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Flow Pattern and Frictional Losses in Pulsating Pipe Flow

Part 4 General Representation of Turbulent Frictional Losses*

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Friction factors in a pulsating turbulent pipe flow are estimated from the flow pattern diagrams shown in a previous paper and the effect of frequency and time-averaged Reynolds number Re_{ta} on the frictional losses is verified with experimental data. The instantaneous friction factor $\lambda_u(t)$ and the time average friction factor $\lambda_{u,ta}$ may be generally expressed as functions of $\omega'/Re_{ta}^{3/4}$, where ω' is a dimensionless frequency. Here, $\lambda_u(t)$ is almost equal to the quasi-steady friction factor λ_q in the quasi-steady region, but in the intermediate and the inertia dominant regions, $\lambda_u(t) \neq \lambda_q$ in both accelerating and decelerating zones. The time average friction factor $\lambda_{u,ta}$ increases with an increase in both $\omega'/Re_{ta}^{3/4}$ and velocity amplitude ratio A_1 . In addition, more adequate values indicating the limits between the above mentioned three regions are proposed.

1. Introduction

Although some experiments on frictional losses in an incompressible pulsating turbulent pipe flow are reported, the behaviors of these frictional losses at any instant in a cycle and of their time average values over one cycle are found to be contradictory with each other. Therefore, it seems that this problem is far from having been solved. This is partly attributable to the fact that the response characteristics of measuring facilities (electro-magnetic flowmeter in particular) used in an unsteady flow are not good enough. In addition, the conventional indirect method for determining frictional losses needs substitution of experimental values of $\Delta p/l$ and u_m into an unsteady integral momentum equation. This leads to a noticeable error as the pulsation frequency increases. This is because the magnitude of viscous term in a higher ω' region become negligible in comparison with those of pressure gradient term and of inertia term.

However, other investigators have not paid any attention to the points mentioned above, so the reliability of their experimental results is questionable. The present authors also used the conventional method for evaluating the frictional losses. Many considerations were

made to improve the response characteristics of the measuring facilities and to minimize the inevitable uncertainties described above and results were satisfactory.

Then, experimental results of the four characteristic parameters were plotted on the flow pattern diagrams, and the curve for each parameter was drawn smoothly through the mean of the data points. By reading the four characteristic parameters off these diagrams, frictional losses can be estimated with a sufficient accuracy in a relatively higher ω' region as functions of dimensionless frequency ω' and time-averaged Reynolds number Re_{ta} .

An outline of this investigation is given below. The four characteristic parameters describing the flow pattern were found to be functions of $\omega'/Re_{ta}^{3/4}$ in the experimental Reynolds number range. Here, at first the instantaneous and the time average friction factors are estimated as functions of $\omega'/Re_{ta}^{3/4}$ by using the four characteristic parameters and then they are compared with experimental results to verify the validity of the estimation. Furthermore, the limits between quasi-steady, intermediate, and inertia dominant regions are determined by referring to the previous experimental results also.

2. Nomenclature

The symbols used in this report are as follows:

A_1 : velocity amplitude ratio = $|u_{m,os,i}|/u_{m,ta}$
 B_1, B_2 : coefficients defined by Eq.(4)
 D : pipe diameter
 f : pulsation frequency
 l : pipe length of test section
 p : pressure
 Δp : pressure drop
 R : radius of pipe

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Re : instantaneous Reynolds number $= u_m D / \nu$
 Re_{ta} : time-averaged Reynolds number
 $= u_{m,ta} D / \nu$
 r : radial distance
 T : period of pulsation $= 2 \pi / \omega$
 t : time
 u : axial velocity component
 $= u_{ta} + u_{os} + u' = \bar{u} + u'$
 u^* : friction velocity $= u_m \sqrt{\lambda_g / 8}$
 x : axial distance
 $\xi_1 = \lambda_{u,ta} / \lambda_s$
 λ_g : quasi-steady friction factor
 λ_s : steady flow friction factor
 $\lambda_u(t)$: instantaneous friction factor
 $\lambda_{u,ta}$: time average friction factor
 ν : kinematic viscosity
 ρ : density of fluid
 σ_1 : modification factor $= 1 + 21 A_1^2 / 64$
 τ_w : wall shear stress
 $\Phi_{t,n}, \Phi_{z,n}, \alpha_{t,n}, \alpha_{z,n}$: characteristic parameters describing the flow pattern
 ω : angular frequency $= 2 \pi f$
 ω' : dimensionless frequency $= R^2 \omega / \nu$
 Subscripts and others
 q : quasi-steady state
 n : n th harmonic in the finite Fourier expansion
 os : oscillating component due to pulsation
 ta : time-averaged value over one cycle
 $-$: short-time-averaged value which is permitted to include the relatively low frequencies for the sinusoidal waves under consideration

