Reattachment Flow Issuing from a Finite Width Nozzle*
(Report 2. Effects of Initial Turbulence Intensity)

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From the experimental results of a two-dimensional free jet flow, it was found that the flow depends on the initial turbulence intensity at the nozzle exit and this dependency diminishes as turbulence intensity becomes large. The relations between the empirical constants contained in the eddy kinematic viscosity and the initial turbulence intensity were obtained comparing the experimental values of a velocity decay on the jet center line with its approximate calculations. These relations were used for the approximate calculations of a two-dimensional reattachment jet flow. The calculations of the reattachment flow having different turbulence intensities agree well with the experimental results in a wide range of offset ratios, except so small offset ratios.

1. Introduction

In the experiments concerning the reattachment flow done by Bourque and Newman(11), Sawyer(2), Perry(3) and Kumada et al.(4), there are some differences in the reattachment distances from each other since the turbulence intensity of the flow is not prescribed. Concerning the effects of the initial turbulence intensity on the free jet, some papers were written by Gray and Shearer(5) and Kamoi and Tamaka(6). Investigations concerning the reattachment flow considering the initial turbulence intensity were carried out by Tagami et al.(7), Wada et al.(8) and Miyata and Hanaoka(9) as a flow in fluidic elements. But, as far as the authors are aware, it seems that few reports are present concerning the reattachment flow for a wide range of offset ratios and the turbulence intensities.

It was reported that a two-dimensional reattachment flow issuing from a finite width nozzle is well expressed by using the results of calculation of a two-dimensional jet flow by Hatta and Nozaki(10) for a flow having special turbulence intensity (11). In the results of approximate calculations of the free jet, the empirical constants contained in the eddy kinematic viscosity appear. Thus the empirical constants as flow parameters must be given in order to carry out the calculations of reattachment flow. It is troublesome to determine the constants for each case. The initial turbulence intensity is used as a measure of extent of diffusion of the jet. Therefore, it is available to formulate the relations between the initial turbulence intensity and the empirical constants contained in the eddy kinematic viscosity for a free jet beforehand, for carrying out the calculations of a reattachment flow having an arbitrary turbulence intensity. By using these relations for the approximate calculations of reattachment flows having a wide range of offset ratios and turbulence intensities, the calculated results are compared with the experimental ones.

It is well known that behaviors of reattachment flow are well expressed by using the results of calculations of a free jet flow. So far, the solutions for the free jet flow by Görtler(12) has been used for the calculations of reattachment flow with some modifications. In those solutions, another diffusion parameter is used instead of the above mentioned constants. The relation between those parameters is also discussed for comparison.

2. Empirical constant contained in eddy kinetic viscosity and turbulence intensity

In the approximate calculations of a two-dimensional jet flow issuing from a finite width nozzle(10), the eddy kinetic viscosity is given by

\[ \nu = \kappa_1 \left( \frac{b_m - b_c}{} \right) U_0 \]  \hspace{1cm} (1)

in the zone of flow establishment and

\[ \nu = \kappa_2 \frac{e m}{U} \]  \hspace{1cm} (2)

in the zone of established flow, where \( U_0 \) is the uniform velocity at the nozzle exit, \( U \) the velocity on the jet center line, \( b_m \) the mean width of the jet, \( b_c \) the core width, and \( \kappa_1 \) and \( \kappa_2 \) the empirical constants, respectively. The values of \( \kappa_1 \) and \( \kappa_2 \) as diffusion parameters of flow are con-
stant when the jet is under the condition of a free turbulence. In the case of a flow issuing from a finite width nozzle, the distance between the nozzle exit and a point where the jet shows a free turbulence state varies according to the initial turbulence intensity. The relations between the initial turbulence intensity and those empirical constants are to be found for convenience. Using the root mean squared value of the streamwise velocity fluctuations $u'$ and the mean velocity $U$, the turbulence intensity $T$ is given by

$$T = \sqrt{\frac{\langle u'^2 \rangle}{U^2}}$$

and the turbulence intensity at the nozzle exit, that is, the initial turbulence intensity is denoted by $T_0$.

