Heat Transfer Around a Tube in a Bank

by

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An experimental study was made to investigate the heat transfer and flow around the third and fourth ones of four cylinders in line in a cross flow of air. The in-line pitch ratio was in the range 1.15 < c/d < 5.0, where c is the center to center distance and d the cylinder diameter, and in the Reynolds number 12000 < Re < 50000. The heat transfer characteristics of the third and fourth cylinders are found to exhibit a strong dependency upon the separation point of their upstream cylinders.

1. Introduction

Prediction of heat transfer and flow around cylinders in tube banks is very important in relation to various engineering aspects and there have been many works(1)-(4). Recent energy crisis has been promoting developments of high performance heat exchangers for the effective use of energy. For that purpose, it is firstly necessary to clarify the detailed heat transfer characteristics of cylinders in tube banks.

It is well known that the heat transfer and flow around the first cylinder in tube banks are almost equal to those around a single cylinder(7)(8), and that the heat transfer rate around the cylinder increases toward downstream because of the increase of turbulence intensity(7)(9).

On the other hand, as far as the in-line tube bank is concerned, the flow behavior around the cylinder may be classified into three groups; that is, a flow around the first cylinder, one around the second and one around the third and downstream cylinders(3). However there have been relatively few investigations of the detailed correlations between the flow and heat transfer characteristics.

From this standpoint, the present authors investigated, in detail, the heat transfer and flow around the second one of three cylinders placed at equal intervals in a cross flow of air(7)(8) so as to obtain the basic data on the heat transfer in a tube bank. However the heat transfer around the third and downstream cylinders is still not investigated. That is very important in connection with the tube bank composes of a large number of cylinders.

The purpose of the present study is to clarify the heat transfer characteristics around the third and downstream cylinders in an in-line tube bank. Four cylinders are placed at equal intervals in a cross flow of air, as schematically shown in Fig. 1. All the four cylinders are heated under the condition of constant heat flux. Measurements are made mainly of the heat transfer rate around the third and fourth cylinders along with the distributions of static pressures, mean velocities and turbulence intensities. Their mutual correlations are discussed in detail in the present paper.

Nomenclature

\( c \) : longitudinal spacing between cylinder centers

\( C_D \) : pressure drag coefficient = \( \frac{1}{2} \frac{C_{D_{s}} A_{c}}{A_{e}} \)

\( C_p \) : pressure coefficient = \( \frac{(P_{\text{avg}} - P_{\text{ref}})}{\frac{1}{2} \rho U_0^2} \)

\( d \) : cylinder diameter

\( I \) : electric current

\( Nu \) : Nusselt number

\( p \) : static pressure

\( q \) : heat flux per unit area and unit time

\( Re_g \) : Reynolds number = \( \frac{U_0 d}{v} \)

\( S \) : area of heated surface

\( T \) : temperature

\( U \) : mean flow velocity

\( U_0 \) : free stream velocity

\( \sqrt{\frac{U_0^2}{g}} \) : r.m.s. of streamwise turbulent fluctuating velocity

\( V \) : electric voltage

\( x, y \) : coordinates

\( \alpha \) : heat transfer coefficient

\( \delta \) : distance normal to cylinder surface

\( \theta \) : angle from forward stagnation point

\( \lambda, \nu, \rho \) : thermal conductivity, kinematic viscosity and density of air

Subscripts

\( c \) : critical
in the present study. The symmetry of the flow field around the cylinders was found to be satisfactory through the measured distributions of local heat transfer coefficients and static pressures. The two-dimensionality of the flow was confirmed from the surface oil flow pattern.

3. Experimental apparatus and technique

The wind tunnel used in the present study is the same one as employed in the previous work(7)(8). The test section is a rectangle 325mm high and 225mm wide. The turbulence intensity of the upstream uniform flow \( \sqrt{\langle u' \rangle} / U_0 \) is about 0.005 to 0.007 through the experiments.