3. Estimation of instantaneous friction factor $\lambda_u(t)$ and time average friction factor $\lambda_{u,ta}$

In the third report⁽¹⁾ of this research series the characteristic parameters $\Phi_{t,1}$, $\Phi_{z,1}$, $\alpha_{t,1}$, and $\alpha_{z,1}$ in the incompressible pulsating turbulent pipe flow are described as functions of $\omega' / Re_{ta}^{3/4}$. Now, when the cross-sectional mean velocity is represented as

$$u_m = u_{m,ta} + |u_{m,os,1}| \cos \omega t \quad (1)$$

the wall shear stress τ_w is written by using these parameters as

$$\tau_w = \lambda_g \frac{\rho_0}{8} u_{m,ta}^2 \sigma_1 + \frac{D}{4} \frac{\Phi_{t,1}}{\Phi_{t,1}} \rho_0 \omega |u_{m,os,1}| \cos(\omega t + \alpha_{t,1} - \alpha_{z,1}) \quad (2)$$

or as

$$\tau_w = \lambda_g \frac{\rho_0}{8} u_{m,ta}^2 \sigma_1 + \frac{D}{4} \frac{\Phi_{t,1}}{\Phi_{t,1}} \rho_0 \omega |u_{m,os,1}| [\cos(\omega t + \alpha_{t,1} - \alpha_{z,1}) + B_1 \cos(2\omega t + \alpha_{t,1} - \alpha_{z,1}) - B_2 \cos(3\omega t + \alpha_{t,1} - \alpha_{z,1})] \quad (3)$$

where

$$B_1 = \frac{21}{64} A_1^2 / \left(\frac{7}{4} A_1 - \frac{21}{512} A_1^3 \right), \quad B_2 = \frac{7}{512} A_1^3 / \left(\frac{7}{4} A_1 - \frac{21}{512} A_1^3 \right)$$

$$\sigma_1 = 1 + \frac{21}{64} A_1^2, \quad A_1 = |u_{m,os,1}| / u_{m,ta} < 1, \quad \lambda_g = 0.3164 / Re_{ta}^{1/4}$$

.....(4)

Equation (3) was derived from Eq.(2). This coincides with the following quasi-steady relation

$$\tau_w = \lambda_g \frac{\rho_0}{8} u_m^2, \quad \lambda_g = 0.3164 / Re^{1/4}$$

as the dimensionless frequency ω' decreases.

The instantaneous friction factor $\lambda_u(t)$ and the time average friction factor $\lambda_{u,ta}$ are defined respectively below

$$\tau_w u_m = \lambda_u(t) \frac{\rho_0}{8} u_m^3 \quad (5)$$

$$\frac{1}{T} \int_0^T \tau_w u_m dt = \lambda_{u,ta} \frac{\rho_0}{8} u_{m,ta}^3 \quad (6)$$

These friction factors $\lambda_u(t)$ and $\lambda_{u,ta}$ can be estimated by substituting Eqs.(2), (3), and (1) into Eqs.(5) and (6), and they are subsequently termed the estimated values.

The instantaneous friction factor $\lambda_u(t)$ calculated from Eqs.(1), (2), and (5) is represented in the form of the ratio to the quasi-steady friction factor λ_g below

$$\frac{\lambda_u(t)}{\lambda_g} = \left[\sigma_1 + 25.28 \frac{\Phi_{t,1}}{\Phi_{t,1}} \frac{\omega'}{Re_{ta}^{3/4}} A_1 \cos(\omega t + \alpha_{t,1} - \alpha_{z,1}) \right] / (1 + A_1 \cos \omega t)^{7/4} \quad (7)$$

and from Eqs.(1), (3), and (5) as

$$\frac{\lambda_u(t)}{\lambda_g} = \left\{ \sigma_1 + 25.28 \frac{\Phi_{t,1}}{\Phi_{t,1}} \frac{\omega'}{Re_{ta}^{3/4}} A_1 [\cos(\omega t + \alpha_{t,1} - \alpha_{z,1}) + B_1 \cos(2\omega t + \alpha_{t,1} - \alpha_{z,1}) - B_2 \cos(3\omega t + \alpha_{t,1} - \alpha_{z,1})] \right\} / (1 + A_1 \cos \omega t)^{7/4} \quad (8)$$