There are some procedures to determine the constants contained in Eqs. (1) and (2). This time they are determined by using the velocity decay on the jet center line. Let the axis of $x$ be taken in the direction along the jet center line and let $2h_0$ denote the nozzle width and $x_1$ the length of the zone of flow establishment, then velocity on the jet center line in the zone of established flow is given by

$$\frac{U}{b_0} = \left( \frac{54912}{20335} \frac{x - x_1}{b_0} + 1 \right)^{1/2}.$$  \hspace{1cm} (4)

Comparing the experimental results having each turbulence intensity with Eq. (4), we get the most probable values of $k_2$ and $x_1$ for each turbulence intensity. After $x_1$ is determined, in the equation

$$\frac{k_2}{b_0} = 1 - \frac{13728}{2345} \frac{x}{b_0},$$

which gives the core width, by putting $b_2 = 0$ at $x = x_1$, $k_1$ is obtained by

$$k_1 = \frac{2345}{13728} \left( \frac{x_1}{b_0} \right).$$

3. Relations between empirical constant $\kappa$ and spread parameter $\sigma$

So far, the solutions of a two-dimensional free jet flow issuing from an infinitesimal width nozzle by Görtler(12) have been used for the approximate calculations of the reattachment flow with some modifications. In those calculations, the spread parameter $\sigma$ is used, which is defined as

$$\eta = \frac{\sigma y}{\zeta},$$

where the axis of $y$ is taken in the direction perpendicular to the jet center line. Therefore, it is available to obtain the relations between the empirical constants contained in the eddy kinematic viscosity $\epsilon$ and the spread parameter $\sigma$, in comparing the experimental results and the different solutions of a free jet. In the case of the jet flow issuing from an infinitesimal width nozzle, $\epsilon$ is defined similar to Eq. (2), and we write $\kappa$ for $\kappa_2$ this time. The relations between $\sigma$ and $\kappa$ can be determined by referring to the center line velocity of the jet, the mean width of the jet and the eddy kinematic viscosity expressed using those parameters, whereas the results differ a little from each other according to the velocity profile assumed. The center line velocity of the jet by Görtler is given by

$$U = \left( \frac{20335}{109824} \frac{J}{\rho x} \right)^{1/2},$$

where $J$ is the momentum of the fluid issuing from the nozzle exit per unit time and $\rho$ the density of the fluid. On the other hand, by carrying out the same calculation as the jet issuing from a finite width nozzle, we obtain

$$U = \left( \frac{20335}{109824} \frac{J}{\rho x} \right)^{1/2}.$$  \hspace{1cm} (8)

Comparing Eq. (7) with Eq. (8), we obtain

$$\sigma = 0.2469/\kappa.$$  \hspace{1cm} (9)

Comparison of the mean widths of the jet, it becomes

$$\sigma = 0.2645/\kappa.$$  \hspace{1cm} (10)

Furthermore, comparing

$$\epsilon = 1.135 \frac{b_{1/2} U^{(a)}}{40},$$

used by Görtler(12), where $b_{1/2}$ denotes half the width at half depth, with Eq. (2), we obtain

$$\sigma = 0.25/\kappa.$$  \hspace{1cm} (11)

The relations between $\sigma$ and $\kappa$ show somewhat different values according to the procedure of comparison. For instance, $\sigma = 7.67$ proposed by Reichardt(14) experimentally cor-

The factor 1.125 which appears in reference(13) is incorrect and it should be 1.135 from the authors reexamination.

Fig.1 Experimental apparatus

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resolves to $k = 0.0322$ when we use Eq. (9).