Four cylinders are set at equal intervals at the center of the test section as indicated in Fig. 1, which includes the coordinate system employed. All the cylinder diameters are 26mm and the in-line pitch ratio \( c/d \) is varied from 1.15 to 5.0 and the Reynolds number \( Re_d \) based on the cylinder diameter from 12000 to 50000.

![Fig. 1 Flow configuration and coordinate](image)

For the heat transfer measurements, a stainless steel ribbon 20mm wide and 0.05mm thick was wound helically around the central section of a hard vinyl chloride tube and was electrically heated(8). Thus the present data were obtained under the condition of constant heat flux. The inside of the tube was filled with a rigid urethane foam to minimize the heat loss. The temperature distributions along the cylinder surface were measured at intervals of 10° with 0.065mm copper-constantan thermocouples which were stuck to the back of the stainless steel ribbon. The present results shown in the following were obtained at about q = 1300 kcal/s/m². Heat transfer coefficient and Nusselt number \( Nu_d \) were defined respectively as follows:

\[
q_0 = \frac{VI}{s(T_w - T_a)}
\]

\[
Nu_d = \frac{q_d}{\lambda}
\]

The surface pressure distributions were measured at intervals of 10° with another four cylinders of the same diameter having static pressure holes of 0.5mm diameter. The mean and streamwise turbulent fluctuating velocities were measured with a constant temperature hot-wire anemometer having a linearising circuit. The hot-wire used was a 0.005mm tungsten wire with 1mm effective length.

Heat transfer measurements were conducted by heating all the four cylinders under the same heat flux. On the other hand, all the cylinders were not heated in the measurements of static pressure, velocity and turbulence intensity. No corrections were made to the data for the tunnel wall effects.

3.1 Local Nusselt number

The local Nusselt number \( Nu_d \) and static pressure coefficient \( C_p \) distributions are described for all the four cylinders in case of \( c/d = 1.8 \) and \( Re_d = 41000 \) in Figs. 2 and 3, respectively.

The results with the first cylinder show qualitatively similar trends to those with a single cylinder at the subcritical Reynolds number. The heat transfer coefficient for the former is, in general, lower than that for the latter. The minimum pressure coefficient \( C_p \) at around \( \theta = 65° \) and also the base pressure coefficient \( C_p\theta = 0° \) are, on the other hand, a little higher than those for the latter. This difference may be due to effects of the downstream cylinders.

As to the second cylinder, both \( Nu_d \) and \( C_p \) attain their maximum values around \( \theta = 70° \) since the shear layer separated from the first cylinder adheres there. Its velocity gradient is much larger than that of the
Separated shear layer from the second and further downstream cylinders, resulting in a high heat transfer rate near the attachment point for the second cylinder\(^{(4)}\).

\(\text{Nu}_0\) and \(C_p\) for the third cylinder represent very similar behaviors to those for the fourth cylinder. Moreover, their characteristic features are much different from those for the first and second cylinders. That is, the angular position of \(\text{Nu}_{\text{max}}\) and \(C_p\text{max}\) locates 50° to 40° upstream as compared to that for the second cylinder and \(\text{Nu}_0\) and \(C_p\) in the neighborhood of the forward stagnation point are higher than those for the second cylinder. This may be due to the fact that on the second and downstream cylinders, the shear layer separates around \(\theta = 110°\) to 120° and consequently the wake width upstream of the third and fourth cylinders becomes smaller\(^{(7)}\). It may also result in an increase of the pressure drag coefficient of the third and fourth cylinders\(^{(9)}\) (see Fig. 9). However, the velocity gradient of the separated shear layer along the second and third cylinders is smaller than that along the first one (see Fig. 12) and then the curve of \(\text{Nu}_0\) becomes flat. The separation point was confirmed with the surface oil flow pattern and it was found to correspond with the position of \(\text{Nu}_{\text{min}}\). It seems from the descriptions noted above that the main flow comes closer to the third and fourth cylinders than to the second cylinder (see Figs. 12, 13 and 14).