By putting $\sqrt{\omega'} = 20$ and $A_1 = 0.4$, for example, in Eqs.(7) and (8) and reading the values of characteristic parameters off the flow pattern diagrams, $\lambda_u(t) / \lambda_g$ can be estimated at several time-averaged Reynolds numbers, as shown in Fig.1. The values of $\sqrt{\omega' / Re_{ta}^{3/4}}$ calculated from the combination of ω' and Re_{ta} , shown in Fig.1, range from 0.249 to 0.543. They are within the intermediate region defined in the third report.⁽¹⁾ It is evident that $\lambda_u(t) / \lambda_g$ becomes unity as the time-averaged Reynolds number increases and that $\lambda_u(t) > \lambda_g$ in the accelerating zone except the first part where $\lambda_u(t) < \lambda_g$. However, $\lambda_u(t) < \lambda_g$ in the decelerating zone except the first part where $\lambda_u(t) > \lambda_g$. As $\omega' / Re_{ta}^{3/4}$ decreases, the flow comes into quasi-steady state over one cycle, then the accuracy of Eq.(7) becomes unsatisfactory at large A_1 value, but that of Eq.(8) is not affected.

It should be noted according to the results discussed in the second report⁽²⁾ that Eq.(7) can be applied in the whole region of ω' when A_1 is rather small and in the inertia dominant region when A_1 is even large. Equation (8) is applicable in the quasi-steady and the intermediate regions regardless of the value of A_1 . However, the applicability of Eq.(8) in the

inertia dominant region cannot be understood at present.

The time average friction factor $\lambda_{u,ta}$ can be estimated from Eqs.(1), (2), and (6) as follows:

$$\zeta_1 = \frac{\lambda_{u,ta}}{\lambda_s} = \sigma_1 + 12.64 \frac{\phi_{t,1}}{\phi_{t,1}'} \frac{\omega'}{Re_{ta}^{3/4}} A_1^2 \cos(\alpha_{t,1} - \alpha_{s,1}) \quad (9)$$

where λ_s is the steady flow friction factor shown in Eq.(4). Replacement of Eq.(2) with Eq.(3) gives the same results as Eq.(9). By using the following momentum balance equation

$$\rho_0 \frac{du_m}{dt} + \frac{4}{D} \tau_w = \frac{\Delta p}{l} \quad (10)$$

Equation (9) is rewritten after simple rearrangement(2) as shown below

$$\zeta_1 = \sigma_1 + 12.64 \frac{\omega'}{Re_{ta}^{3/4}} \frac{A_1^2}{\phi_{t,1}} \cos \alpha_{t,1} \quad (11)$$

The estimated value of ζ_1 is shown in Fig. 2 for A_1 as a parameter. It increases monotonically with an increasing $\omega'/Re_{ta}^{3/4}$. Since the reading uncertainties of the characteristic parameters from the flow pattern diagrams are large when $\sqrt{\omega'/Re_{ta}^{3/4}} > 2$, the estimation of ζ_1 becomes practically impossible in this region.

4. Comparison of experimental and estimated values of $\lambda_u(t)$ and $\lambda_{u,ta}$

Details on the experimental facilities, procedure, and conditions are reported elsewhere.(1)

4.1 Instantaneous friction factor $\lambda_u(t)$

Experimental values of u_m , $\Delta p/l$, and resultant $\lambda_u(t)$ in the intermediate region are shown together in Figs.3 and 4. It is noteworthy that the time-averaged Reynolds number is much different in each figure, while $\sqrt{\omega'}$ has almost a constant value of 22. The solid lines in Figs.3(a) and 4(a) denote that the finite Fourier expansions with fundamental wave for $\Delta p/l$, u_m , and τ_w . It is seen in Fig.3 that u_m is well fitted by the fundamental wave while $\Delta p/l$ and τ_w distribute around an individual finite Fourier expansion with a small scattering. However, the experimental values of $\lambda_u(t)$ are well fitted by the fundamental wave(2) and agree favourably with the estimated values. The behaviour of $\lambda_u(t)$ in the accelerating and decelerating zones described in the preceding chapter is observed here clearly.

