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Flow Field Determination in Nearly Isotropic Turbulence of Any Intensity by Hot-wire Anemometry*¹

By Hiroyuki KIDO*², Yutaro WAKURI*³, Shinsuke ONO*⁴,
and Keiji HATANAKA*⁵

In order to discuss the influence of turbulence on turbulent burning velocity and to compare the experimental results of many workers with each other quantitatively, it is necessary to represent a turbulent field using the parameters with general definition. Some arbitrariness was unavoidable in the definition of turbulence parameters especially in the study of turbulent combustion in a closed bomb, in which the turbulence was measured usually with a hot-wire anemometer.

A procedure for the analysis of a turbulent field of any turbulence intensity by processing the DC and AC indication of a squared anemometer output signal of the effective cooling velocity, is described and the results of the analysis of the turbulence parameters in a closed bomb, such as intensity, temporal scale and spatial scale, in comparison with those by conventional method, are shown in this paper.

1. Introduction

In order to study the effect of turbulence on the burning velocity of a combustible mixture quantitatively, it is indispensable to represent the turbulence field using turbulence parameters of general definition in addition to the measurement of turbulent burning velocity. Semenov⁽¹⁾ has measured some turbulence parameters by a hot-wire anemometer, etc. in a closed bomb, in which four rotary mixers were used to create an artificial turbulence. Andrews, et al.⁽²⁾ and Abdel-Gayed, et al.⁽³⁾ have measured turbulent burning velocities in a cylindrical explosion vessel equipped with four fans, in which turbulence parameters were measured by hot-wire anemometers, and they discussed the correlation between the turbulent burning velocity and the turbulence parameters such as the turbulent Reynolds number and the r.m.s. turbulent velocity. Ballal, et al.⁽⁴⁾ have also measured the turbulent burning velocity in a steady flow combustion chamber and proposed a 3-region model of turbulent flames. Turbulence intensity and scale in the chamber could be varied in their experiments independently by suitable modifications of

mean flow velocity and geometry of grid, which was located upstream of the combustion chamber. Turbulence parameters were measured by using hot-wire anemometers in their experiment too. In order to discuss the effect of turbulence on the burning velocity and to compare these experimental results quantitatively, which are obtained in extremely different turbulence fields, that is, with mean directional flow in one and with almost no directional flow in the other, it becomes indispensable to obtain the turbulence parameters of common definition to any turbulence field. Some ambiguity has been, however, inevitable in the definition of turbulence parameters especially in the experiments in a closed bomb, because a hot-wire anemometer detects essentially the so-called effective cooling velocity which is vectorially composed of the velocity components.

From the above point of view, an improved method of obtaining the turbulence parameters of general definition in a flow of any turbulence intensity, by processing the DC⁽⁵⁾ and r.m.s. AC indications, autocorrelation and two point correlation of the AC indications of the squared anemometer output signals of effective cooling velocity, is described in this paper.

Nomenclature

e : base of natural logarithm
 F_r : cross-correlation function of fluctuations of squared effective cooling velocities
 F_T : autocorrelation function of fluctuation of squared effective cooling velocity
 f : longitudinal cross-correlation function
 G : directional sensitivity coefficient of hot-wire probe

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*2 Associate Professor, Faculty of Engineering, Kyushu University, Higashi-ku, Fukuoka.

*3 Professor, Faculty of Engineering, Kyushu University

*4 Professor, Faculty of Engineering, Kyushu University

*5 Graduate Student, Faculty of Engineering, Kyushu University.

g : lateral cross-correlation function
 L_f : longitudinal mean spatial scale
 L_{re} : mean spatial scale obtained from fluctuations of effective cooling velocities
 L_T : mean temporal scale
 L_{Te} : mean temporal scale obtained from fluctuation of effective cooling velocity
 P_1 : Eq.(16)
 P_2 : Eq.(17)
 R_{re} : cross-correlation function of fluctuations of effective cooling velocities
 R_T : autocorrelation function
 R_{Te} : autocorrelation function of fluctuation of effective cooling velocity
 r : distance from reference point r_0
 t : time
 U : mean directional velocity
 u : instantaneous velocity
 $\tilde{u}$: instantaneous fluctuation velocity
 u' : turbulence intensity
 V : anemometer output voltage
 θ : angle between linearized output voltage of anemometer and flow velocity
 τ : time interval
 Subscripts
 a : values at point a
 B : values binormal to hot-wire (Fig.5)
 b : values at point b
 e : values based on effective cooling velocity
 I : values for I-Type probe
 L : values for L-Type probe
 N : values normal to hot-wire (Fig.5)
 T : values tangential to hot-wire (Fig.5)
 w : values in one of three directions x, y, z
 x : values in x -direction
 y : values in y -direction
 z : values in z -direction

