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Analysis of Forced Vibration with Reduced

Impedance Method*

(Part 1. Introduction of Analytical Procedure)

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Forced vibrations of mechanical structures are analyzed by the reduced impedance method the authors proposed. A structure is divided into some components, and the mechanical impedances of these components are calculated by the finite element method. These impedances are reduced and combined to get a united equation of the forced vibration.

The forced vibrations of a plate, a box specimen and an actual cylinder block are analyzed by this method, changing the damping coefficient and the number of the finite elements of the components. On the other hand, the steady state vibration is measured under a harmonic exciting force, and the mechanical impedance is determined from the measured wave of the force and the acceleration. The calculated results are compared with the experimental ones in order to check the accuracy of the theoretical analysis, and both results agree well with each other.

1. Introduction

As the structures of many kinds of machines are so complex that their vibrations can be hardly analysed by the usual methods, for instance the finite element method (FEM in brief) or the transfer matrix method. Klosterman⁽¹⁾ proposed a new method named the building block analysis in order to conquer this difficulty. According to this method, the total structure of a machine is divided into some substructures, and the dynamic characteristics of each substructure are obtained experimentally by a harmonic vibration test or an impact test. The dynamic characteristics of all substructures are combined to make an equation of motion for the total structure. In this method, it is possible to determine the dynamic characteristics of the substructure by FEM instead of an experiment. But both the eigenvalue analysis and the modal analysis must be conducted in this case, and a vast calculation time is necessary to do these two analyses of all substructures.

One of the authors proposed a general method of vibrational analysis named the reduced impedance method (RIM in brief) in order to analyze theoretically the vibration of complex mechanical structures. The dynamic characteristics of substructures can be determined directly and simply by the authors' method without doing the eigenvalue analysis and the modal analysis. The authors analysed the natural frequencies and

the natural modes of the impellers of the compressor⁽²⁾ and the cylinder block of the diesel engine⁽³⁾⁽⁴⁾ by RIM, and as the result, proved that RIM is a useful method for the eigenvalue analysis.

In the present report, the problem of forced vibration is tentatively solved by RIM. First, the basic procedure of analysis is explained. Next, dynamic responses of several kinds of model structures forced in harmonic vibration are calculated by RIM, and accuracy of calculation is checked by experiments. Finally, forced vibrations of two kinds of cylinder blocks are calculated.

2. Basic procedure of analysis of forced vibration by the reduced impedance method.

The total structure of a complex machine is divided into some substructures each of which can be analysed by the usual method, for instance, FEM or the transfer matrix method. In a substructure (which is written sub. later for simplification), the region which connects with the adjacent subs. is called the region b. The regions where the vibrational force acts and where the dynamic response should be detected are also included in the region b. The part except the region b is called the region a. FEM is applied to one sub. which is cut off from all other subs., and the following equation of motion is obtained.

$$[M]\{\ddot{\delta}\} + [C]\{\dot{\delta}\} + [K]\{\delta\} = \{F\} \quad (1)$$

where $[M]$, $[C]$ and $[K]$ are the mass, the damping and the stiffness matrices of this sub. respectively. $\{\delta(t)\}$ and $\{F(t)\}$ are the nodal displacement and the nodal force vec-

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tor respectively. When the force is expressed with a harmonic function, the displacement takes the same shape, Thus

$$\begin{cases} \{F\} = \{F_0\} e^{j\omega t} \\ \{\delta\} = \{\delta_0\} e^{j\omega t} \end{cases} \quad (2)$$

where $\{F_0\}$ is the amplitude of force, and $\{\delta_0\}$ is the complex amplitude of displacement. Substitute eq. (2) into eq. (1),

$$\begin{cases} [A]\{\delta_0\} = \{F_0\} \\ [A] = [K] + j\omega[C] - \omega^2[M] \end{cases} \quad (3)$$

As no force acts on the region a, eq. (3) becomes

$$\begin{bmatrix} A_{aa} & A_{ab} \\ A_{ba} & A_{bb} \end{bmatrix} \begin{bmatrix} \delta_{0a} \\ \delta_{0b} \end{bmatrix} = \begin{bmatrix} 0 \\ F_{0b} \end{bmatrix} \quad (4)$$