An example of measurement in the inertia dominant region is not shown here, because the uncertainty of measuring τ_w in this region is large.

4.2 Time average friction factor $\lambda_{u,ta}$

Comparison of the experimental values of $\lambda_{u,ta}$ with the estimated values in a man-

Fig.1 Estimated values of $\lambda_u(t)/\lambda_s$ over one cycle as a parameter Re_{ta}

Fig.2 Estimated values of $\lambda_{u,ta}/\lambda_s$ as a function of $\omega'/Re_{ta}^{3/4}$

(a) $\Delta p/l$, u_m , τ_w

(b) Instantaneous friction factor $\lambda_u(t)$

(c) Comparison of estimated and experimental values of $\lambda_u(t)/\lambda_s$

Fig.3 Instantaneous friction factor in the intermediate region

ner shown in Fig.2 has no advantage, because the individual velocity amplitude ratio in each run is different. Therefore, by paying attention to the second term of the right hand side of Eq.(9), the experimental values of $(\xi_1 - \delta_1)/A_1^2$ are compared with the estimated values of

$$12.64 \frac{\Phi_{z,1}}{\Phi_{t,1}} \frac{\omega'}{Re_{ta}^{3/4}} \cos(\alpha_{t,1} - \alpha_{z,1})$$

and the results are shown in Fig.5. Although the experimental values are slightly scattered, the agreement of experimental and estimated value is good. Accordingly, if the time-averaged Reynolds number Re_{ta} , dimensionless frequency ω' , and velocity amplitude ratio A_1 are given, then the frictional losses over one cycle can be evaluated readily.

Baird, et al.⁽³⁾ measured frictional losses over one cycle for $Re_{ta} = (1 \sim 2) \times 10^5$, $\sqrt{\omega'} = 44 \sim 57$, and shows that the experimental values agreed well with the values calculated from the quasi-steady relation between u_m and τ_w . On the basis of these results they concluded that the quasi-steady state held within their experimental range.

Now, combination of Re_{ta} and ω' in their experiments gives the maximum value of $\sqrt{\omega'}/Re_{ta}^{3/4}$ as about 0.6. This means that the quasi-steady state holds at least in the $\sqrt{\omega'}/Re_{ta}^{3/4}$ region of less than 0.6, and this result coincides with the present experimental results as seen in Fig.5.

5. Relation between the distribution of axial velocity components $\bar{u}$ and the behavior of instantaneous friction factor during accelerated and decelerated motions

The distribution of $\bar{u}$ versus r/R and the universal velocity distribution for different values of $\sqrt{\omega'}/Re_{ta}^{3/4}$ are shown in Figs.6(a), 7(a), and 8(a) and Figs.6(b), 7(b), and 8(b) respectively. In addition, the changes of u_m over one cycle are summarized in Fig.9, where $\omega t/(\pi/6) = 0 \sim 6$ and $6 \sim 12$ denote the accelerating and the decelerating zones respectively. It should be noted that all data were obtained from experiments with air and that data in Fig.6, 7, and 8 belong to the quasi-steady, the intermediate, and the inertia dominant regions respectively. In these figures the friction velocity was not calculated from the experimental values of τ_w , but according to the quasi-steady relationship $u^* = u_m \sqrt{\lambda_g/8}$.

The experimental values of $\bar{u}$ for 0.00217 Hz are well fitted by the universal velocity distribution as seen in Fig. 6(b). That leads to a conclusion that the quasi-steady state holds at any instant in a cycle.⁽⁴⁾ It can slightly be seen in Fig.7(b) that the measured distribution becomes flat in the accelerating zone where $\omega t/(\pi/6) = 0 \sim 6$ while it becomes sharp in the decelerating zone where $\omega t/(\pi/6) = 6 \sim 12$. Here "flat" means that the gradient of measured velocity distribution

(a) $\Delta p/l, u_m, \tau_w$

(b) Instantaneous friction factor $\lambda_u(t)$

(c) Comparison of estimated and experimental values of $\lambda_u(t)/\lambda_g$

Fig.4 Instantaneous friction factor in the intermediate region

Fig.5 Comparison of experimental values of $(\xi_1 - \delta_1)/A_1^2$ and estimated values of $12.64 (\Phi_{z,1}/\Phi_{t,1}) (\omega'/Re_{ta}^{3/4}) \cos(\alpha_{t,1} - \alpha_{z,1})$