2. Experimental apparatus and procedure

Perpendicularly intersecting cylinders with a diameter of 100mm constitute a closed bomb in this experiment. Two transparent side walls with a diameter of 80mm face each other and the remaining four surfaces are fitted with perforated plates. This bomb is practicable for combustion test. A cross-sectional view of the bomb is shown in Fig.1. Four centrifugal fans ② behind the plates ① can be driven independently. Air jets issuing from inner ports of the plates generate an almost uniform turbulence in the central region of the bomb and then the air is re-circulated into the fan chambers from outer ports of the plates. The perforated plates used in this experiment are shown in Fig.2. Figure 3 shows an orthogonal coordinate system and the points for turbulence measurement. The origin of the coordinate system coincides with the center of the bomb and x -axis is directed to the transparent walls, whereas y and z -axes pass through the centers of the

plates. The arrangement of the measuring points is as follows. They are located on the intersections of planes 1, 2 and 3 and a cylinder of 15mm radius, whose central axis is the x -axis, and on the x -axis. Total number of measuring points is 27 as shown in Fig.3. Plane 2 is one containing y and z -axes, and planes 1 and 3 are parallel ones to plane 2, which are +15mm and -15mm distant from it respectively. The points will be named hereafter (2, 4) for example. The former figure designates the plane number and the latter represents the direction angle. Namely, direction 1 makes an angle of 45° to both positive y and z -axes, and the following direction numbers are given clockwise at intervals of 45° . The points on the x -axis are named 0. According to this system of naming, point

Fig.1 Cross-sectional view of closed bomb

Fig.2 Perforated plates (port diameter 5mm, pitch 20mm)

Fig.3 Coordinate system and measuring points

(2, 0) means the center of bomb. Relative position of measuring points to perforated plates is shown in Fig.4. A plate of 7 holes (Fig.2(a)) is placed at No.1 position and ones of 8 holes (Fig.2(b)) are placed at the other positions in Fig.4 to make an almost uniform turbulence in the central region of the bomb.

I-type and L-type probes, which are shown in Figs.5(a) and 5(b) respectively, are used in the experiment, and the diameter and the length of the hot-wire are $7\mu\text{m}$ and 3mm respectively in either case. To designate the flow direction to a hot-wire, N (normal), B (binormal) and T (tangential) directions are defined as shown in Fig.5. Typical examples of the linearized output of a hot-wire anemometer are shown in Figs.6(a) and 6(b). Calibrations of hot-wires in N and T directions for I-type and in N and B directions for L-type are carried out by using a pendulum⁽⁶⁾, and the others are done with a wind tunnel. A typical example of the linearized output of squared effective cooling velocity, whose analysis is the main subject of this paper, is shown in Fig.7. To analyze the mean directional velocity, r.m.s. turbulent velocity and autocorrelation function at each point, measurements are made in three different ways of probe placement so that T or wire direction becomes parallel to each of the three coordinate axes, at each point and in each flow condition. A probe is inserted from the positive x -direction in every case mentioned above. Two point correlations are measured with one of two probes fixed at points (2,0), (2,2), (2,4), (1,0), (1,2) and (1,4). The placement of

Fig.4 Relative position of measuring points to perforated plates

(a) I-type probe (b) L-type probe

Fig.5 Schematic shape of probes

the probes in the measurement at point (2,0) is shown in Figs.8(a) ~ (c). One of the two probes is fixed at point (2,0), and the other is shifted toward the positive x -direction in the case of Fig.8(a), toward the negative y -direction in Fig.8(b) and toward the negative z -direction in Fig.8(c). The probe placement at the other points is the same as that at point (2,0).

3. Analysis of output of hot-wire anemometer

The linearized output voltage V_e of a hot-wire anemometer, when the hot-wire detects an instantaneous velocity u whose components in the N , B and T directions are u_N , u_B and u_T respectively, is given by the following equation⁽⁷⁾.

$$V_e = \{ (u_N \tan \theta_N)^2 + (u_B \tan \theta_B)^2 + (u_T \tan \theta_T)^2 \}^{1/2} \dots \dots \dots (1)$$

where θ_N , θ_B and θ_T are the angles between the linearized output voltages and the

(a) I-type probe

(b) L-type probe

Fig.6 Relationship between linearized output of hot-wire anemometer and flow velocity, and directional sensitivity of hot-wire probe

Fig.7 Relationship between linearized output of hot-wire anemometer and squared flow velocity