The complex amplitude of the region a is eliminated, using the upper part of eq. (4), and the lower part of eq. (4) becomes

$$[Z]\{\delta_{0b}\} = \{F_{0b}\} \quad (5)$$

where

$$[Z] = [A_{bb} - A_{ba}A_{aa}^{-1}A_{ab}] \quad (6)$$

The matrix $[Z]$ defined with eq. (6) is called the reduced impedance in this sub., though strictly speaking it is a dynamic stiffness. Calculation of the inverse matrix $[A_{aa}]^{-1}$ in eq. (6) is troublesome and is liable to yield errors usually. In order to avoid this trouble, Gauss's method of elimination is applied only to the upper part of eq. (4), thus

$$\begin{bmatrix} A'_{aa} & A'_{ab} \\ 0 & A'_{bb} \end{bmatrix} \begin{bmatrix} \delta_{0a} \\ \delta_{0b} \end{bmatrix} = \begin{bmatrix} 0 \\ F_{0b} \end{bmatrix} \quad (7)$$

The lower part of eq. (7) is the same as eq. (5), and $[A'_{bb}] = [Z]$. $[Z]$ is obtained far more simply and accurately by this new process than by eq. (6).

After getting the reduced impedance of all subs., these are combined to make a united equation of motion for the total structure. Here, we consider a structure composed of 3 subs. as shown in Fig. 1.

Fig. 1 Structure composed of 3 subs.

- 1, 2, 3: No. of subs.
- b1: Connecting part between subs. 1 and 2.
- b2: Connecting part between subs. 2 and 3.
- b3: Connecting part between subs. 3 and 4.
- b4: Generated part with the external force.
- b5: Part where reaction should be detected.

Three subs. are connected with one another in the regions b1, b2 and b3. The external force acts on the region b4, and the displacement on the region b5 should be detected. Eq. (6) for these 3 subs. is rewritten in the following 3 equations.

$$\begin{bmatrix} Z_{b1}^1 & Z_{b1b3}^1 & Z_{b1b5}^1 \\ Z_{b3b1}^1 & Z_{b3}^1 & Z_{b3b5}^1 \\ Z_{b5b1}^1 & Z_{b5b3}^1 & Z_{b5}^1 \end{bmatrix} \begin{bmatrix} \delta_{0b1} \\ \delta_{0b3} \\ \delta_{0b5} \end{bmatrix} = \begin{bmatrix} -F_{0b1} \\ F_{0b3} \\ 0 \end{bmatrix} \text{ for sub. 1} \quad (8)$$

$$\begin{bmatrix} Z_{b1}^2 & Z_{b1b2}^2 \\ Z_{b2b1}^2 & Z_{b2}^2 \end{bmatrix} \begin{bmatrix} \delta_{0b1} \\ \delta_{0b2} \end{bmatrix} = \begin{bmatrix} F_{0b1} \\ -F_{0b2} \end{bmatrix} \text{ for sub. 2} \quad (9)$$

$$\begin{bmatrix} Z_{b2}^3 & Z_{b2b3}^3 & Z_{b2b4}^3 \\ Z_{b3b2}^3 & Z_{b3}^3 & Z_{b3b4}^3 \\ Z_{b4b2}^3 & Z_{b4b3}^3 & Z_{b4}^3 \end{bmatrix} \begin{bmatrix} \delta_{0b2} \\ \delta_{0b3} \\ \delta_{0b4} \end{bmatrix} = \begin{bmatrix} F_{0b2} \\ -F_{0b3} \\ F_{0e} \end{bmatrix} \text{ for sub. 3} \quad (10)$$

Combining these 3 eqs., the united equation becomes

$$\begin{bmatrix} Z_{b1}^1 + Z_{b1}^2 & Z_{b1b2}^2 & Z_{b1b3}^3 & 0 & Z_{b1b5}^1 \\ Z_{b2b1}^2 & Z_{b2}^2 + Z_{b2}^3 & Z_{b2b3}^3 & Z_{b2b4}^3 & 0 \\ Z_{b3b1}^1 & Z_{b3b2}^3 & Z_{b3}^3 + Z_{b3}^4 & Z_{b3b4}^3 & Z_{b3b5}^1 \\ 0 & Z_{b4b2}^3 & Z_{b4b3}^3 & Z_{b4}^3 & 0 \\ Z_{b5b1}^1 & 0 & Z_{b5b3}^1 & 0 & Z_{b5}^1 \end{bmatrix} \begin{bmatrix} \delta_{0b1} \\ \delta_{0b2} \\ \delta_{0b3} \\ \delta_{0b4} \\ \delta_{0b5} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ F_{0e} \\ 0 \end{bmatrix} \quad (11)$$