Distributions of the local Nusselt numbers with the Reynolds number are shown for \(c/d = 1.3, 1.8\) and 2.8 in Figs. 4(a) through 4(c), respectively. In cases of \(c/d = 1.8\) and 2.8, the variation is small. Especially at \(c/d = 1.8\), \(\text{Nu}_{\text{max}}\) and \(\text{Nu}_{\text{min}}\) always occur at about \(\theta = 45°\) and 110°, respectively, independently of the Reynolds number. At a wider spacing with \(c/d = 2.8\), the entrainment of the main flow into the front surface increases and it causes the position of \(\text{Nu}_{\text{max}}\) to become identical with the forward stagnation point. The position of \(\text{Nu}_{\text{min}}\) locates at about \(\theta = 110°\), which is nearly equal to that for \(c/d = 1.8\), and it suggests that the flow separates there.

In case of \(c/d = 1.3\), the Nusselt number distribution changes its pattern at about \(\text{Re}_d = 21000\). In the region of \(\text{Re}_d > 21000\), \(\text{Nu}_0\) distribution is not much different from that at \(c/d = 1.8\) though the position of \(\text{Nu}_{\text{max}}\) locates at about \(\theta = 50°\), which is larger than that at \(c/d = 1.8\). It is to be noticed that the separation point exists at about \(\theta = 110°\) similarly to the cases of \(c/d = 1.8\) and 2.8 in spite of the smaller spacing in the other hand. In the range of \(\text{Re}_d = 21000\), the position of \(\text{Nu}_{\text{max}}\) shifts downstream, \(\theta = 60°\) to 65°. The heat transfer on the front face deteriorates and the location of \(\text{Nu}_{\text{min}}\) becomes obscure. Therefore it becomes difficult to determine the separation point by the local Nusselt number distribution. The results at \(c/d = 1.3, 1.8\) and 2.8 for the fourth cylinder show similar trends to those for the third one. The separation point locates at about \(\theta = 115°\) independently of the cylinder spacing and of the Reynolds number, excluding the case of \(c/d = 1.3\) and \(\text{Re}_d = 21000\).

Fig. 5 shows the variation of the angular position \(\theta_\alpha\) of \(\text{Nu}_{\text{max}}\) with \(c/d\) at \(\text{Re}_d = 41000\) for the second, third and fourth cylinders. As to the second cylinder, the so-called jumping phenomenon\(^{(10)}\) occurs at about \(c/d = 4.0\) and \(\theta_\alpha\) tends to zero. This spacing is almost equal to that for two cylinders of in-line arrangement. Included for comparison in Fig. 5 is the angular position \(\theta_\alpha\) at which the mass transfer coefficient reaches maximum according to Hiwada, et al\(^{(11)}\) for the downstream cylinder of two cylinders. Their results exhibit a similar feature to the present one. On the other hand, \(\theta_\alpha\) for the third and fourth cylinders is always smaller than that for the second one. It decreases monotonically with \(c/d\) and becomes...
zero at about $c/d = 2.2$. It may be concluded from these results that the separated shear layers from the second and third cylinders are entrained into the upstream of the third and fourth cylinders, respectively.

3.2 Mean Nusselt number

Variation of the mean Nusselt number with the cylinder spacing is described for the third and fourth cylinders in Figs. 6 and 7, respectively. The present data show a weak dependency on $c/d$ except the case of $c/d = 1.3$ and Re$_d < 21000$. Similar feature was found for the second cylinder, in which the tripping-wires set on the first cylinder decreased its wake width. These observations are about the same as the results obtained in the previous work.

