Turbulent Near Wake of a Flat Plate*  
(Part 3. Effect of Pressure Gradients)

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The turbulent near wakes starting from the turbulent boundary layers on both sides of a flat plate and subjected to pressure gradients have been investigated. The experiments have been conducted in zero, adverse, favourable and mixed pressure gradients, and the results show that the total pressure along a streamline of the near wake in a pressure gradient changes at a rate nearly equal to that along the same streamline in zero pressure gradient, which suggests that the transverse gradients of Reynolds stress are little affected by pressure gradients in the turbulent near wakes. On the basis of the experimental results a model to predict the development of a near wake in a pressure gradient has been presented. The calculated results agree well with the experimental data except for a strong adverse pressure gradient case.

1. Introduction

The near wakes are of great importance for engineering applications and a number of works on the phenomena have been reported. The flows are concerned with the vortex formation and shedding for bluff bodies and with the development of the near wakes from boundary layers for thin bodies. The near wakes behind thin bodies may be classified into two types: the laminar near wakes and the transition from laminar boundary layers, and the turbulent near wakes from turbulent boundary layers. From the practical point of view, since the boundary layers are mostly turbulent, the turbulent near wakes are more important.

The previous experimental and numerical works on the problem were mostly conducted on the flows in zero pressure gradient. The wakes behind cascades of turbomachines and aerfoils of aircrafts are often subjected to pressure gradients, which seem to affect the development of the wakes. A few works dealing with the effect by Frabu, Gartshore and Narasimha were limited to the far wakes which have similar solutions. On the other hand, on the near wakes there is no similar solution because of a significant influence of boundary layers and the pressure gradient effect not being clear.

In the present study, the turbulent near wakes starting from the turbulent boundary layers on both sides of a flat plate and subjected to pressure gradients have been investigated experimentally, and a model to predict the development of a near wake in a pressure gradient has been presented on the basis of the experimental results. Comparisons of calculated and experimental results have been also shown.

2. Nomenclature

\[ b_0 \]: Half width of wake  
\[ C_1 \]: Pressure coefficient, \( (2-p_0)/2p_0 \)  
\[ H \]: Shape factor of boundary layer, \( h^*/\delta \)  
\[ n \]: Distance normal to streamline  
\[ \delta \]: Total pressure  
\[ \rho _p \]: Normalized total pressure, \( (P_r-p_0)/2p_0 \)  
\[ \rho _t \]: Total pressure in zero pressure gradient  
\[ \rho _0 \]: Normalized value of \( \rho _t \), \( (P_r-p_0)/2p_0 \)  
\[ (\hat{P})_c \]: \( \hat{P} \) at wake center  
\[ p \]: Static pressure  
\[ \rho _b \]: Static pressure of boundary layer at the trailing edge  
\[ \delta_b \]: Reynolds number, \( \rho _b u_0 /v \)  
\[ s \]: Distance along streamline  
\[ s_0 \]: Normalized streamline distance, \( s/v \delta_b \)  
\[ u \]: Streamwise velocity of wake  
\[ u_c \]: Streamwise velocity at wake center  
\[ u_s \]: Free-stream velocity of wake  
\[ u_b \]: Streamwise velocity of boundary layer  
\[ u_m \]: Free-stream velocity of boundary layer  
\[ u_v \]: Velocity defect, \( u_s-u \)  
\[ u_{fs} \]: Maximum velocity defect  
\[ z \]: Streamwise distance from the trailing edge  
\[ y \]: Transverse distance from wake center  
\[ \delta_s \]: Displacement thickness of boundary layer, \( \int_{s_0}^{s_0} [1-(u_s/u_m)] \, dy \)
\(a\): Momentum thickness of boundary layer, 
\[ \theta = \int_{0}^{y} \frac{\nu}{\nu_{w}} \left( 1 - \frac{\nu}{\nu_{w}} \right) dy \]

\(\theta_{w}\): Total momentum thickness of boundary layers on both sides of a flat plate

\(\theta_{t}\): Momentum thickness of wake

\(\alpha\): Momentum transfer coefficient

\(\nu\): Kinematic viscosity

\(\rho\): Density

\(\tau\): Reynolds stress

\(\psi\): Stream function

\(\phi\): Normalized stream function, 
\[\int_{\psi}^{\phi} u dy = U(\psi)\]

3. Experimental procedures

The experimental apparatus is shown in Fig. 1. The air delivered by an axial blower (driven by a 5.5 kw motor) enters the test section through a nozzle via the settling chamber (1000 x 1000). The streamwise turbulence intensity at the exit of the nozzle is about 0.3%. A steel plate of 1.0 mm thickness and 1200 mm length was installed, of which the rear end was machined symmetrically on both sides so that it had a straight taper of 1.5 degrees and a thickness of less than 0.1 mm at the trailing edge. Tripping rods were installed on both sides of the plate at 100 mm downstream from the leading edge in order to produce thick and stable turbulent boundary layers near the trailing edge. The flexible walls were so adjusted that various pressure gradients could be attained downstream of the plate. Mean velocities in boundary layers and wakes were determined from measured total and static pressures. It is confirmed that there is little error due to the discrepancy between the flow direction and the axes of the pressure tubes.