4. Experiments

4.1 Experimental apparatus and methods

The experimental apparatus of the two-dimensional reattachment flow is shown in Fig. 1. The experiments with the two-dimensional free jet flow are carried out using this apparatus removing the reattachment wall. The aspect ratio of the nozzle is $AR = 8$ and the nozzle width is 25mm. From the results reported previously, it was found that behaviors of a two-dimensional reattachment flow are independent of Reynolds number when it is larger than $1.0 \times 10^4$. Therefore, the experiments are done at $Re = 3.0 \times 10^4$ for a flow having a large initial turbulence intensity and at $Re = 5.0 \times 10^4$ for the other conditions on account of the capacity of the blower. In order to get a high turbulence intensity of the flow, Gray and Shearer (5) and Wada et al. (6) used the method of supplying a secondary flow to the main flow, but this time the method of venting the flow to the atmosphere is used in addition to the grid and the stainless screens (8 to 30 mesh), to increase the turbulence intensity. By combination of these methods the measurements of the turbulence intensity and the velocity on the jet center line are done from $T_0 = 0.0042$ to 0.0846. The mean velocity of the flow is detected by a Pitot tube and the turbulence intensity by a hot wire probe.

In the experiments of the reattachment flow, the offset ratio is expressed by $D/d_0 (D$ denotes the distance from the edge of the nozzle to the reattachment wall).

4.2 Experimental results and discussion

The mean velocity and the turbulence intensity distributions near the nozzle exit for different turbulence intensities are shown in Fig. 2. Both the mean velocity and the turbulence intensity are uniform over a wide range of the nozzle exit, except the edges. Thus we let the turbulence intensity at the center of the nozzle exit represent the initial turbulence intensity $T_0$.

4.2.1 Free jet flow

The velocity decays on the jet center line are shown in Fig. 3. As $T_0$ becomes large, the velocity decays rapidly, but it be-
comes not to vary as $T_b$ increases. That behaviours of the flow do not change in spite of the variation of $T_b$ relates to the variation of the turbulence intensity on the jet center line. From a point of view of self-preservation of the jet, which means that its fluid dynamic characteristics are similar, Henkestad(46) proposed that the jet reaches approximately a self-preserving state at $x/b_0=130$. But it is seen from Fig.4 that the point where the flow reaches a self-preserving state shifts downstream gradually as $T_b$ decreases. Furthermore, it is found from the variation of the turbulence intensity on the jet center line that there exists a transition region, where the jet is not self-preserving, fairly downstream of the nozzle exit as $T_b$ becomes so small.

Equations (4) and (5) were deduced after an approximate calculation by considering that the jet is self-preserving by nature and also $k_1$ and $k_2$ are independent of $x$ and $y$. Therefore, in determining the most probable values of $k_1$ and $k_2$ by comparing for each initial turbulence intensity the experimental values of the velocity on the jet center line shown in Fig.3 with Eq.(4), it may be appropriate to use the values of $U/U_0$ and $x/b_0$ in the zone where the turbulence intensity is kept constant. From the experimental results of the two-dimensional free jet issuing from a finite width nozzle, it was found that the velocity profile is self-preserving even in the transition zone experimentally, and Eqs. (4) and (5) were derived on the assumption that the time mean velocity profile is similar. From these facts and considering the aspect of variation of $U/U_0$ shown in Fig.3 and the range of offset ratios of the reattachment flow treated in this investigation, it may be allowed to use the data for somewhat upstream range where the turbulence intensity is not yet constant in determining the most probable values of $k_1$ and $x_1$ for each initial turbulence intensity. This time, the values for $U/U_0$ between 0.5 and 0.9 were adopted. The values of $k_2$ and $x_1$ are presented in Figs. 5 and 6 for each value of $T_b$. Substituting the values of $x_1$ shown in Fig.6 into Eq. (6), $k_1$ is obtained as shown in Fig.5. These figures show that $k_1$ and $k_2$ do not change considerably in the range of large values of $T_b$. Furthermore, the length of the zone of flow establishment also becomes constant as $T_b$ increases. The results by Kamoi and Tanaka(2) and Wada et al.(3) are also shown for comparison. The values differ from each other according to different definitions of $x_1$. For example, Eq.(4) becomes

$$\frac{U}{U_0} = (0.0802 - 0.123)^{1/2}$$

for $T_b=0.0053$, the value of $x_1$ is larger than the actual length, so the experimental results do not agree with Eq.(12) in the transition zone where $U/U_0 > 0.9$.