Fig.6 Measured velocity distribution in the quasi-steady region ($\sqrt{\omega'/Re_{ta}^{3/4}} = 0.0120$, $f = 0.00217$ Hz)

Fig.7 Measured velocity distribution in the intermediate region ($\sqrt{\omega'/Re_{ta}^{3/4}} = 0.359$, $f = 2.079$ Hz)

Fig.8 Measured velocity distribution in the inertia dominant region ($\sqrt{\omega'/Re_{ta}^{3/4}} = 1.71$, $f = 48.0$ Hz)

Fig.9 Cross-sectional mean velocity u_m over one cycle

is smaller than 5.75 and "sharp" does that it is larger. In Fig.8(b) this trend is recognized more clearly. That is, in the accelerating zone a sharp distribution in the first part changes suddenly into a flat distribution in the rest while in the decelerating zone the change is from flat to sharp. On the other hand, u^* is calculated according to the relationship $u^* = u_m \sqrt{\lambda_q/8}$, so that the instantaneous wall shear stress τ_w cannot be evaluated quantitatively from these figures. However, a qualitative evaluation of $\tau_w/\tau_{w,q}$ at a given instant is possible. The flat velocity distribution means that the wall shear stress τ_w is larger than that for the equivalent steady state, i.e., the quasi-steady value $\tau_{w,q}$. Conversely, sharp distribution means that $\tau_w < \tau_{w,q}$.

Since the relationship $\tau_w/\tau_{w,q} = \lambda_u(t)/\lambda_q$ is derived from the definition of instantaneous friction factor, it is concluded that in the accelerating zone $\lambda_u(t) < \lambda_q$ in the first part and $\lambda_u(t) > \lambda_q$ in the rest while in the decelerating zone $\lambda_u(t) > \lambda_q$ in the first part and $\lambda_u(t) < \lambda_q$ in the rest. This agrees well with the result obtained in the preceding chapter.

6. The limits between the quasi-steady, the intermediate, and the inertia dominant regions

6.1 The limit between the quasi-steady and the intermediate regions

The method of determining the values of $\sqrt{\omega'/Re_{ta}^{3/4}}$ indicating the limit between the quasi-steady and the intermediate regions, i.e., $\sqrt{(\omega'/Re_{ta}^{3/4})_q}$ on the basis of the characteristic parameters $\Phi_{t,1}$, $\Phi_{z,1}$ is proposed in the third report.⁽¹⁾ Then it was shown that $\sqrt{(\omega'/Re_{ta}^{3/4})_q} = 0.06$ when $\Phi_{t,1} = 0.05$ and 0.145 when $\Phi_{z,1} = 0.95$, but its applicability was not mentioned. Here the determination about which criterion is better is made by observing the relationship between the experimental values of $\lambda_u(t)$ and λ_q . The experimental values of $\lambda_u(t)$ when $\sqrt{\omega'/Re_{ta}^{3/4}}$ lies around 0.06 and 0.145 are shown in Fig.10. Although it should be noted that the velocity amplitude ratio A_1 in each run is different and that the ef-

Fig.10 Instantaneous friction factor near quasi-steady and intermediate regions

fects of ω' and Re_{ta} on $\lambda_u(t)$ become unnoticeable as A_1 is decreased, it is likely from Fig.10 that the quasi-steady state holds approximately up to $\sqrt{\omega'/Re_{ta}^{3/4}} = 0.145$. Accordingly it would be appropriate to adopt the limit where $\sqrt{\omega'/Re_{ta}^{3/4}} = 0.145$.

6.2 The limit between the intermediate and the inertia dominant regions

As the pressure gradient term and the inertia term balance each other in the inertia dominant region, the momentum balance equation for the oscillating component becomes

$$\frac{\Delta p - \Delta p_{ta}}{l} = \rho_0 \frac{du_m}{dt} \quad (12)$$

Let's examine whether or not the data in the region above $\sqrt{(\omega'/Re_{ta}^{3/4})_t} = 1.50$ satisfy Eq. (12). Equation (12) is nondimensionalized by dividing its both members with $\rho_0 u_m^2 / (2D)$. Experimental values of $2D(\Delta p/l - \Delta p_{ta}/l) / (\rho_0 u_m^2)$ are plotted against $2D(du_m/dt) / u_m^2$ in Fig.