Fig.8 Relative probe positions to coordinate system

velocity, as shown in Fig.6. When the directional sensitivity coefficients of a hot-wire are defined as $G_N=1.0$, $G_B = \tan\theta_B/\tan\theta_N$ and $G_T = \tan\theta_T/\tan\theta_N$, the effective cooling velocity $u_e (= V_e/\tan\theta_N)$ is given by

$$u_e = (u_N^2 + G_B^2 u_B^2 + G_T^2 u_T^2)^{1/2} \quad (2)$$

From the above equation, the squared value u_e^2 of the effective cooling velocity and its time averaged value $\overline{u_e^2}$ are given by the following equations.

$$u_e^2 = u_N^2 + G_B^2 u_B^2 + G_T^2 u_T^2 \quad (3)$$

$$\overline{u_e^2} = \overline{u_N^2} + G_B^2 \overline{u_B^2} + G_T^2 \overline{u_T^2} \quad (4)$$

3.1 Analysis of mean directional velocity and turbulence intensity

The effective cooling velocity u_{ex} , when an L-type probe is placed such that the wire direction is parallel to the x -axis and the N direction is to the z -axis as shown in Fig.8(b), is given by

$$u_{ex}^2 = G_{TL}^2 u_z^2 + G_{BL}^2 u_y^2 + u_x^2 \quad (5)$$

where u_x , u_y and u_z are the axial components of an instantaneous velocity u parallel to x , y and z -axes respectively. $G_{BL}=0.91$ and $G_{TL}=0.22$ are obtained in the calibration. When an L-type probe is placed at the same point so that the wire direction is parallel to the y -axis and the N direction is to the z -axis as shown in Fig.8(c), the following equation is obtained.

$$u_{ey}^2 = G_{BL}^2 u_z^2 + G_{TL}^2 u_x^2 + u_y^2 \quad (6)$$

where $G_{BL}=1.06$ and $G_{TL}=0.17$ are obtained in the calibration. When an L-type probe too is placed at the same point so that the wire direction is parallel to the z -axis and the N direction is to the y -axis, the equation is as follows.

$$u_{ez}^2 = G_{BL}^2 u_x^2 + u_y^2 + G_{TL}^2 u_z^2 \quad (7)$$

When the instantaneous velocity u is broken up into the mean velocity U and the fluctuation velocity $\tilde{u}$ and the turbulence intensity u' is defined by the root mean square of $\tilde{u}$, that is, $u' = (\tilde{u}^2)^{1/2}$, $U_x^2 + (u'_x)^2$, $U_y^2 + (u'_y)^2$ and $U_z^2 + (u'_z)^2$ can be obtained by time-averaging of Eqs. (5), (6) and (7).

$$\begin{bmatrix} G_{TL}^2 & G_{BL}^2 & 1 \\ G_{BL}^2 & G_{TL}^2 & 1 \\ G_{BL}^2 & 1 & G_{TL}^2 \end{bmatrix} \begin{bmatrix} U_z^2 + (u'_z)^2 \\ U_y^2 + (u'_y)^2 \\ U_x^2 + (u'_x)^2 \end{bmatrix} = \begin{bmatrix} \overline{u_{ex}^2} \\ \overline{u_{ey}^2} \\ \overline{u_{ez}^2} \end{bmatrix} \quad (8)$$

In the next place, the AC indications of the squared value of the effective cooling velocity are analyzed. Eqs.(5), (6) and (7) can be expressed generally by the following equation,

$$u_{ew}^2 = G_x^2 u_x^2 + G_y^2 u_y^2 + G_z^2 u_z^2 \quad (9)$$

where G_x , G_y and G_z are the directional sensitivity coefficients parallel to the x , y and z -axes respectively. They can be decided depending on the type of probe and on its placement in respective cases. From the above equation, the fluctuation of u_{ew}^2 is given by

$$u_{ew}^2 - \overline{u_{ew}^2} = G_x^2 [2U_x \tilde{u}_x + (\tilde{u}_x)^2] + G_y^2 [2U_y \tilde{u}_y + (\tilde{u}_y)^2] + G_z^2 [2U_z \tilde{u}_z + (\tilde{u}_z)^2] - [G_x^2 (u'_x)^2 + G_y^2 (u'_y)^2 + G_z^2 (u'_z)^2] \quad (10)$$

From Eq.(10), the root mean square of the fluctuation of u_{ew}^2 , that is, $(u_{ew}^2)'$ = $\{(\overline{u_{ew}^2} - \overline{u_{ew}^2})^2\}^{1/2}$ is given by the following equation.

