mm, $V_p = 5$ m/s.

diameter d_p , and probe velocity V_p .

If it could be assumed that the ion current to the probe depended only on the ion density at the location where the probe axis was passing, the ion current fluctuation when the probe traversed the flame from the unburned gas side would be exactly the same as the inverse (with respect to the time) of that when it traversed the flame in opposite direction under the same conditions. By comparing the ion current fluctuations recorded in the present experiments for both cases, it was found that the former was somewhat different from the latter. At the same time, however, it could be confirmed that under the present experimental conditions the difference between them was not so large, i.e., the relations between the ion current fluctuation and the potential, size, and velocity of the probe were very similar in both cases.

Thus, in this paper, the results on the ion current fluctuations only when the probe passed the flame from the burned gas side will be described in detail. The difference in the ion-current fluctuations due to the difference in the traversing directions of probe will be discussed in detail and the results will be presented elsewhere.

The variation of j_m with ψ_p is shown in Fig. 4. In the present study, the absolute value $|\psi_p|$ ($= -\psi_p$) was used for convenience because the value of ψ_p was kept negative throughout the experiments. j_m increases with $|\psi_p|$. For $|\psi_p| \geq 2$ V, the relation between j_m and $|\psi_p|$ can be expressed as

$$j_m \propto |\psi_p|^{0.65 \pm 0.1} \dots \dots \dots (1)$$

This relation is similar to those obtained from the ion-current measurements in uniform ion-density gas flows (21)~(29). For $1 \text{ V} \leq |\psi_p| \leq 20 \text{ V}$, t_h is found to be almost independent of ψ_p , while for $\psi_p \geq 20 \text{ V}$, it increases with ψ_p (Fig. 5). These phenomena are considered to be closely related to the aspects of the ion-sheath-thickness variation surrounding the probe with $|\psi_p|$, as will be discussed in the next section.

It is seen that j_m is almost proportional to L_p (Fig. 6) and t_h is independent

Fig. 5 Variation of half-value period with absolute value of probe potential. $L_p = 2$ mm, $V_p = 5$ m/s.

Fig. 6 Variation of maximum ion current with probe length.
 $\psi_p = -10$ V, $V_p = 5$ m/s.

Fig. 7 Variation of half-value period with probe length.
 $\psi_p = -10$ V, $V_p = 5$ m/s.

Fig. 8 Variation of maximum ion current with probe diameter.
 $L_p = 2$ mm, $V_p = 5$ m/s.

of L_p (Fig. 7). These results indicate that the phenomena near the probe such as the ion diffusion and ion-sheath formation, which are related to the ion current, can be assumed to be two-dimensional, i.e., to be the same at any section perpendicular to the probe axis.

As d_p is increased, j_m increases (Fig. 8). When d_p is smaller than 0.1 mm, the increasing rate of j_m with d_p is small. In the range of d_p from 0.1 to 0.2 mm the rate becomes large, and then it decreases gradually with increasing d_p . t_h also increases with d_p (Fig. 9). The increasing rate of t_h with d_p is very small for $d_p \lesssim 0.15$ mm, and then it becomes large with increasing d_p . These phenomena may be understood by considering the change of the structure surrounding the probe with d_p .

The ion current fluctuation is largely affected by V_p . j_m increases with V_p (Fig. 10), while t_h is inversely proportional to V_p (Fig. 11). The relation between j_m and V_p is approximately expressed as

$$j_m \propto V_p^{0.45 \pm 0.05} \dots \dots \dots (2)$$

Since V_p is not exactly equal to the gas velocity with respect to the probe in the

Fig. 9 Variation of half-value period with probe diameter.
 $L_p = 2$ mm, $V_p = 5$ m/s.

Fig. 10 Variation of maximum ion current with probe velocity.
 $L_p = 2$ mm, $\psi_p = -10$ V.

high-ion-density region, this relation must be different from that between the gas velocity and the ion current to a cylindrical, electrostatic probe immersed in a gas of uniform-ion-density. Nevertheless, it seems to indicate the ion current increase with the gas velocity with respect to the probe.

The product of t_h and V_p can be assumed to represent the half-value thickness δ_h of the high-ion-density region. If the values of ϕ_p and d_p are the same, the value of δ_h obtained in the present experiments agrees well with that obtained by moving an electrostatic probe in a quasi-steady manner⁽³⁾. The results shown in Figs. 5, 7, 9, and 11 indicate that in the ranges of L_p from 1 to 14 mm and V_p from 1 to 22 m/s, δ_h decreases and approaches about 0.4 mm as $|\phi_p|$ and d_p decreases. This limiting value (0.4 mm) of δ_h is considered to be intrinsic.

