Wave Power Absorption Characteristics of a Rocking Body

By Hirohisa TANAKA** and Makoto SAITO-O***

We test the performance of a wave power absorber of a rocking body of Salters Duck of 37cm in diameter and 150cm in width combined with a hydro-static power conversion mechanism consisting of a hydraulic cylinder, check valves and an accumulator, clarify the effect of both the shape of the body and the load characteristics on the efficiency, and propose a control method of hydraulic power conversion mechanism for efficient storage and absorption of wave energy.

1. Introduction

A rocking body called Salters Duck(1,2) is one of the promising bodies for absorbing wave power. It absorbs wave power efficiently due to its asymmetrical shape of exponential front section and circular rear section. Recent linear theories(3,4) estimate its efficiency of wave power absorption and hydrodynamic forces acting on the body. In this paper, we examine the effect of the shape of front section on the efficiency and also the effect of load characteristics on the efficiency by making a prototype of a body of 37cm in diameter and 150cm in width combined with a hydro-static power conversion mechanism, and propose a control method of the mechanism for efficient absorption and storage of wave energy.

2. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>a</td>
<td>amplitude of incident wave</td>
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<tr>
<td>B</td>
<td>width of the body</td>
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<tr>
<td>C</td>
<td>restoring force constant</td>
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<td>d</td>
<td>draft of the body</td>
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<tr>
<td>Dp</td>
<td>displacement of hydraulic cylinder</td>
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<td>Gp</td>
<td>hydraulic conductance of throttle valve</td>
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<tr>
<td>g'</td>
<td>acceleration of gravity</td>
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<tr>
<td>Hf</td>
<td>freeboard of the body</td>
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<tr>
<td>Ιν</td>
<td>mechanical inertia</td>
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<tr>
<td>Ιιι</td>
<td>added inertia due to oscillation of the body</td>
</tr>
<tr>
<td>k</td>
<td>wave number</td>
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<tr>
<td>Nt</td>
<td>damping coefficient of fluid reaction force</td>
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<tr>
<td>Nl</td>
<td>damping coefficient of load reaction force</td>
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<tr>
<td>P</td>
<td>pressure</td>
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<tr>
<td>Q</td>
<td>flow rate</td>
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<tr>
<td>R</td>
<td>a measure of effectiveness for wave power absorption</td>
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<tr>
<td>r</td>
<td>radius of front section of the body</td>
</tr>
<tr>
<td>ro</td>
<td>radius of rear section of the body</td>
</tr>
<tr>
<td>a</td>
<td>length along the body surface</td>
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<tr>
<td>W</td>
<td>wave period</td>
</tr>
<tr>
<td>Vn</td>
<td>fluid velocity normal to the wetted surface of the body</td>
</tr>
<tr>
<td>vn</td>
<td>body elements velocity normal to the wetted surface of the body</td>
</tr>
<tr>
<td>W</td>
<td>incident wave power</td>
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<tr>
<td>ζn</td>
<td>normalized damping coefficient of oscillating body</td>
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<tr>
<td>ζn''</td>
<td>normalized damping coefficient of load</td>
</tr>
<tr>
<td>η''</td>
<td>efficiency</td>
</tr>
<tr>
<td>η''n</td>
<td>efficiency estimated by simplified theory</td>
</tr>
<tr>
<td>θ</td>
<td>rocking amplitude of body</td>
</tr>
<tr>
<td>λ</td>
<td>wave length</td>
</tr>
<tr>
<td>τL</td>
<td>reaction torque of load</td>
</tr>
<tr>
<td>τW</td>
<td>exciting torque of incident wave</td>
</tr>
<tr>
<td>ωn</td>
<td>angular frequency of incident wave</td>
</tr>
<tr>
<td>ω0</td>
<td>resonant angular frequency of the body</td>
</tr>
</tbody>
</table>

3. Estimation of efficiency

3.1 Estimation by simplified theory

Recent linear theory estimates the efficiency and hydrodynamic forces induced by the oscillation of body in sinusoidal wave trains. However, as it takes complex calculations of Greens functions to solve the diffraction and radiation problems, it has not been successful to apply the theory to the comparison of efficiencies. Then we apply a simplified method(5) to the estimation of efficiency.

\[
\eta_n = (1-R_{inv})(W-W_f)/W \quad (1)
\]

\[
R = \int (u_n - u) da / \int u_n da \quad (2)
\]

\[
W_f/W = e^{-rt/\zeta''} \quad (3)
\]

where \( u_n \) and \( u \) are unperturbed fluid velocity

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and body velocity (normal to the surface of the body in Fig.1). \( R_m \) is the minimum of \( R \), and integrals are time averaged.

