Theory of the Cut-size of a Rotary Flow Dust Collector

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The flow pattern in a rotary flow dust collector was changed on a large scale by the flow rates of the primary flow $Q_1$ and the secondary flow $Q_2$. Considering the flow characteristics of a turbulent rotational flow, one of the authors newly derived theoretical formulae for the cut-sizes which were determined by another conception in comparison with the derivative means of the cut-size of the ordinary types of cyclones. In these results, the authors confirmed that the relationship between the experimentally determined cut-size $Xc_{E}$ and the theoretically calculated cut-size $Xc$ showed good coincidence in the range of flow rates $Q_2/Q_1 = 1.0 \sim 1.5$.

1. Introduction

The separation mechanism of fine solid particles and the flow pattern of air in a rotary flow dust collector shows very different conditions from those of the ordinary types of the tangential cyclones. Fig. 1 illustrates the separation mechanism of fine solid particles, the flow systems of the primary dust-laden gas and of the secondary circulating flow and also the total construction of the rotary flow dust collector.

The primary dust-laden gas is thrown as an upward swirling jet into the main cylindrical separation chamber from the primary vortex chamber, and then during drawing into the inner pipe of the diameter D2 (29.5 mm), fine solid particles are separated by the centrifugal force in the primary vortex flow, finally the separated solid particles settle to the dust-bunker following the descending air of the secondary rotational flow.

On the other hand, in order to increase the magnitude of the rotation of the primary vortex flow and the effect of the centrifugal separation, a secondary pure air is introduced into the main separation chamber of the diameter Ds (150 mm) through two nozzles. And this secondary air flows in through the co-axial slit between the inner pipe and the outer pipe, then it re-circulates...
again to the secondary air nozzles by passing through a blower.

In general the cut-size $X_{c0}$ can be defined as the particle diameter which corresponds to the value of the fractional collection efficiency $\eta_X$ to be 0.5 (50%), and also the theoretical cut-size $X_c$ can be defined as the imaginary particle diameter which can be attained at the edge of the inner pipe located at the height $H$ from the primary vortex chamber.

In this paper, the correlation between a newly derived theoretical cut-size $X_c$ and an experimentally determined cut-size $X_{c0}$ and also the effects which are influenced by the flow rates of the primary dust-laden gas $Q_1$ and of the secondary pure air $Q_2$ on the cut-size $X_{c0}$ are discussed in detail.

2. Symbols (SI units are used)

- $D_1$: outer diameter of the rotary flow dust collector (m)
- $D_2$: (2$R_1$) diameter of the inner pipe (m)
- $D_s$: diameter of the slit (m)
- $H$: height from the primary vortex chamber to the inner pipe (m)
- $k_1$: supplementary value of the tangential velocity (1)
- $Q_1$: flow rate of the primary dust-laden gas (m$^3$/s)
- $Q_2$: flow rate of the secondary pure air (m$^3$/s)
- $r$: radius from the center axis (m)

Fig.1 Illustration for the flow system and for the separation mechanism of a rotary flow dust collector
Fig. 2 Illustration for defining the cut-size

- $R_m$: representative radius of the primary vortex chamber (m)
- $R_{n1}$: radius of the exit nozzle of the primary vortex chamber (m)
- $V_i$: mean inlet velocity in the inlet pipe (m/s)
- $V_z$: mean axial velocity of the primary vortex chamber (m/s)
- $V_{t}$: tangential velocity of air (m/s)
- $V_{t2}$: tangential velocity of air at the radius $r = R_{n1}$ (m/s)
- $V_i$: mean exit velocity from the exit nozzle of the primary vortex chamber (m/s)
- $V_{z2}$: mean axial velocity of the primary vortex flow near the inner pipe (m/s)
- $V_{z5}$: mean axial velocity of the secondary air flow in the slit (m/s)
- $V_{z59}$: representative mean axial velocity (m/s)
- $X_c$: theoretically calculated cut-size (μm)
- $X_{cs9}$: experimentally determined cut-size

Fig. 3 Illustration of the primary vortex chamber and of the main axial velocities

- $\eta$: fractional collection efficiency (%)
- $\eta$: viscosity of air (Pa·s)
- $\delta$: shape factor of the solid particles (1)
- $\rho$: density of the test dust (Kg/m$^3$)
- $\omega$: angular velocity of the primary vortex flow (rad/s)

3. Theoretical equation of the cut-size

3.1 Fundamental equation for the cut-size
velocity of the primary vortex flow. This equation (1) will be derived in Appendix in detail.