Applying Gauss's method of elimination to the upper three lines of eq. (11),

$$[Z_c] \begin{bmatrix} \delta_{0b4} \\ \delta_{0b5} \end{bmatrix} = \begin{bmatrix} F_{0e} \\ 0 \end{bmatrix} \quad (12)$$

Equation (12) can be solved easily because $[Z_c]$ is a small matrix, and

$$\begin{bmatrix} \delta_{0b4} \\ \delta_{0b5} \end{bmatrix} = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \{F_{0e}\} \quad (13)$$

where $[G_1]$ is the self compliance of the region b4, and $[G_2]$ is the mutual compliance between the regions b4 and b5. Even when the number of subs. is larger than in Fig. 1, the same analytical process can be adopted.

Though FEM is used in the above process, the transfer matrix method or a simple theory of mass-spring system can be used instead, and the reduced impedance matrix $[Z]$ can be obtained experimentally in a harmonic vibration test or an impact test just as in the building block analysis.

When some of the subs. are in the same shape, this RIM is especially efficient,

because the same reduced impedance matrix $[Z]$ can be used repeatedly.

3. Experimental procedure

The experimental procedure is shown in Fig.2. The harmonic voltage of a arbitrary frequency is generated by the low

Fig.2 Schematic diagram of experimental system.

- A: Low frequency generator of automatic sweep type.
- B, H, I, J: Amplifier.
- C: Electro-dynamic vibrator.
- D: Impedance head.
- E: Specimen.
- F: Acceleration pick-up.
- G: Displacement pick-up non-contact type.
- K, L: Tracking filter.
- M: Operator.
- N: Oscilloscope.
- P: 2 channel X-Y recorder.

frequency generator A. The specimen E is vibrated by an electro-magnetic vibrator. The dynamic load is detected by the impedance head D. The acceleration and the displacement of the specimen are detected by the piezo-electric acceleration pick-up F and the displacement pick-up of non-contact type G respectively. All detected signals are filtered by the tracking filters K and L, and sent to the calculator M where the compliance is calculated. The gain and the phase of the compliance are recorded in the frequency domain by two-channel X-Y recorder P. The mode of vibration is obtained by synthesizing the compliances between many measured points on the specimen.

4. Discussion of the results

4.1. Flat plate

A rectangular flat plate of steel with one fixed end is divided into 2 subs. of equal size as in Fig.3. Each sub. is di-

Fig.3 Division of rectangular flat plate with one fixed end

Steel. 2.9mm thickness.

F: Forced point.

A, B: Measured point.

vided again into triangular elements of 18 degrees of freedom (3 nodes). Six types of finite division B1~B6 in Fig.3 are adopted, but 2 subs. in a plate are divided in equal type in every case. The harmonic force is given at the point F, and the displacement is measured at the points A and B.

First, undamped compliances are calculated between 1Hz and 691Hz in every 10Hz, and the same calculation is done in every 1Hz near the natural frequencies. As shown in Table 1, the calculative errors of the natural frequencies are smaller in B1 than in B2, and also smaller in B4 than in B5, though the same number of finite elements is used in each of these two cases. The accuracy of calculation is the best in B4 at least in the lowest 6 natural modes. Table 1 shows also that the calculation time is proportional to about 2.4 power of the number of the nodes.

M.P.	B1	B2	B3	B4	B5	B6
Time 1	10.4 _s	10.4 _s	37.5 _s	88.0 _s	73.9 _s	109.2 _s
Time 2	8.1 _s	8.1 _s	26.2 _s	65.1 _s	51.8 _s	80.7 _s
O.N.	Exp.	Cal.				
1	27	27	27	27	25	27
2	167	181	182	169	186	172
3	196	194	209	194	204	194
4	456	561	562	476	603	496
5	580	623	685	605	606	653
						608

Table 1 Time of calculation and natural frequency (Hz) of plane plate

Time 1: Calculation between 1Hz~691Hz in every 10Hz.

Time 2: Calculation near natural frequency in every 1Hz.

M.P.: Method of calculation.

O.N.: No. of order.

Exp.: Experimental result.