However, in the case of a very small spacing such as $c/d = 1.3$, the heat transfer around the third and fourth cylinders deteriorates at low Reynolds number and Nu$_m$ for them decreases discontinuously at about Re$_d = 21000$. Similar results were also obtained for the first and second cylinders. This may be due to the very low heat transfer rate on both the front and rear surfaces of the cylinder at low Reynolds number. Such Reynolds number, at which Nu$_m$ varies discontinuously, is hereafter called the critical Reynolds number Re$_{dc}$. It increases as the cylinder spacing decreases and is expressed as

$$\text{Re}_{dc} = 1.14 \times 10^5 (c/d)^{-5.84}$$

Fig. 6 Variation of mean Nusselt number with Reynolds number for the third cylinder

Fig. 7 Variation of mean Nusselt number with Reynolds number for the fourth cylinder

in the present experimental range. The flow pattern changes drastically at Re$_{dc}$, and the flow between neighboring cylinders becomes very stagnant. More detailed descriptions in connection with the critical Reynolds number may be found in another paper by the present authors.

Fig. 8 shows the variation of Nu$_m$ with c/d at Re$_d = 41000$. The results with the first and second cylinders exhibit a complicated variation. The previous data on the second one of the three cylinders, when the other cylinders except the measured one were not heated, are a little higher than those on the second cylinder in the present study but show a very similar trend to the present one. On the other hand, the results with the third and fourth cylinders do not exhibit a dependency on c/d. Similar results are found in those on the second cylinder of the previous work, in which the separation point was observed to shift towards downstream by the tripping-wires set on the first cylinder.

At about $c/d = 4.0$ where the jumping phenomenon occurs, Nu$_m$ increases suddenly for the first and second cylinders but decreases somewhat for the third and fourth ones. As the cylinder spacing increases beyond the jumping one, a vortex street is formed in the wake of the first cylinder and then the heat transfer behavior of the first cylinder approaches that of a single cylinder. The separated shear layer from the first cylinder is entrained into the upstream of the second one and it results in a sudden increase of the heat transfer coefficient for the second cylinder. These situations may be to those for the single cylinder in a strong turbulent flow. On the other hand, the entrainment of the main flow around the third and fourth cylinders decreases after the jumping phenomenon and Nu$_m$ decreases, as discussed later.

At a very small cylinder spacing such as $c/d = 1.15$, Nu$_m$ decreases for all the four cylinders. Re$_d = 41000$ is lower than the critical Reynolds number for $c/d = 1.15$. 

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Fig. 9 Variation of pressure drag coefficient with c/d at Re_d = 41000. -- -- -- C_d for the second cylinder at Re_d = 40000(8); --- --- mean value of C_d for all the four cylinders

(a) c/d = 1.3

(b) c/d = 1.8

(c) c/d = 2.8

(d) c/d = 4.5

Fig. 10 Mean Nusselt number

The average Nusselt number of the four cylinders is included as a dash-dotted line in the figure and it exhibits a maximum at about c/d = 1.5. As shown in Fig. 9, the mean pressure drag coefficient of the four cylinders becomes maximum at a specific value of c/d, which is a little larger than c/d = 1.5.

The variation of C_d with c/d is shown at Re_d = 41000 in Fig. 9. The data on the first and second cylinders exhibit complicated variations similar to those on two cylinders of in-line arrangement(10). The results with the third and fourth cylinders, on the other hand, are nearly equal to each other and they show small changes with c/d as compared with those for the first and second cylinders. This may suggest that the heat transfer characteristics are almost the same for the third and fourth cylinders. The previous data(8) on the second one of the three cylinders of in-line arrangement agree well with the present ones on the second cylinder though some quantitative deviation can be detected. It is interesting to notice that the mean value of C_d for all the four cylinders is almost equal to that for the third and fourth cylinders in the range of c/d = 4.0.

The mean Nusselt numbers for all the four cylinders at c/d = 1.3, 1.8, 2.8 and 4.8 are summarized in Fig. 10. It has been generally accepted that the heat transfer rate from the cylinder in tube banks increases with the number of cylinders in the downstream(10). However in the present results, such feature is not necessarily observed except in the case of c/d = 1.3 and Re_d = 21000. That is, in the case of c/d = 1.8, Nu_m for the third cylinder is larger than that for the fourth one, and in the case of c/d = 4.5, the results with the third and fourth cylinders are always smaller than that with the second cylinder in the range of Reynolds numbers examined in the present study.