3.2 Relationship between the tangential velocity of the primary vortex flow and the construction of the primary vortex chamber

As shown in Fig.3, denoting by $R_m$ the representative radius of the inlet pipe of the vortex chamber and by $V_i$ the mean inlet velocity in the inlet pipe, the relation between the radius $r$ and the tangential velocity $V_r$ near the exit radius $R_v$ of the vortex chamber may be described in the region where the theorem of the conservation of angular momentum is applicable, as follows;

$$k_l V_r r = R_m V_i$$  \hspace{1cm} (2)

where $k_l$ is a constant which can

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4}
\caption{Main sizes of a rotary flow dust collector}
\end{figure}

At a talk in the congress \textit{(1979, July)} of fluid engineering and fluid machine (JSME) at Muroran city, the cut-size is defined as the particle diameter that can be reached at the edge ($r = R_2$) of the inner pipe of height $H$ from the radius $r$ of the vortex chamber as shown in Fig.2.

The equation for this can be described as follows:

$$X_c(r) = \sqrt{(187 \xi V_r / \mu \omega^5 H)} \cdot \ln(R_2 / r) \quad \hspace{1cm} (1)$$

where $\xi$ is the shape factor of the solid particle, $\mu$ is the viscosity of gas, $\rho_p$ is the density of solid particles, and $\omega$ is the angular

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5}
\caption{Calculated results of the cut-sizes $X_c$ for $H=466$ mm}
\end{figure}
be determined by the ratios of the primary flow rate $Q_1$ and the secondary flow rate $Q_2$. Then the relationship between $k_1, Q_1$ and $Q_2$ can be written experimentally as follows:

$$k_1 = \frac{Q_1}{Q_2}$$  \hspace{1cm} (3)

Therefore the tangential velocity $V_{\text{tang}}$ of the primary vortex flow at the exit nozzle of the radius $R_{\text{v}}$ can be written as follows:

$$V_{\text{tang}} = \frac{R_{\text{m}} V_1}{k_1 R_{\text{v}}} = \frac{R_{\text{m}} Q_2 V_1}{Q_1 R_{\text{v}}}$$  \hspace{1cm} (4)

so the equation for the angular velocity $\omega$ of the primary vortex flow can be written as

$$\omega = \frac{1}{R_{\text{v}}} \frac{R_{\text{m}} Q_2 V_1}{R_{\text{v}} Q_1 R_{\text{v}}}$$  \hspace{1cm} (5)

3.3 Representative axial velocity $\bar{V}_z$ of the primary vortex flow

As shown in Fig.3, the mean axial velocity $V_1$ at the exit nozzle of the primary vortex chamber can be written as $V_1 = 4 \frac{Q_1}{\pi D_{\text{m}}}$ and the mean axial velocity $V_{2\text{m}}$ of the primary flow entering into the inlet pipe of the diameter $D_2$ can be written as $V_{2\text{m}} = 4 \frac{Q_1}{\pi D_2}$. Then denoting the flow rate of the secondary air by $Q_2$, the mean axial velocity $V_{2\text{s}}$ of the secondary air flowing through the slit between the diameters $D_2$ and $D_2$ can be written as $V_{2\text{s}} = 4 \frac{Q_2}{\pi (D_2 - D_2)}$.

Consequently, the mean axial velocity of the primary vortex flow near the inner pipe may be assumed as $\bar{V}_{z\text{m}} = (V_{2\text{m}} + V_{2\text{s}}) / 2$. This equation can be re-written as

$$\bar{V}_{z\text{m}} = \frac{V_{2\text{m}} + V_{2\text{s}}}{2} =$$

- Fig.6: Relationships between $X_c$ and $X_{c50}$ for a rotary flow dust collector

$$= \frac{2Q_1}{\pi D_2} \left( 1 + \frac{Q_2}{Q_1} \right) \left( \frac{D_2}{D_2} - 1 \right)$$  \hspace{1cm} (6)