$$\begin{aligned} ((u_{ew}^2)')^2 = & G_x^4 \{4U_x^2 \overline{(\tilde{u}_x)^2} + 4U_x \overline{(\tilde{u}_x)^3} + \overline{(\tilde{u}_x)^4}\} + G_y^4 \{4U_y^2 \overline{(\tilde{u}_y)^2} + 4U_y \overline{(\tilde{u}_y)^3} + \overline{(\tilde{u}_y)^4}\} + G_z^4 \{4U_z^2 \overline{(\tilde{u}_z)^2} \\ & + 4U_z \overline{(\tilde{u}_z)^3} + \overline{(\tilde{u}_z)^4}\} + 2G_x^2 G_y^2 \{4U_x U_y \overline{\tilde{u}_x \tilde{u}_y} + 2U_x \overline{(\tilde{u}_x)^2} \overline{\tilde{u}_y} + 2U_y \overline{(\tilde{u}_y)^2} \overline{\tilde{u}_x} + \overline{(\tilde{u}_x)^2} \overline{(\tilde{u}_y)^2}\} \\ & + 2G_y^2 G_z^2 \{4U_y U_z \overline{\tilde{u}_y \tilde{u}_z} + 2U_y \overline{(\tilde{u}_y)^2} \overline{\tilde{u}_z} + 2U_z \overline{(\tilde{u}_z)^2} \overline{\tilde{u}_y} + \overline{(\tilde{u}_y)^2} \overline{(\tilde{u}_z)^2}\} + 2G_x^2 G_z^2 \{4U_x U_z \overline{\tilde{u}_x \tilde{u}_z} \\ & + 2U_x \overline{(\tilde{u}_x)^2} \overline{\tilde{u}_z} + 2U_z \overline{(\tilde{u}_z)^2} \overline{\tilde{u}_x} + \overline{(\tilde{u}_x)^2} \overline{(\tilde{u}_z)^2}\} - 2[G_x^2 (u'_x)^2 + G_y^2 (u'_y)^2 + G_z^2 (u'_z)^2] [G_x^2 \overline{2U_x \tilde{u}_x} + \overline{(\tilde{u}_x)^2}] \\ & + G_y^2 \overline{2U_y \tilde{u}_y} + \overline{(\tilde{u}_y)^2}] + G_z^2 \overline{2U_z \tilde{u}_z} + \overline{(\tilde{u}_z)^2}] + [G_x^2 (u'_x)^2 + G_y^2 (u'_y)^2 + G_z^2 (u'_z)^2]^2 \end{aligned} \quad (11)$$

where $\overline{u_x} = \overline{u_y} = \overline{u_z} = 0$ and $\overline{(\tilde{u}_x)^2} = (u'_x)^2$, $\overline{(\tilde{u}_y)^2} = (u'_y)^2$ and $\overline{(\tilde{u}_z)^2} = (u'_z)^2$. Then, we introduce an assumption that the turbulence is nearly isotropic and a hypothesis⁽⁸⁾ that the fourth-order velocity cumulants vanish. From the assumption of isotropy, the third order correlations and the second order correlations of the velocity fluctuations, whose directions are right-angled with each other, may be nearly equal to zero. Furthermore, as $(\tilde{u}_x)^4 = 3(u'_x)^4$, $(\tilde{u}_y)^4 = 3(u'_y)^4$, $(\tilde{u}_z)^4 = 3(u'_z)^4$, $(\tilde{u}_x)^2(\tilde{u}_y)^2 = (u'_x)^2(u'_y)^2$, $(\tilde{u}_y)^2(\tilde{u}_z)^2 = (u'_y)^2(u'_z)^2$ and $(\tilde{u}_z)^2(\tilde{u}_x)^2 = (u'_z)^2(u'_x)^2$ can be obtained from the hypothesis of the zero fourth-order cumulants, we derive the following equation from Eq.(11).

$$((u_{ie}')^2)^2 = 2[G_x^4(u_{s'}')^2[2U_s^2 + (u_{s'}')^2] + G_y^4(u_{y'}')^2[2U_y^2 + (u_{y'}')^2] + G_z^4(u_{z'}')^2[2U_z^2 + (u_{z'}')^2]] \dots\dots\dots(12)$$

For the same placement of the probes as in the cases of Eqs.(5)~(7), the following equation can be obtained from Eq.(12).

$$\begin{vmatrix} G_{TL}^4 & G_{BL}^4 & 1 \\ G_{BL}^4 & G_{TI}^4 & 1 \\ G_{BI}^4 & 1 & G_{TI}^4 \end{vmatrix} \begin{vmatrix} 2(u_{s'}')^2[2U_s^2 + (u_{s'}')^2] \\ 2(u_{y'}')^2[2U_y^2 + (u_{y'}')^2] \\ 2(u_{z'}')^2[2U_z^2 + (u_{z'}')^2] \end{vmatrix} = \begin{vmatrix} ((u_{ie}')^2)^2 \\ ((u_{ie}')^2)^2 \\ ((u_{ie}')^2)^2 \end{vmatrix} \dots\dots\dots(13)$$