It will be convenient to formulate the relations between $k_2$ and $T_b$, and also $k_1$ and $T_b$ shown in Fig.5 in case of calculating reattachment flows having different initial turbulence intensities. In the zone of established flow, $k_1$ is given by

$$k_1 = 0.0283 + 0.264 T_b - 1.24 T_b^2$$

approximately, and $k_1$ is given as follows:

$$k_1 = 0.1012 + 0.393 T_b - 1.61 T_b^2$$

$x_1$ is determined as

$$x_1 = \frac{2345}{b_0} = 13728 (0.0102 + 0.393 T_b - 1.61 T_b^2)^{1/2}$$

from Eqs.(6) and (14) and shown in Fig.6.

4.2.2 Reattachment flow

Figure 7 shows the pressure distributions along the reattachment wall for the

![Fig.5](image_url)

**Fig.5** Empirical constants contained in the eddy kinematic viscosity

![Fig.6](image_url)

**Fig.6** Length of the zone of flow establishment
offset ratio. As $T_0$ increases, the flow tends to reattach at upstream position gradually. This agrees with the tendency that the velocity decay on the jet center line depends on the turbulence intensity of the flow and it is shown that the structure of a free jet flow strongly correlates to that of a reattachment flow. Just as in the case of a free jet flow, behaviors of the reattachment flow hardly depend on $T_0$ in the range larger than 0.06. As an example of such a flow, Fig.8 shows the pressure distributions along the reattachment wall for $T_0 = 0.0785$ using the offset ratio as a parameter. In order to detect a similarity of the flow, the pressure is made non-dimensional using the apparent dynamic pressure on the jet center line just before reattachment instead of the dynamic pressure at the nozzle exit and it is denoted by $Q_p^*$, and the distance $z$ is measured by the scale $D + b_0$ instead of $b_0'$. As the result, in the range of offset ratios of this experiment, the influence of a finite width of the nozzle appears, and so the similarity of the flow can not be recognized. Nevertheless, as $D/b_0$ becomes relatively large, it seems that there appears a tendency of similarity in the flow. The same experiments are done for $T_0 = 0.0042$. Figure 9 shows the mean bubble pressure determined by the proposal of Sawyer(2) for $T_0 = 0.0042$ and 0.0785. It is seen from this figure that there occur wide differences as $T_0$'s differ.

The calculations of reattachment flow have been carried out applying the relations Eqs.(13) and (14) to the approximate calculations as reported previously(11). The calculations have been done both for the zone of flow establishment and for the zone of established flow, separately, and the limit of the offset ratio, for which the results of calculation for the zone of flow establishment are applicable, is determined in the following manner. Denoting by $\theta$ the angle made by the jet center line with the reattachment wall and by $R$ the radius of curvature of the jet center line, the distance from the center of the nozzle exit to the reattachment wall measured along the jet center line is given by

$$\frac{z}{b_0} = \frac{R}{b_0} \frac{\theta}{\pi / 2}$$

Fig.7 Pressure distributions along the wall

Fig.8 Pressure distributions along the wall

Fig.9 Coefficients of mean bubble pressure

The calculations in the zone of flow establishment is applicable for $x_c'$ defined by the above equation smaller than $x_c'$, the length of the zone of flow establishment.
measured along the curved jet center line. From the geometrical relations, \( R \) contained in Eq.(16) is given by

\[
\frac{R}{b_0} (1 - \cos \theta) = \frac{D}{b_0} + 1 \quad (17)
\]

and

\[
\frac{x_i'}{b_0} = \frac{D}{b_0} + 1 \cdot \frac{\theta}{1 - \cos \theta} \quad (18)
\]

is obtained from Eqs.(16) and (17). The value of \( \theta \) is given by

\[
(1 - \cos \theta)^2 = 0.8248 \kappa_1 \left( \frac{D}{b_0} + 1 \right) \theta \quad (19)
\]

for each offset ratio and initial turbulence intensity. By putting \( x_1 = x_i' \) in Eq.(6), \( x_i' \) is obtained by

\[
\frac{x_i'}{b_0} = 2345 \cdot \frac{1}{13728 \kappa_1} \quad (20)
\]