The measurements have been conducted in zero (FG 0), adverse (FG 1-3), favourable (FG 4, 5), and mixed (FG 6, 7) pressure gradients as shown in Fig. 2. The transverse gradients of static pressures in the wakes were small in all the experiments. In order to avoid the initial condition effect on the near wake development, the boundary layer profiles 2 mm upstream from the trailing edge were kept nearly the same in all the experiments as shown in Fig. 3. The characteristics of the boundary layers are as follows: shape factor \(H = 1.36\), momentum thickness \(\theta = 2.7 \text{ mm}\) and Reynolds number \(Re = 2500-8500\), which show the turbulent boundary layers are fully developed.

4. Numerical analysis

The equation for the turbulent flows, neglecting the viscous stress and the streamwise gradient of Reynolds stress, is

\[ \frac{\partial}{\partial x} \left( x^{-1/2} \rho u^2 \right) = \frac{\partial \tau_{\phi}}{\partial x} \tag{1} \]

Equation (1) shows that the streamwise gradient of total pressure equals the transverse gradient of Reynolds stress. In the outer region of the turbulent boundary layer, the Reynolds stress term in Eq. (1) is little affected by pressure gradients: the total pressure along a streamline in a pressure gradient changes at a rate nearly equal to that along the same streamline in zero pressure gradient. It suggests that the turbulent near wake having a similar structure to the flow in the outer region might have the characteristics described above. The measured total pressure distributions are shown in Fig. 4, where the total pressure and the stream function are normalized as follows:

\[ P = \frac{P}{P_{0}} \tag{2} \]

\[ \psi = \int_{\psi}^{\phi} u dy \tag{3} \]

We can put \(\psi = \psi(x)\) and \(\psi = \psi(y)\) since the transverse deviations of streamlines are small for all the pressure gradients. Figure 4 shows the similar changes of the total pressure distribution, suggesting that the Reynolds stress term is independent of pressure gradients along each streamline. Since the Reynolds stress is related to the transverse velocity gradient \(\partial u / \partial y\), it seems to be important to discuss the effect of pressure gradients upon \(\partial u / \partial y\). Consider the normalized transverse velocity gradient

\[ \frac{\partial u}{\partial y} \tag{4} \]
theory of turbulence due to Reichardt is

\[ \frac{\partial (w/w_u)}{\partial y/y_u} - \frac{\partial (w_{max}/w_u)}{\partial y/y_u} = \frac{(w_{max}/w_u)}{\partial x/x_u} - \frac{(w_{max}/w_u)}{\partial y/y_u} \]

The initial and boundary conditions are

\[ t = 0: \frac{w}{w_u} = \frac{w_{max}}{w_u} \]

\[ y/y_u = 0: \frac{w}{w_u} = 1 \]

where \( w/w_u \) is the velocity distribution of the boundary layer at the trailing edge. We solve Eq. (11) with the momentum integral equation for the wake in zero pressure gradient. \( \rho_1(\theta, \psi) \) is obtained by the following equations.

\[ \rho_1(\theta, \psi) = \frac{w}{w_u} \]

The momentum transfer coefficient \( C_1 \) must be determined by experimental results.

The procedure to calculate the velocity profiles of the near wake in a pressure gradient is summarized as follows:

1. \( \rho_1(\theta, \psi) \) is obtained by solving Eq. (11) with the initial condition at the trailing edge.

![Fig. 4: Total pressure distributions](image)

![Fig. 5: Velocity defect distributions](image)

![Fig. 6: Transverse velocity gradients](image)

Defined by

\[ \frac{\partial (w/w_u)}{\partial y} = \frac{\partial (w_{max}/w_u)}{\partial y} \]

The measured distributions of \( w/w_u \) and \( (w_{max}/w_u)/(w_{max}/w_u) \) show little dependency of pressure gradients, which suggests that the transverse velocity gradients in the wakes in pressure gradients are nearly the same as that in zero pressure gradient.

On the basis of the above-mentioned consideration we propose a model to predict the development of a near wake in a pressure gradient. Equation (1) leads to

\[ \rho_1(\theta, \psi) = \rho_1(0, \psi) + \int_0^x \frac{\partial \rho_1}{\partial y} dx \]

Since the distribution of \( \rho_1(\theta, \psi) \) is independent of pressure gradient as mentioned above, we get

\[ \rho_1(\theta, \psi) = \rho_1(\theta, \psi) \]

where \( \rho_1(\theta, \psi) \) is the total pressure in zero pressure gradient. Using Eqs. (2) and (3), we get

\[ \frac{w}{w_u} = \frac{\rho_1(\theta, \psi) - C_0}{C_0} \]

\[ \frac{\partial w}{\partial y} = \int_0^x \frac{\partial \rho_1}{\partial y} dx \]

The velocity profile \( w/w_u \) is obtained by the following equations.

\[ \frac{w}{w_u} = \frac{\rho_1(\theta, \psi) - C_0}{C_0} \]

\[ \frac{\partial w}{\partial y} = \int_0^x \frac{\partial \rho_1}{\partial y} dx \]

Equations (9) and (10) show that the velocity profile of the wake in a pressure gradient is predictable from \( \rho_1(\theta, \psi) \).