There have been several experiments on the heat transfer in the tube banks(10,11,12),
in which only a measured cylinder was heated. Fig. 11 shows a difference between the results obtained under such condition and those measured under heating of all the four cylinders. In order to investigate effects of heat flux, q was varied from about 880 to 2350 kcal/m²h. Its effects on Nu_m, however, were found to be negligible. The downstream cylinder exists in the thermal wake of the upstream one, and consequently Nu_m for the downstream cylinder becomes smaller as compared with that for the same cylinder when its upstream one is not heated. Its decreasing rate is maximum for the second cylinder, about 10%, and several percent for the third and fourth cylinders. Such difference increases furthermore at lower Reynolds number than Re_c. This corresponds to the case of c/d = 1.3 and Re_d = 18000 in the figure.

\[ \text{Nu}_m \]

\( \text{Re}_d = 1 \times 10^5 \)

\( \text{Re}_d = 1.8 \times 10^5 \)

\( 1 \text{st}, 2 \text{nd}, 3 \text{rd}, 4 \text{th} \)

(a) c/d = 1.3

\[ \text{Nu}_m \]

\( \text{Re}_d = 1 \times 10^5 \)

\( \text{Re}_d = 1.8 \times 10^5 \)

\( 1 \text{st}, 2 \text{nd}, 3 \text{rd}, 4 \text{th} \)

(b) c/d = 1.8

Fig. 11 Effects of not heating the cylinders except the measured one, heating all the cylinders, and heating only the measured cylinder.

3.3 Temperature, velocity and turbulence intensity

Fig. 12 describes the temperature, velocity and streamwise turbulence intensity distributions in case of c/d = 1.8 and Re_d = 39100. In the wake of the first cylinder, for example at x/d = 0.77, the temperature is quite higher than \( T_m \) and it results in a decrease of Nu_m for the second cylinder by heating the upstream cylinder as described in Fig. 11. In the downstream from the second cylinder, the temperature field diffuses rapidly and the temperature difference between the wake and the free stream decreases.

As far as the velocity distribution is concerned, a considerable velocity defect occurs in the separated flow region between the first and second cylinders but it decreases toward the downstream. The turbulence field diffuses to the downstream similarly to the temperature profile but its intensity increases in the same direction.

Half the wake width is, now, defined as a distance from the wake axis to a point of maximum turbulence intensity. It is clear from Fig. 12 that the wake width in the downstream of the second and third cylinders is narrower than that of the first one. This may be due to the downstream shift of the separation point of the second and third cylinders. Therefore the main flow comes closer to the third and fourth cylinders than to the second one. These conjectures may be confirmed from the velocity and turbulence intensity distributions along the
cylinder surface measured at $\delta = 0.5\text{mm}$, as shown in Fig. 13. That is, the velocity on the third and fourth cylinders is, in general, higher than that on the second one and it becomes nearly equal to $U_w$ at about $\delta = 80\%$.

On the other hand, in case of a large cylinder spacing such as $c/d = 4.5$, the velocity on the second cylinder is found to be higher than that on the third and fourth cylinders. It results in a larger $Nu_d$ value for the former than for the latter, as exhibited in Figs. 8 and 10. The turbulence intensity attains its maximum near an attachment point of the separated shear layer from the upstream cylinder onto the third and fourth ones. Similar behavior can be detected in the results with a roughness element attached to a flat surface\(^{16}\). On the contrary, the turbulence intensity becomes minimum near the attachment point on the second cylinder. This may suggest that the inner part of the separated shear layer having a low turbulence intensity adheres the second cylinder.