Denoting by \( Dc/b_0 \) the offset ratio giving \( x_i' = x_i' \), it becomes

\[
\left( \frac{Dc}{b_0} + 1 \right) \cdot \frac{\theta}{1 - \cos \theta} = 2345 \cdot \frac{1}{13728 \kappa_1} \quad (21)
\]

from Eqs.(18) and (20). The value of \( \theta \) is obtained as 0.5373 independent of \( \kappa_1 \) and \( Dc \) using Eqs.(19) and (21), and the limiting offset ratio for which the calculation of the flow in the zone of flow establishment is applicable is given by

\[
\frac{Dc}{b_0} = 0.04479 \frac{1}{\kappa_1} - 1 \quad (22)
\]

The relations between the reattachment distance and the offset ratio is shown in Fig.10 for various values of \( T_0 \). The results by Bourque and Newman(17), Sawyer(22) and Kumada et al.(23), of which the turbulence intensity is not prescribed, are also shown for comparison. In the case of flows having small offset ratios, the authors' experimental results for \( T_0 = 0.0042 \) and 0.0785 are shown in this figure. The reattachment distance expressed by the solid lines in Fig.10, which are calculated using the values of \( \kappa_2 \) and \( x_i' \) obtained by the data in the zone of established flow, disagree with the experimental results as the offset ratio becomes small. This comes from the fact that the region of \( U/U_0 > 0.9 \) is the transition zone, as it can be seen from the example for \( T_0 = 0.0053 \) in Fig.3.

It is supposed that all the experiments by the others in the case of flows having comparatively large offset ratios were done at small turbulence intensities. Figure 11 shows the results of calculations and experiments of reattachment distance with the offset ratios 5 and 11. The results of calculations are smaller than those of experiments in the case of \( Dc/b_0 = 11 \), but a-

Fig.10 Reattachment distances

\[
AR = 8
\]

\[
\bullet \quad Dc/b_0 = 5
\]

\[
\bullet \quad 11
\]

Fig.11 Reattachment distances

\[
T_0 = 0.0042
\]

\[
0.01
\]

\[
0.02
\]

\[
0.03
\]

\[
0.05
\]

\[
0.0785
\]

5. Conclusions

From the experimental results of a two-dimensional jet flow issuing from a finite width nozzle, it was found that the de-
velopment of the jet becomes slower as the initial turbulence intensity at the nozzle exit becomes smaller and that the turbulence intensity on the jet center line approaches a constant value far downstream of the nozzle exit. But, for the initial turbulence intensity greater than 0.06, the flow reaches an approximately self-preserving state fairly rapidly and the turbulence in the downstream region is independent of the initial turbulence intensity, and the velocity on the jet center line, the turbulence intensity and the length of the zone of flow establishment do not change any more.

Comparing the experimental results with the calculated results of a two-dimensional jet flow, the relations between the empirical constants contained in the eddy kinematic viscosity and the initial turbulence intensity are formulated. By applying the method of calculation of a free jet issuing from a finite width nozzle for the flow issuing from an infinitesimal width nozzle, the relations between the empirical constants which appear in the calculation of authors and the spread parameter σ in the solutions by Görtler were presented for comparison.

In the case of a two-dimensional reattachment flow, it was found that the initial turbulence intensity influences the reattachment flow. By applying the relations between the initial turbulence intensity and the empirical constant contained in the eddy kinematic viscosity to the approximate calculation of the reattachment flow, the calculated results are found to agree with the experimental ones in a wide range of offset ratios.

References

(12) Görtler, H., Z. AAM, Bd.22, Nr.5(1942-10), S.244.
(14) Reichardt, H., VDI-Forsch.-h., 414 (1942).