Therefore the mean axial velocity $\bar{V}_z$ along the effective height $H$ can be written as follows:

$$\bar{V}_{z\text{m}} = \frac{2Q_1}{\pi D_{\text{m}}} \left( 1 + \frac{1}{2} \left( \frac{D_2}{D_2} \right) \left( \frac{Q_2}{Q_1} \right) \right)$$  \hspace{1cm} (7)

3.4 Theoretical formulae for the cut-sizes

Substituting eqs. (5) and (7) into eq. (1), and arranging eq. (1), the following equation can be obtained as

$$X_c(\tau) = \frac{R_{\text{v}}}{R_{\text{m}}} \frac{Q_2}{V_1} \frac{3}{4} \times$$

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Here defining the starting radius positions at the exit nozzle of the primary vortex chamber as \( r = Rv_1 / 2 \) and \( r = 3Rv_1 / 4 \), the following cut-size equations can be obtained, respectively,

\[
X_c \left( \frac{1}{2} Rv_1 \right) = \Phi \frac{Rv_1}{Rm_1} \cdot \frac{Q_1}{Q_2} \frac{3}{V_1} \cdot \ln 2
\]

\[
X_c \left( \frac{3}{4} Rv_1 \right) = \Phi \frac{Rv_1}{Rm_1} \cdot \frac{Q_1}{Q_2} \frac{3}{V_1} \cdot \ln 4 \cdot \frac{1}{3}
\]

4. Numerical example of the cut-size

A detailed construction of the rotary flow dust collector is shown in Fig.4, and the values of the cut-sizes can be calculated by eqs. (9) and (10). The necessary numerical values are as follows; \( Rm_1 = 28.8 \) mm, \( Dv_1 = 29.5 \) mm, \( D_2 = 29.5 \) mm, \( D_3 = 80 \) mm, \( H = 466 \) mm, the density of fly-ash \( \rho = 2.14 \) g/cm\(^3\) and the viscosity of air \( \eta = 1.814 \times 10^2 \) Pa·s, and the value of \( \Phi \) could be determined as \( \Phi = 1.6 \) by the experimental results of the tangential velocities. And also a shape factor \( \xi \) of the solid particle is supposed as \( \xi = 1 \). The calculated results of the cut-sizes are shown in Fig.5 for the cases of \( Q_2 / Q_1 = 0.2, 0.5, 1.0, 1.5 \) and 2.0 and also in this figure the cut-sizes \( X_c \) (\( r = Rv_1 / 2 \)) and \( X_c \) (\( r = 3Rv_1 / 4 \)) are shown as solid lines and dotted lines, respectively.

5. Comparison of the experimental results of the cut-sizes \( X_{C_90} \) with those of \( X_c \).

Here the relationships between the calculated cut-size \( X_c \) and the experimentally determined cut-size \( X_{C_90} \) which can be estimated by the fractional collection efficiency are discussed. Fig.6(a) and (b) show the cut-sizes for \( H = 466 \) mm and 266 mm, respectively. The starting radii positions in the exit nozzle of the primary vortex chamber are \( r = Rv_1 / 2 \) and \( r = 3Rv_1 / 4 \), respectively. The comparisons of the calculated cut-size \( X_c \) with the experimentally determined cut-size \( X_{C_90} \) for fly-ash as the test dust are shown in these figures. From these results, it is sufficiently possible to estimate the cut-size \( X_c \) by eqs. (9) and (10) in comparison with the experimentally determined cut-size \( X_{C_90} \) in the domain \( Q_2 / Q_1 = 1.0 \sim 1.5 \).

Because the actual flow pattern in this collector coincides with the assumed flow pattern by the authors in the domain \( Q_2 / Q_1 = 1.0 \sim 1.5 \) in connection with the tangential velocities \( \omega \) and the axial velocities \( V_z \) which are concerned with the flow rates \( Q_1 \) and \( Q_2 \). Especially, it is made clear that the separation mechanism for fine solid particles is largely hindered and also the pressure drop is increased by the generation of eddies due to the velocity differences at the mixing plane, where the flow of the primary vortex layer contacts with the secondary rotational air flow.