11. As they are well fitted by a straight solid line when $\sqrt{\omega'/Re_{ta}^{3/4}} \geq 1.50$, it seems appropriate to indicate the limit between the intermediate and the inertia dominant regions by $\sqrt{(\omega'/Re_{ta}^{3/4})_t} = 1.50$.

Fig.11 Relation between pressure gradient and inertia terms near intermediate and inertia dominant regions

6.3 Comparison of the present results with others on the limits between the three regions

Brown, et al.⁽⁵⁾ showed that the pulsating turbulent pipe flow is classified into three kinds with respect to frequency f : namely low, intermediate, and high frequency regions on the basis of the experimental results available for steady turbulent flow. These three regions are considered to correspond to the regions defined in the present third report.⁽⁷⁾ The limits between these three regions are indicated by the relationships

$$\omega_q' = 0.014 Re_{ta}^{0.84} \dots (13)$$

$$\omega_t' = 0.00025 Re_{ta}^{1.6} \dots (14)$$

Funk, et al.⁽⁶⁾ determined ω_q' using a single boundary layer model analysis as follows:

$$\omega_q' = 0.00105 Re_{ta}^{1.5} \dots (15)$$

Baird, et al.⁽³⁾ proposed ω_q' as

$$\omega_q' = 0.025 Re_{ta} \dots (16)$$

referring to the results of Brown, et al.⁽⁵⁾ One of the present authors obtained ω_q' numerically on the basis of the analysis of velocity distribution with time-dependent friction velocity which varies in a quasi-steady manner.⁽⁷⁾

The present results are summarized as follows:

$$\omega_q' = 0.00360 Re_{ta}^{3/4} (\phi_{t,1} = 0.05) \dots (17)$$

$$\omega_q' = 0.0210 Re_{ta}^{3/4} (\phi_{t,1} = 0.95) \dots (18)$$

$$\omega_t' = 2.25 Re_{ta}^{3/4} (\phi_{t,1} = 0.95) \dots (19)$$

for the time-averaged Reynolds number smaller than 10^5 . For the region where $Re_{ta} > 10^5$ it is reasonable to express Eqs. (17)~(19) in the following forms:

$$\omega_q' = 0.0114 Re_{ta} \lambda_s \dots (20)$$

$$\omega_q' = 0.0665 Re_{ta} \lambda_s \dots (21)$$

$$\omega_t' = 7.11 Re_{ta} \lambda_s \dots (22)$$

where λ_s must be calculated from the well-known relationship

$$1/\sqrt{\lambda_s} = 2.0 \log_{10} (Re_{ta} \sqrt{\lambda_s}) - 0.8$$

Figure 12 shows the results described above. The limits of Brown, et al.⁽⁵⁾ are similar to those of the present work. However, ω_q' of Funk, et al.⁽⁶⁾ and of Baird, et al.⁽³⁾ are seen to correspond to ω_q' proposed here. It is found from Fig.12 also that Eq.(18) should be used for the limit between the quasi-steady and the intermediate regions.

Fig.12 Limits between quasi-steady, intermediate, and inertia dominant regions

7. Conclusions

A summary of the present investigation is given below.

(1) The instantaneous friction factor $\lambda u(t)$ and the time average friction factor $\lambda_{u,ta}$ are described systematically as

functions of $\omega'/Re_{ta}^{3/4}$.

(2) Closer agreement between experimental and estimated values was obtained for each friction factor.

(3) The instantaneous friction factor $\lambda_u(t)$ and the quasi-steady friction factor λ_g are almost equal in the quasi-steady region which is denoted by $\sqrt{\omega'/Re_{ta}^{3/4}} \leq 0.145$. With an increasing $\omega'/Re_{ta}^{3/4}$, $\lambda_u(t)$ becomes smaller than λ_g in the first part and larger in the rest of the accelerating zone. On the other hand, $\lambda_u(t) > \lambda_g$ in the first part and $\lambda_u(t) < \lambda_g$ in the rest of the decelerating zone.

(4) The time average friction factor $\lambda_{u,ta}$ is larger than the steady flow friction factor λ_s and it increases with an increasing $\omega'/Re_{ta}^{3/4}$.

(5) It is appropriate to express the limit between quasi-steady and intermediate regions by $\sqrt{(\omega'/Re_{ta}^{3/4})_g} = 0.145$ and the limit between intermediate and inertia dominant regions by $\sqrt{(\omega'/Re_{ta}^{3/4})_t} = 1.50$, at $Re_{ta} \leq 10^5$.

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