The development of a near wake in zero pressure gradient is calculated using Toyoda and Hiroyama's model. The governing equation obtained on the inductive
(2) By comparing the calculated and measured total pressures along the wake center line the value of \( \varepsilon \) is determined, and with \( x/\theta_a = t/\varepsilon \) the total pressure function \( \bar{p}(x/\theta_a, \beta) \) is calculated.

(3) Substituting \( \beta \) and \( C_\theta \) into Eqs. (9) and (10), the velocity profile \( u/w_a - y/\theta_a \) of the near wake in a pressure gradient is calculated.

5. Comparisons of calculated results with experimental data

The total pressure \( \bar{p}(x/\theta_a, \beta) \) is obtained by solving Eq. (11) with the measured initial condition in Fig. 3. Comparing the calculated total pressure along the wake center line (\( \bar{p} \)) in Fig. 7 with the experimental data, we get the relation between \( t \) and \( x/\theta_a \) as shown in Fig. 8. From Fig. 8 we obtain

\[
\frac{t}{\theta_a} = 0.021 (x/\theta_a) \quad \cdots \cdots \cdots \cdots \cdots (14)
\]

and the value of \( \varepsilon \) as 0.021. The difference between the present value and 0.032 in the previous report may be due to the differences of turbulence characteristics in the turbulent boundary layers and of free-stream turbulence, the tunnels used in the present and previous reports being different. The calculated distributions of \( \bar{p}(x/\theta_a, \beta) \) are compared with the experimental data in Fig. 9, which shows a good agreement. The velocity profiles of the near wakes in PG 1-7 are calculated with \( \bar{p}(x/\theta_a, \beta) \).

The calculated and measured velocities along the center line of the wake are shown in Figs. 10(a) and (b), where the free-stream velocity distributions are also shown. Figure 10(a) shows that with an increasing pressure gradient the center line velocity \( u/\theta_a \) decreases both in the adverse and favourable pressure gradient cases. The agreement of the calculated results with the experimental data is good except for PG 3. In Figs. 5 and 6 the values in PG 3 deviate from the curve of PG 0 for downstream, suggesting that the transverse velocity gradients and the Reynolds stress distribution differ from those of PG 0. The difference leads to a disagreement of PG 3 in Fig. 10(a). Although the difference between the total pressure distributions of PG 3 and PG 0 is small as shown in Fig. 4, the disagreement in Fig. 10(a) is large, for which the reason is to be explained as follows. From Eq. (9) we get

\[
d(\frac{u}{w_a}) = \frac{1}{2(1-C_\theta)(\bar{p}-C_\theta)} \times dp_i \quad \cdots \cdots \cdots (15)
\]

Equation (15) shows that even if \( dp_i \) is small \( (\delta u/\theta_a) \) becomes large in case of PG 3 where \( C_\theta \) is very large. Fig. 10(b) shows a good agreement in the mixed pressure gradient cases.

The comparisons of the calculated and measured half widths are shown in Figs. 11(a) and (b). Figure 11(a) shows that with an increasing pressure gradient the half width increases in the adverse and favourable pressure gradient cases. The agreement of the calculated results with the experimental data is good except for PG 3. The disagreement in PG 3 is understood by
considering Fig. 4, Eq. (15) and Eq. (10). Figure 11(b) shows a good agreement in the mixed pressure gradient cases.

The comparisons of the calculated and measured velocity profiles are shown in Figs. 12(a) and (b). The agreement is good except for PG 3, the disagreement in PG 3 being due to the above-mentioned reason.

Fig. 10 Velocities along wake center line

Fig. 11 Half widths
6. Conclusions

The turbulent near wakes starting from the turbulent boundary layers on both sides of a flat plate and subjected to pressure gradients have been investigated. The results are summarized as follows:

1. The total pressure along a stream line of the near wake in a pressure gradient changes at a rate nearly equal to that along the same streamline in zero pressure gradient.

2. A model to predict the development of a turbulent near wake in a pressure gradient has been presented on the basis of the experimental results, and the calculated results agree with the experimental data except for PG3.

3. The reason for the disagreement between the calculated and experimental data in PG3 is discussed with the experimental results.

4. With an increasing pressure gradient, the velocity along the center line of the wake \( \left( \frac{u}{U_c} \right) \) decreases and the half width \( \left( \frac{h_{1/2}}{U_c} \right) \) increases both in the adverse and favourable pressure gradient cases.

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