Consequently, the mean directional velocities and turbulence intensities parallel to the three coordinate axes can be calculated from Eqs.(8) and (13) by measuring the mean values and the root mean square

values of the fluctuations of the squared effective cooling velocities. Because of the finiteness of the number of data and of the uncertainty of the above two assumptions, U and u' were calculated as follows in this study. In the results calculated by using Eqs.(8) and (13), we wrote $B = U^2 + (u')^2$ and $C = 2(u')^2[2U^2 + (u')^2]$ and introduced a discriminator $D = B^2 - C/2$. Then, the flow parameters were calculated as $U=0$ and $u' = \sqrt{B}$ when $D \leq 0$, and as $U = (D)^{1/4}$ and $u' = \sqrt{B - U^2}$ when $D > 0$. Typical examples of the flow parameters thus obtained are shown in Fig.9 for the fan speed of 15000 rpm. It can be known from the figure that the variations of the turbulence intensities for respective points relative to the three coordinate axes are not large. In plane 2, the mean directional velocities were detected on the z -axis and this may be attributed to the placement of a perforated plate in Fig.2(a) perpendicular to the positive z -axis. Mean directional velocities parallel to the x -axis were detected at every point in planes 1 and 3. Basically in this study, the direction of the directional velocity, that is, positive or negative, can not be known, but it is well estimated that the flow will move toward the positive x -direction in plane 1 and toward the negative x -direction in plane 3. The qualitative features of the flow parameters described so far did not change with the fan speed. The mean value of the turbulence intensities of all points and of the directions

Fig.9 Spatial distribution of mean directional velocities and turbulence intensities (fan speed 15000 rpm)

Fig.10 Variation of u' , ue' and Ue with fan speed

relative to the three coordinate axes is shown against the fan speed in Fig.10. It can be seen from the figure that the mean turbulence intensity is in proportion to the fan speed. The mean value of the time-averaged values U_e of the effective cooling velocities and the mean value of the root mean square values u_e' of their fluctuations of all points and of the three different placements of the probes are shown also in Fig.10. The value of U_e is a little larger than the value of u' , because $(G_x^2 + G_y^2 + G_z^2)$ is nearly equal to unity.

3.2 Analysis of temporal scale of turbulence

In order to analyze the temporal scale of turbulence at each point, the product of the fluctuation of the squared effective cooling velocity at the time t and that at the time $(t+\tau)$ is time-averaged. Assuming the isotropy and the zero fourth-order cumulants to apply as well as in the preceding section, the following equation can be obtained.

$$\begin{aligned} & \overline{\{u_{ew}^2(t) - \overline{u_{ew}^2}\} \{u_{ew}^2(t+\tau) - \overline{u_{ew}^2}\}} \\ &= 2[G_x^4(u_x')^2 \{2U_x^2 R_r(\tau) + (u_x')^2 R_r^2(\tau)\} \\ &+ G_y^4(u_y')^2 \{2U_y^2 R_r(\tau) + (u_y')^2 R_r^2(\tau)\} \\ &+ G_z^4(u_z')^2 \{2U_z^2 R_r(\tau) + (u_z')^2 R_r^2(\tau)\}] \end{aligned} \quad (14)$$

where $R_r(\tau)$ is defined as $R_r(\tau) = \overline{\tilde{u}_x(t)\tilde{u}_x(t+\tau)} / (u'_x)^2 = \overline{\tilde{u}_y(t)\tilde{u}_y(t+\tau)} / (u'_y)^2 = \overline{\tilde{u}_z(t)\tilde{u}_z(t+\tau)} / (u'_z)^2$, and it is just the autocorrelation function of general definition. By feeding a correlator with a linearized output of the squared effective cooling velocity, we can obtain the following autocorrelation function $F_r(\tau)$.

$$F_r(\tau) = \overline{\{u_{ew}^2(t) - \overline{u_{ew}^2}\} \{u_{ew}^2(t+\tau) - \overline{u_{ew}^2}\}} / \{(\overline{u_{ew}^2})'\}^2 \quad (15)$$

where $\{(\overline{u_{ew}^2})'\}^2$ is given by Eq.(12). We write P_1 and P_2 to replace the following expressions at each measuring point.

$$P_1 = [G_x^4(u_x')^4 + G_y^4(u_y')^4 + G_z^4(u_z')^4] \quad (16)$$

$$P_2 = 2[G_x^2 U_x^2 (u_x')^2 + G_y^2 U_y^2 (u_y')^2 + G_z^2 U_z^2 (u_z')^2] \quad (17)$$