Calculated efficiencies of three types of bodies are compared in Fig.3, where the shapes of front sections are shown in Fig.2, and every efficiency is calculated under the assumption that the rocking amplitude of body at fore perpendicular (front section) is equal to the incident wave amplitude.

3.2 Effect of load dynamics

We can formulate the following equation of motion of a rocking body which is permitted to roll about a fixed axis in response to the sinusoidal incident waves.

\[
(I + \dot{I}) \ddot{\theta} + \dddot{\theta} + C_{\theta} \dot{\theta} + D_{\theta} \theta = 0
\]

where \( I \) is mechanical inertia, \( C_{\theta} \) is restoring force constant due to buoyancy, \( D_{\theta} \) and \( N_{\theta} \) are parameters of reaction fluid \( F \)-force induced by an oscillation of the body. \( \tau_L \) is wave exciting torque and \( \tau_{\theta} \) is reaction torque from an energy absorber.

If the energy absorber consists of a hydro-static pump with displacement \( D_p \) and of a viscous frictional load with hydraulic conductance \( G_p \), the reaction torque of a load will be

\[
\tau_{\theta} = D_p \ddot{\theta} + \frac{G_p}{\omega} \dot{\theta}
\]

The equation of motion is

\[
(I + \dot{I}) \ddot{\theta} + \dddot{\theta} + C_{\theta} \dot{\theta} + D_{\theta} \theta = 0 + \tau_{\theta}
\]

The rocking amplitude of the body which is excited by sinusoidal wave trains is

\[
T_{\theta} = \sqrt{\frac{\omega^2}{2\pi} + \frac{\omega^2}{2\pi} \left( \frac{\omega^2}{\omega^2 + (\omega_0^2 - \omega^2)} \right)}
\]

where \( \omega = 2\pi f \) and \( \omega_0 = \sqrt{\frac{D_{\theta}}{I + \dot{I}}} \).

The efficiency of a two-dimensional wave power absorbing body is represented by the proportion of the power absorbed by load \([ (\omega/2\pi)^2 (N_{\theta}, \dot{\theta})] \) to the incident wave power \([ (\omega/2\pi)^2 \theta^2 (\omega_0^2 - \omega^2)] \).

\[
\eta = \frac{\tau_{\theta}}{2\omega^3 \left( \frac{\omega_0^2}{\omega_0^2 - \omega^2} \right)}
\]

where \( \delta \) is radiation wave ratio of the body (Appendix 1).

The wave power absorption efficiency becomes maximum when the body is oscillated at resonant frequency \( \omega = \omega_0 \) and also the load damping force of absorber is controlled to be equal to the fluid damping force induced by an oscillation of the body \( \omega = \omega_0 \).

The maximum efficiency \( \eta_{\max} \) oscillating amplitude \( \theta_0 \) and rate of load torque to wave exciting torque become \( \omega = \omega_0 \), \( \theta_0 \approx \theta_0' / (2\omega_0) \) and \( \tau_{\theta} / \tau_{\theta_{\max}} = 1 / 2 \), respectively.

At other frequencies than the resonant one, the load damping should be controlled to be \( \tau_{\theta} = \theta_0^2 (1/2) (v^2 - \omega_0^2)^2 \) due to \( \theta_0 \).

4. Experimental characteristics

4.1 Wave power absorbing system

We make a system consisting of a rocking body of 37cm in diameter and 150cm in width, and of a hydro-static power conversion mechanism which can absorb wave power of about 0.1kw at wave amplitude of 0.1m and wave period of 1.6s. Hydro-static power conversion system is appropriate for absorbing wave power because it has features of easy storage and control of absorbed energy and of high-torque low-speed power characteristics. Fig.4 shows a configuration of the wave power absorber with a hydro-static power conversion mechanism. The mechanism consists of a rocking body(1), a hydraulic cylinder(2), a four-way check valve(3), a hydraulic throttle valve(4) and a hydraulic accumulator(5).