Fig. 11 Variation of half-value period with probe velocity.
 $L_p = 2$ mm, $\phi_p = -10$ V.

Fig. 12 Model for discussing ion current fluctuation recorded by a cylindrical, electrostatic probe passed by a premixed flame.

3.2 Ion currents to cylindrical, electrostatic probes

Even in the high-ion-density region near a flame, the ion density is much lower than the neutral particle density^{(1)~(4)}, so that the gas near the flame can be considered to be a weakly ionized, high density gas. Phenomena when a probe passes a stationary flame with a velocity V_p can be assumed to be the same as those when a flame passes a stationary probe with a velocity $V_f (= V_p)$, because the relative relation between the probe and the flame is the same in both cases. Therefore, based on the results of the previous studies of electrostatic probes in weakly ionized, high density gas flows^{(27)~(29)}, a model shown in Fig. 12 is assumed for discussing the ion current fluctuation recorded with an electrostatic probe passed by a premixed flame. When a negatively biased, electrostatic probe is immersed in a weakly ionized, high density gas flow, an ion sheath is established around the probe and an ambipolar-diffusion region surrounds the ion sheath. In this regime, a large portion of ions which attain the ambipolar-diffusion-region edge can be assumed to reach the probe surface through the sheath region.

If the ratio of the number of ions which attain the probe surface to that of ions which pass the ambipolar-diffusion-region edge is C_1 , the ion current i per unit length of the probe is expressed as

$$i = C_1 e \int_A^B u_n n_a ds, \dots\dots\dots (3)$$

where e is the charge of an electron, u_n the velocity component normal to the ambipolar-diffusion-region edge, n_a the ion number density at the edge, and s the distance along the edge. A and B are the points on the ambipolar-diffusion-region edge between which all the streamlines passing the edge are involved.

Assuming that n_a can be replaced by the average ion number density $\bar{n}_a$ of the gas passing the ambipolar-diffusion-region edge, Eq. (3) can be approximately rewritten as

$$i = C_1 e \bar{n}_a q, \dots\dots\dots (4)$$

where

$$q = \int_A^B u_n ds, \dots\dots\dots (5)$$

which is the gas flow rate across the ambipolar-diffusion-region edge.

It is obvious that $\bar{n}_a$ and q depend on the thickness of the ambipolar diffusion region and ion sheath, which are closely related to ϕ_p , d_p , and the freestream gas velocity U (21)~(29). Since the sheath thickness increases with $|\phi_p|$, the increase of the ion current $j = L_p i$ with $|\phi_p|$ (Fig. 4) is attributable to the increase of q due to the increase of ion sheath thickness (27)(29). For $|\phi_p| \gtrsim 20$ V, the diameter of the ambipolar-diffusion-region edge might become comparable to or larger than the width of the high-ion-density region. Consequently, t_h is observed to increase as $|\phi_p|$ is increased (Fig. 5).

In the case when d_p is much smaller than the diameter of the ambipolar-diffusion-region edge, j_m and t_h only slightly increase with d_p (Figs. 8 and 9). These results show that $\bar{n}_a$ and q might be slightly influenced by changing d_p . When d_p is increased and becomes comparable to the diameter of the ambipolar-diffusion-region edge, the increase of j_m with d_p might be caused by the increase of q with d_p . When d_p is increased further and becomes comparable to the width of the high-ion-density region, the maximum value of $\bar{n}_a$ might decrease though q might still increase with d_p . Thus, in this case t_h increases rapidly with d_p , but the increasing rate of j_m with d_p becomes small.

It is well known that the ambipolar-diffusion-region edge approaches to the probe surface with increasing U (21) (27) (29), so that the distance between the points A and B along the line s might decrease as U increases. Despite the reduction of this distance, q increases with U because the increase of u_n with U covers the reduction of the integral span. The result that j_m increases with V_p indicates this fact (Fig. 10).

As V_p increases, the diameter of the ambipolar-diffusion-region edge might decrease so that the increase of spatial resolution power would be expected. In other words, δ_h is supposed to decrease with increasing V_p . Nevertheless, such a trend could not be recognized in the present experiments (Fig. 11). This result is consistent with the aforementioned relation between V_p and j_m , but it suggests that further discussion on the interaction between an electrostatic probe and the high-ion-density region is needed.

3.3 Optimum conditions for cylindrical, electrostatic probe measurements

When a cylindrical, electrostatic probe is used for measuring a fluctuating flame, it is desirable that the intrinsic ion current fluctuation of the measured flame is exactly the same as the recorded ion-current fluctuation or the former can be obtained from the latter by a simple correction.