Then, $R_r(\tau)$ can be calculated from the measured autocorrelation function $F_r(\tau)$ as follows.

$$P_1 R_r^2(\tau) + P_2 R_r(\tau) - (P_1 + P_2) F_r(\tau) = 0 \quad (18)$$

From the above equation, it can be concluded that $F_r(\tau) = R_r^2(\tau)$ is valid in the measurement of turbulence without mean directional flow ($P_2 = 0$) and $F_r(\tau) = R_r(\tau)$ is approximately valid in the measurement of turbulence with a strong directional flow. Let e be the base of the natural logarithm, and if we define the mean temporal scale L_r of turbulence as $R_r(L_r) = 1/e$, then the following equation can be obtained from Eq.(18).

$$F_r(L_r) = \frac{1}{P_1 + P_2} \left(\frac{P_1}{e^2} + \frac{P_2}{e} \right) \quad (19)$$

Fig.11 Variation of mean temporal scales of turbulence with turbulence intensity

Fig.12 Variation of $u' L_\tau$ and $U_e L_{\tau e}$ with turbulence intensity

From the measured curve of $F_r(\tau)$, the mean temporal scale L_r can be decided by using Eq.(19). The average value of the mean temporal scales of all points and of the three different placements of the probes is shown against the turbulence intensity in Fig.11. From the figure, it can be known that L_r decreases inversely with an increase of u' . $L_{\tau e}$, which is defined as $R_{\tau e}(L_{\tau e}) = 1/e$ in the autocorrelation function $R_{\tau e}(\tau) = \overline{\tilde{u}_e(t)\tilde{u}_e(t+\tau)} / (u'_e)^2$ of the fluctuation of the effective cooling velocity itself, is shown also in Fig.11. As the fluctuation of the effective cooling velocity is composed of the fluctuation components relative to the three coordinate axes, $L_{\tau e}$ shows a little smaller value than L_r .

In the next place, a kind of spatial scale $u' L_r$ is plotted against u' in Fig.12. From the figure, it can be known that the spatial scale $u' L_r$ increases with an increase of u' and shows a value of about 2mm at $u' = 1$ m/s and about 3mm at $u' = 4$ m/s. $U_e L_{\tau e}$, which is obtained by the analysis of the effective cooling velocity itself, is shown also in the same figure and its value is found nearly equal to the value of $u' L_r$.

3.3 Analysis of spatial scale of turbulence

In order to analyze the spatial

scale of turbulence at each point mentioned previously, the instantaneous product of the fluctuations of the squared effective cooling velocities at points a and b is time-averaged. Assuming the iso-

tropy and the zero fourth-order cumulants to apply as well as in the preceding sections, and for brevity assuming that the points a and b are on the x -axis, the following equation can be obtained.

$$\begin{aligned} \overline{(u_{xa}^2(r_0) - \overline{u_{xa}^2})(u_{xb}^2(r_0+r) - \overline{u_{xb}^2})} &= 2[G_{xa}^2 G_{xb}^2 u_{xa}' u_{xb}' (2U_{xa} U_{xb} f(r) + u_{xa}' u_{xb}' f^2(r)) \\ &+ G_{ya}^2 G_{yb}^2 u_{ya}' u_{yb}' (2U_{ya} U_{yb} g(r) + u_{ya}' u_{yb}' g^2(r)) + G_{za}^2 G_{zb}^2 u_{za}' u_{zb}' (2U_{za} U_{zb} g(r) + u_{za}' u_{zb}' g^2(r))] \end{aligned} \quad (20)$$

where $f(r)$ is the longitudinal cross-correlation function defined by $f(r) = \overline{u_{xa}(r_0) u_{xb}(r_0+r)} / (\overline{u'_{xa} u'_{xb}})$ and $g(r)$ is the lateral cross-correlation function defined by $g(r) = \overline{u_{ya}(r_0) u_{yb}(r_0+r)} / (\overline{u'_{ya} u'_{yb}}) = \overline{u_{za}(r_0) u_{zb}(r_0+r)} / (\overline{u'_{za} u'_{zb}})$. By feeding a correlator with a linearized output of the squared effective cooling velocities at points a and b , we can obtain the following cross-correlation function $Fr(r)$.

$$Fr(r) = \overline{(u_{xa}^2(r_0) - \overline{u_{xa}^2})(u_{xb}^2(r_0+r) - \overline{u_{xb}^2})} / (\overline{(u_{xa}')^2 (u_{xb}')^2}) \quad (21)$$