Shown in Fig. 14 is a typical example of visualization studies, which were conducted in a water channel using floating aluminium powders. The photograph corresponds to a case of $c/d = 1.8$ and about $Re_d = 1300$, which is much lower than the present Reynolds number. However it may be of help to understand the flow pattern in the tube bank. It can be detected that the wake in the downstream of the second and third cylinders is narrower than that of the first one and thus the main flow comes closer to the former than to the latter. Such approach of the main flow to the third and fourth cylinders may be considered to be one of the most important factors to determine their heat transfer and flow characteristics. That is, the pressure on the front surfaces of the third and fourth cylinders is more recovered as compared to that of the second cylinder and the position of $C_{p_{\text{max}}}$ and $Nu_{\text{max}}$ for the former locates at farther upstream than that for the latter. This results in a high heat transfer rate on the front surfaces of the third and fourth cylinders. Though not clear in Fig. 14 the

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Fig. 13 Velocity and streamwise turbulence intensity distributions along cylinder surface for $c/d = 1.8$ and $Re_d = 40000$ ($\delta = 0.5\text{mm}$)

Fig. 14 Flow visualization photograph for $c/d = 1.8$ and $Re_d = 1300$. Exposure time $1/30\text{ sec.}$
separated shear layer from the first cylinder is entrained into the wake in the downstream of the second cylinder. In case of large cylinder spacing of \(c/d > 4.0\), that is entrained into the upstream of the second cylinder. Thus the heat transfer rate decreases slightly (see Fig. 8), since the velocity and turbulence intensity of an oncoming flow to the third and fourth cylinders decrease somewhat as compared with those of one to the second cylinder.

3.4 Correlation between heat transfer and flow

It is clear from the descriptions noted above that the heat transfer characteristics of the third and fourth cylinders are much different from those of the first and second cylinders. In consideration of the present results along with the previous ones\(^{(7)}\)\(^{(8)}\), their differences may be summarized as follows. The cases of \(c/d < 4.0\) are especially described in the following by considering their applications to industrial heat exchangers having small cylinder spacing.

(i) In spite of a relatively large velocity defect, a high turbulence intensity of the oncoming flow to the third and fourth cylinders causes an increase of the heat transfer on their front surfaces as compared with that on the first cylinder. A flow around the third and fourth cylinders becomes similar to one around a single cylinder in the transcritical flow region\(^*\) because of such a high turbulence intensity. Consequently their boundary layer thickness may be small as compared with that on the first cylinder, resulting in an increase of the heat transfer rate noted above. Furthermore the downstream shift of the separation point decreases the wake width and it produces a high heat transfer coefficient on the rear surface of those cylinders\(^{(8)}\).

(ii) Higher heat transfer rate for the second cylinder than for the first one may be due to a large velocity gradient of the separated shear layer from the first cylinder. On the other hand, in cases of the third and fourth cylinders, the oncoming flow to them maintains a low temperature and a high turbulence intensity. Furthermore the downstream shift of the separation point on their upstream cylinders, with a decrease of the wake width, results in a closer approach of the free stream. These factors yield a high heat transfer rate for the third and fourth cylinders as compared with that for the second one.

4. Concluding remarks

The heat transfer and flow around the third and fourth ones of four cylinders in

\* In the present paper, the definition of the flow regime is based on Achenbach\(^{(17)}\).

in-line arrangement were measured. The main results obtained are summarized as follows.

(1) Their heat transfer and flow characteristics are very close to each other.

(2) Their mean heat transfer coefficient shows a weak dependency on the cylinder spacing, as compared with that for the first and second cylinders.

(3) The angular position of maximum heat transfer coefficient locates farther upstream than that for the second cylinder in the range of \(c/d < 4.0\).

(4) The separation point locates at about \(\varphi = 110^\circ\) to \(115^\circ\) independently of the Reynolds number in the range of \(c/d < 4.0\).

(5) The wake width in the downstream of the second and further downstream cylinders is narrower than that of the first one, and this may be one of the most important factors to determine the heat transfer characteristics for the third and fourth cylinders.

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