6. Conclusions

New formulae for the cut-sizes of a rotary flow dust collector which is different in the separation mechanism and in the flow pattern from the ordinary types of cyclones are derived. For the derivation of these equations, the ratios \( Q_2 / Q_1 \) of the flow rates \( Q_2 \) and \( Q_1 \) are the most
important factor concerning the tangential and axial velocities. And then one of the interesting facts is that the cut-sizes $X_c$ are independent of the outer diameter $D_1$ by eqs. (9) and (10). And also the coincidence between the calculated values $X_c$ and the experimentally determined values $X_c^o$ is very good in the range of $Q_2/Q_1 = 1.0 \sim 1.5$.

Moreover, from a point of view of the pressure drop, this range of the ratios of the flow rates is very useful.

References

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(4) Nickel, W., Staub, Bd. 23, Nr. 11 (1963) s. 509/512.

APPENDIX

From a fluid dynamical point of view, assuming that Reynolds' number $Rex(Ur \cdot Xp/V)$ around a sphere may be regarded as $Rex \leq 4$, so that a sphere accepts Stokes' drag for radial direction, the equation (1) can be derived as follows. Here denoting the velocity vector of fluid by $\mathbf{V}$ ($V_\Phi$, $V_r$, $V_z$) and that of the solid particle by $\mathbf{U}$ ($U_\Phi$, $U_r$, $U_z$), the equation of motion for the solid particle in the radial direction can be written as

$$\frac{\rho \pi X_p^2}{6} \left( \frac{dU_r}{dt} - \frac{U_r}{r} \right) = -3\pi \gamma X_p \frac{V_r}{r} (Ur \sim V_r)$$

(A-1)

where suffixes $\Phi$, $r$ and $z$ denote the tangential, radial and axial components, respectively.

In order to simplify the above equation, assumed that the solid parcles are in a quasi-steady motion in the radial direction, and the term $dU_r/dt$ is nearly zero. Therefore eq. (A-1) becomes

$$\frac{\rho \pi X_p^2}{6} \frac{V_r^2}{r} = 3\pi \gamma X_p \frac{V_r}{r} (Ur \sim V_r)$$

(A-2)

Then the rotation of the primary vortex flow mainly depends on the ratios of flow rates $Q_2/Q_1$, but considering a flow model of the rotational cylindrical shell which flows upwards, the radial velocity of flow in the primary vortex flow is assumed zero as $V_r \approx 0$. Then eq. (A-2) becomes

$$\frac{\rho \pi X_p^2}{6} \frac{V_r^2}{r} = 3\pi \gamma X_p \frac{U_r}{r}$$

(A-3)

In this equation, the radial velocity $U_r$ of the solid particle can be transformed as

$$U_r = \frac{dr}{dt} = \frac{dr}{dz} \frac{dz}{dt} = U_z \frac{dr}{dz}$$

(A-4)

so eq. (A-3) becomes

$$U_z \frac{dr}{dz} = \frac{\rho \pi X_p}{18 \gamma \frac{\pi}{r}} \frac{V_r^2}{r}$$

(A-5)

Still more, it is assumed that the tangential and axial velocities of the solid particles are equal to those of the fluid velocities, and also that the axial velocity $V_z$ of the primary vortex flow goes up nearly with a constant velocity $V_z$ from the vortex chamber to the edge of the inner pipe. Now from the experimental results of the tangential velocities in the primary vortex flow, the tangential velocity $V_\Phi$ can be written approximately as follows,

$$V_\Phi = \omega r$$

(A-6)

Substituting eq. (A-6) into eq. (A-5), the equation of the cut-size $X_c$ which satisfies the boundary conditions (starting position, $z=0$, $r=r$; final position, $z=H$, $r=R$) can be obtained as follows,

$$ln \frac{R^2}{r} = \frac{\rho \pi X_p \omega^2 H}{18 \gamma \frac{\pi}{r} V_z}$$

(A-7)
Therefore eq. (A-7) can be transformed as

\[ X_c(r) = \frac{1876 V_z}{N \beta_0 f H} \ln \frac{R_e}{r} \quad (A-8) \]

This equation shows a good coincidence with eq. (1), where \( X_c(r) \) denotes the cut-size corresponding to the starting radial positions at \( r = r_e \) and \( z = 0 \).