In the isotropic turbulence, as $g(r)$ is related to $f(r)$ by $g(r) = f(r) + (r/2)\{\partial f(r)/\partial r\}$, the longitudinal cross-correlation function $f(r)$ can be calculated from the measured cross-correlation function $Fr(r)$ as follows.

$$\begin{aligned} G_{xa}^2 G_{xb}^2 u_{xa}' u_{xb}' (2U_{xa} U_{xb} f(r) + u_{xa}' u_{xb}' f^2(r)) &+ G_{ya}^2 G_{yb}^2 u_{ya}' u_{yb}' (2U_{ya} U_{yb} g(r) + u_{ya}' u_{yb}' g^2(r)) \\ &+ G_{za}^2 G_{zb}^2 u_{za}' u_{zb}' (2U_{za} U_{zb} g(r) + u_{za}' u_{zb}' g^2(r)) = Fr(r) [G_{xa}^4 (u_{xa}')^2 (2U_{xa}^2 + (u_{xa}')^2) \\ &+ G_{ya}^4 (u_{ya}')^2 (2U_{ya}^2 + (u_{ya}')^2) + G_{za}^4 (u_{za}')^2 (2U_{za}^2 + (u_{za}')^2)]^{1/2} [G_{xb}^4 (u_{xb}')^2 (2U_{xb}^2 + (u_{xb}')^2) \\ &+ G_{yb}^4 (u_{yb}')^2 (2U_{yb}^2 + (u_{yb}')^2) + G_{zb}^4 (u_{zb}')^2 (2U_{zb}^2 + (u_{zb}')^2)]^{1/2} \end{aligned} \quad (22)$$

If the turbulence parameters at point a are nearly equal to those at point b , then $U_{xa} = U_{xb}$, $u'_{xa} = u'_{xb}$, $U_{ya} = U_{yb}$, $u'_{ya} = u'_{yb}$, $U_{za} = U_{zb}$ and $u'_{za} = u'_{zb}$ and if the turbulence is measured by a pair of probes in the same shape as shown in Fig.8, then $G_{xa} = G_{xb}$, $G_{ya} = G_{yb}$ and $G_{za} = G_{zb}$. Eq.(22) can be simplified by using the relations above as follows.

$$\begin{aligned} G_{xa}^4 (u_{xa}')^2 (2U_{xa}^2 f(r) + (u_{xa}')^2 f^2(r)) &+ G_{ya}^4 (u_{ya}')^2 (2U_{ya}^2 g(r) + (u_{ya}')^2 g^2(r)) \\ &+ G_{za}^4 (u_{za}')^2 (2U_{za}^2 g(r) + (u_{za}')^2 g^2(r)) = Fr(r) [G_{xa}^4 (u_{xa}')^2 (2U_{xa}^2 + (u_{xa}')^2) \\ &+ G_{ya}^4 (u_{ya}')^2 (2U_{ya}^2 + (u_{ya}')^2) + G_{za}^4 (u_{za}')^2 (2U_{za}^2 + (u_{za}')^2)] \end{aligned} \quad (23)$$

By using the turbulence parameters at each point and the directional sensitivity coefficients which depend on the placement of a hot-wire, $f(r)$ can be computed without difficulty from the measured $Fr(r)$. As the points a and b are on the x -axis in the discussion above, Eq.(23) holds in that case (Fig.8(a)). When the points a and b are on the y -axis (Fig.8(b)) the following equation is obtained.

$$\begin{aligned} G_{xa}^4 (u_{xa}')^2 (2U_{xa}^2 g(r) + (u_{xa}')^2 g^2(r)) &+ G_{ya}^4 (u_{ya}')^2 (2U_{ya}^2 f(r) + (u_{ya}')^2 f^2(r)) \\ &+ G_{za}^4 (u_{za}')^2 (2U_{za}^2 g(r) + (u_{za}')^2 g^2(r)) = Fr(r) [G_{xa}^4 (u_{xa}')^2 (2U_{xa}^2 + (u_{xa}')^2) \\ &+ G_{ya}^4 (u_{ya}')^2 (2U_{ya}^2 + (u_{ya}')^2) + G_{za}^4 (u_{za}')^2 (2U_{za}^2 + (u_{za}')^2)] \end{aligned} \quad (24)$$