As for L_p , the ion current is proportional to L_p , and any particular correction for this is not necessary. According to the results that the ion current is proportional to $V_p^{0.45}$ and its rate of variation is inversely proportional to V_p , the correction for the probe velocity V_p can be made rather simply. The spatial resolution power of a cylindrical, electrostatic probe is found to be independent of L_p and V_p .

When $|\phi_p|$ is not large, t_h is almost independent of $|\phi_p|$, and the recorded ion-current fluctuation can be corrected rather easily based on the fact that the ion current is proportional to $|\phi_p|^{0.65}$. When $|\phi_p|$ is large, however, an exact correction for the probe potential is impossible because the increase of t_h results in a reduction of the spatial resolution power.

For a small value of d_p , both j_m and t_h are almost independent of d_p , so that any correction for d_p is not necessary. However, for a large value of d_p , the correction is impossible because in addition to a complicated variation of j_m a considerable increase of t_h occurs. Consequently, an intrinsic ion-current fluctuation of flame can be obtained only when an electrostatic probe is used in the ranges, $|\phi_p| \lesssim 20$ V and $d_p \lesssim 0.1$ mm.

On the other hand, practically $|\phi_p|$ should be kept high enough for recording an ion current fluctuation because for its measurement, the potential difference (less than a few tenths volt for a usual burner flame) of the reference electrode (burner port in this case) from the plasma potential is desirable to be negligible as compared to $|\phi_p|$. Furthermore, d_p should be kept large enough for enduring in a high temperature combustion gas. Meeting these requirements is contrary to attaining a high spatial resolution power.

If a practical requirement for the measurements of fluctuating premixed flames by using a cylindrical, electrostatic probe is assumed that the estimated half-value thickness δ_h is less than 0.8 mm, two times the intrinsic value (0.4 mm), the conditions that d_p is smaller than 0.4 mm and that $|\phi_p|$ is less than 40 V should be satisfied. Although the value of δ_h measured under these conditions is in the range from 0.4 to 0.8 mm, the measurements of fluctuating premixed flames can be performed without significant reduction of the spatial resolution power. As mentioned previously, d_p and $|\phi_p|$ cannot be made smaller than limiting values, but to make them smaller obviously improves the spatial resolution power of a probe.

4. Conclusions

When a cylindrical, electrostatic probe passed a premixed propane-air flame parallel to the probe axis, the relations between the ion current fluctuation and the potential ϕ_p , length L_p , diameter d_p , and velocity V_p of the probe were examined, and the basic characteristics of the probe for the measurements of fluctuating premixed flames were discussed in detail.

The results are summarized as follows:

(1) The maximum value j_m of the ion current fluctuation is proportional to L_p , $|\phi_p|^{0.65}$ (for $|\phi_p| \geq 2$ V), or $V_p^{0.45}$. For d_p smaller than 0.1 mm, the increasing rate of j_m with d_p is small, and it becomes large with increasing d_p .

(2) The half-value period t_h of the ion current fluctuation is independent of L_p and inversely proportional to V_p . For $1 \text{ V} \leq |\phi_p| \lesssim 20 \text{ V}$, t_h is almost independent of $|\phi_p|$, while for $|\phi_p| \gtrsim 20 \text{ V}$, it increases with $|\phi_p|$. The increasing rate of t_h with d_p is very small for $d_p \lesssim 0.15 \text{ mm}$, and it becomes large with increasing d_p .

(3) In the ranges of L_p from 1 to 14 mm, and V_p from 1 to 22 m/s, the estimated half-value thickness $\delta_h (= t_h \cdot V_p)$ of the high-ion-density region near the flame decreased and approached about 0.4 mm as $|\phi_p|$ and d_p were decreased. This limiting value (about 0.4 mm) of δ_h was considered to represent the intrinsic value of the half-value thickness.

(4) The effects of ϕ_p , d_p , and V_p on the ion current $i (= j/L_p)$ per unit length of the probe were discussed by analyzing the phenomena near the probe based on the continuum probe theories. Consequently, such effects were found to be interpretable by considering the variations of the average ion number density $\bar{n}_a$ and flow rate q of the gas passing the ambipolar-diffusion-region edge with ϕ_p , d_p , and V_p .

(5) If a practical requirement for the measurements of fluctuating premixed flames by using a cylindrical, electrostatic probe is assumed that the estimated half-value thickness δ_h is less than 0.8 mm, two times the intrinsic value, the conditions that d_p is smaller than 0.4 mm and that $|\phi_p|$ is less than 40 V should be satisfied. Under these conditions, the measurements of fluctuating premixed flames can be performed without significant reduction of the spatial resolution power. Although d_p and $|\phi_p|$ cannot be made smaller than limiting values, to make them smaller improves the spatial resolution power. The spatial resolution power of cylindrical, electrostatic probes was confirmed to be independent of L_p and V_p .

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