4.2 Power characteristics and efficiency

Fig.5 shows one of wave power absorption tests at wave period of 1.2s.
In the chart, cylinder displacement (X), cylinder pressure (P), transmitted and incident waves are recorded.

\[ \frac{\eta}{F_0} = \frac{Q}{W} = \frac{B_0}{B_0g^2a^2/4\sigma} \] ... (9)

In Appendix II, we show the theoretical power characteristics and experimental ones which are compensated for incident wave height at wave period of \( T = 1.4s \). In the figure, we can see flow rates decrease nearly in proportion to an increase of absorbing pressure at homogeneous incident wave trains.

5. Conclusions

We examine both the power characteristics and the efficiency of a wave power absorber consisting of a rocking body and a hydro-static power conversion mechanism. Efficiency of the wave power absorber depends both on the shape of front section of the body and on the load characteristics of the power taking off mechanism. We confirm that the rocking body absorbs wave power with high efficiency when it is oscillated at resonant frequency and the load damping torque is controlled to be equal to half the wave exciting torque.

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References

Appendix I
radiation wave ratio

The efficiency of a wave power absorber becomes maximum \( (\eta = 2-6) \) when the load system is optimized \( (\omega_0 = \text{in Eq.}(8)) \). The radiation wave ratio \( \delta \) is defined by

\[ \delta = \frac{A_0^2}{\eta (\eta_0^2 + A_0^2)} \] ... (10)

where \( A_0 \) and \( \eta_0 \) are amplitude of radiation waves to infinity \((\omega_0 \rightarrow \infty)\) generated by a forced oscillation of the body in calm waters. The characteristics of \( \delta \) are estimated either from the calculation of Kochin-function or from a forced oscillation test of the body in calm waters. Fig.8 shows the calculated results of two types of Salters Ducks. Two pairs of broken and solid lines are amplitude ratios of radiation waves defined by the following formula.

\[ \eta_0 = \frac{A_0}{\eta_0} \] ... (11)

The broken lines indicate calculated results of Salters Duck I of 37cm in diameter. Its shape of front section is designed at wave length of 3.06m \([=\eta_0 \exp(2\pi y_d/\lambda)]\). and the other solid lines those of Salters Duck II of 12cm in diameter. It is designed at wave length of 56cm. Marks ○ and ● denote test results of a forced oscillation of Salters Duck II in calm waters.
Appendix II

Power characteristics and efficiency

Theoretical power characteristics, relation between angular velocity \( \omega \), and torque \( \tau \), of the absorber with a viscous damping load of \( N_\tau \) are represented by

\[
\omega = \frac{\tau}{N_\tau} = \frac{\sqrt{\frac{1}{1 - \nu^2}} \nu \tau}{N_\tau} \]

and

\[
\tau = N_\tau \omega = \frac{\nu}{1 - \nu^2} \tau \]

where \( N_\tau = \frac{Z \omega}{C} \) in Eq. (7).

When the absorbed power is stored in a hydraulic accumulator, the reaction torque of cylinder (in Fig. 4) is

\[
\tau_{LE} = \frac{SIP_0 B}{\nu} \]

where \( S \) is cross sectional area of cylinder, \( l \) is distance between the cylinder and the rocking center of body, and \( P_C \) is accumulating pressure.

When the rocking body is oscillated sinusoidally at amplitude of \( \theta_0 \) without stop motion, the equivalent damping coefficient of load is

\[
\tau_{LE} = \frac{2B \theta_0}{\sqrt{1 - \nu^2}} \]

The power characteristics of the absorber with an accumulator is

\[
\omega_{LE} = \frac{1}{\sqrt{1 - \nu^2}} \nu \tau \]

and

\[
\tau_{LE} = \frac{4B \theta_0}{\nu \tau_{LE}} \]

The efficiency of the absorber with an accumulator is

\[
\eta_{LE} = \frac{2\theta_0^2}{\sqrt{1 - \nu^2}} [4B \theta_0/(\tau \theta_0)] (\theta_0^2/\nu) \]

where \( \theta_0 \) is width of the rocking body, \( B = 1.5 \).