Analogously, when they are on the z -axis (Fig.8(c)), we obtain :

$$\begin{aligned} G_{xa}^4 (u_{xa}')^2 (2U_{xa}^2 g(r) + (u_{xa}')^2 g^2(r)) &+ G_{ya}^4 (u_{ya}')^2 (2U_{ya}^2 g(r) + (u_{ya}')^2 g^2(r)) \\ &+ G_{za}^4 (u_{za}')^2 (2U_{za}^2 f(r) + (u_{za}')^2 f^2(r)) = Fr(r) [G_{xa}^4 (u_{xa}')^2 (2U_{xa}^2 + (u_{xa}')^2) \\ &+ G_{ya}^4 (u_{ya}')^2 (2U_{ya}^2 + (u_{ya}')^2) + G_{za}^4 (u_{za}')^2 (2U_{za}^2 + (u_{za}')^2)] \end{aligned} \quad (25)$$

It can be known from Eqs.(23) ~ (25) that $Fr(r)$ is nearly equal to $f(r)$ when there exists a strong directional flow parallel to a line connecting the two measuring points and moreover the sensitivity coefficient for the flow direction is close to unity, and that $Fr(r)$ is nearly equal to $g(r)$ when there exists a strong directional flow perpendicular to the line connecting the two measuring points and under the same condition of the sensitivity coefficient. In the case of a turbulence without any mean directional flow and with the condition of $u'_{xa} = u'_{ya} = u'_{za}$, the following equation can be obtained from Eq.(23).

$$Fr(r) = \frac{1}{G_{xa}^4 + G_{ya}^4 + G_{za}^4} [G_{xa}^4 f^2(r) + (G_{ya}^4 + G_{za}^4) g^2(r)] \quad (26)$$

Similar equations to Eq.(26) can be easily obtained from Eqs.(24) and (25). In the next place, if the longitudinal cross-correlation function may be approximated by $f(r) = \exp(-r/L_f)$ in the same manner as in the case of the mean temporal scale, the longitudinal integral scale of turbulence is L_f and $f(L_f) = 1/e$ is valid. Then, $g(L_f) = 1/(2e)$ is valid too. Sub-

stituting these relations into Eqs.(23) ~ (25), which correspond to Figs.8(a) ~ (c) respectively, the values of $Fr(L_f)$ are calculated and the longitudinal mean spatial scales L_f to the three coordinate axes can be obtained from the measured curves of $Fr(r)$ at respective points. The values of L_f are somewhat different from each other in the three directions. The mean

Fig.13 Variation of mean spatial scale of turbulence with turbulence intensity

Fig.14 Comparison of various spatial scales

value for the three directions is about 3.4~5.3mm and it increases slightly with an increase of u' as shown in Fig.13.

The mean spatial scales obtained so far are L_f and $u'L_\tau$ of general definition and $U_e L_{\tau e}$ calculated by the effective cooling velocity itself. In addition to them, a mean spatial scale L_{re} , which is defined as $R_{re}(L_{re}) = 1/e$ in the cross-correlation function $R_{re} = \overline{u'_{ea}(r_0)u'_{eb}(r_0+r)}$ $/ (\overline{u'_{ea}u'_{eb}})$ of the fluctuation of the effective cooling velocity itself, is obtained in this paper. The ratios of $u'L_\tau$, $U_e L_{\tau e}$ and L_{re} to L_f are plotted against u' in Fig.14. It can be seen from the figure that those values of $u'L_\tau$, $U_e L_{\tau e}$ and L_{re} are nearly the same and their ratio to L_f is about 0.5 irrespective of a change in u' .

4. Conclusions

(1) A procedure for calculating the turbulence parameters of general definition is described in this paper. It uses the time-averaged value of the squared effective cooling velocity, the root mean

square value of its fluctuation, the auto-correlation function of the fluctuation and the cross-correlation function of the fluctuations. This procedure is successful, whether a main flow exists or not and enables one to express any turbulent field by the turbulence parameters of common definition. Hence it may become feasible to compare quantitatively the results of the combustion tests in turbulent fields of different natures from each other.

(2) Assuming the isotropy and the zero fourth-order cumulants, stationary turbulent fields in a bomb were analyzed by the procedure. Consequently, the spatial distribution of the mean directional velocities and of the turbulence intensities and the variation of the mean temporal scale and of the mean spatial scale with turbulence intensity were obtained. Moreover, the conventional turbulence parameters, which were obtained by the analysis of the effective cooling velocity itself, were compared with the turbulence parameters of general definition and the quantitative differences among them were clearly shown in this paper.

(3) According to the calculated results, the mean temporal scale varies from 2.7 to 0.8ms with the variation of the turbulence intensity from 0.5 to 3.8m/s and it decreases with an increase in the turbulence intensity. The mean spatial scale varies from 3.4 to 5.3mm in the same range of turbulence intensities and it increases slightly with an increase in the turbulence intensity in this experiment.

(4) A mean spatial scale defined by the product of the turbulence intensity and the mean temporal scale is about one half of the longitudinal mean spatial scale of turbulence at the same position.

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