Cal.: Calculated value.

Next, the gain and the phase of damped compliances are calculated under the proportional damping, namely

$$[C] = \alpha [M] \quad (14)$$

In Fig.4., the compliance near the 1st order natural frequency is shown in various values of the proportional coefficient α . The calculated compliance (the solid lines) agrees best with the experimental one (the dotted lines) when $\alpha = 3$. From the same kind of calculation, the best values of α are known to be 3, 10, 30, 40, in the 2nd, 3rd, 4th and 5th orders of natural frequencies respectively. The solid line of Fig.5 represents the damped compliance between the points F and A calculated with these values of α . Meanwhile the circular marks

and the dotted lines are the calculated undamped compliance and the experimental result respectively. The calculated damped

Fig.4 Damped compliance of flat plate between F and A.

Vicinity of 1st natural frequency.
Solid line: Calculated result
[C] = α [M].
Dotted line: Experimental result.

Fig.5 Compliance of flat plate between F and A.

Solid line: Calculated result
(Damped value [C] = α [M]).
Circular mark: Calculated result
(Undamped value).
Dotted line: Experimental result.

compliance agrees well with the experimental one except the higher frequency part. Though undamped compliance becomes infinite at the natural frequencies, it agrees also well with the experimental one except the vicinities of the natural frequencies. Fig.6 shows the compliance between the points F and B, and the calculated undamped one agrees well with the experimental one.

Fig.6 Compliance of flat plate between F and B.

Solid line: Calculated result
(Undamped value).
Dotted line: Experimental result.

4.2. L-shaped plate

An L-shaped steel plate with one fixed end is divided into 2 subs., and each sub. is divided again into 24 triangular finite elements of 18 degrees of freedom, as shown in Fig.7. Table 2 lists the 1st~5th orders natural frequencies, and the calculated ones are in good agreement with the experiment ones. Figures 8 and 9 show the frequency spectrums of the mutual compliance.

Fig.7 Shape and dimension of L-shaped plate with one fixed end.

Steel. 2.9mm thickness.
F: Forced point.
A, B: Measured point.

Fig.8 Compliance of L plate between F and A.

Solid line: Calculated result
(Undamped value).
Dotted line: Experimental result.

Fig.9 Compliance of L plate between F and B.

Solid line: Calculated result
(Undamped value).
Dotted line: Experimental result.

O.N.	Exp.	Cal
1	20	23
2	55	55
3	250	262
4	390	384
5	594	629

Table 2 Natural frequency (Hz) of L plate

O.N.: No. of Order.
Exp.: Experimental result.
Cal.: Calculated value.

The calculated undamped compliance agrees with the experimental compliances except in the vicinities of the natural frequencies.

4.3. Box model

The third example is a box modes of steel with 2 partitions. This model is divided into 4 subs., namely 2 side plates and 2 partitions. The vertical ends of each side plate are fixed. All subs. are divided into triangular finite elements of 18 degrees of freedom as shown in Fig.10.

Fig.10 Shape and dimension of box model

3mm thickness.
F: Forced point.
A, B: Measured point.
Box model is divided into 4 subs.
Solid marks are fixed nodes.
Subs. are connected with each other at hollow marks.

Table 3 lists the 1st~8th orders natural frequencies, and the calculated values agree with the experimental ones within errors of 6%. Figure 11 shows the mutual undamped compliance between the points F and A. and Fig.12 does the compliance between the points F and B. In these 2 figures, the differences between the calculated and the experimental ones are a little larger than in case of the flat plate and of the L-shaped plate. The reason for these differences seems to be effect of the non-linear characteristic of welding part. In spite of these partial differences, the calculated results agree with the experimental ones.

O.N.	Exp.	Cal.
1	92	93
2	234	238
3	330	327
4	436	449
5	494	501
6	566	531
7		554
8	633	628

Table 3 Natural frequency (Hz) of box model

O.N.: No. of Order.
Exp.: Experimental result.
Cal.: Calculated value.

Fig.11 Compliance of box model between F and A

Solid line: Calculated result
(undamped value).
Dotted line: Experimental result.

4.4. Cylinder blocks of 4 and 6 cylinders

As the RIM was proved to be an effective method to analyze the vibration of the above 3 kinds of model specimens, this method is adopted to analyze the forced vibration of two kinds of cylinder blocks of diesel engines in industrial vehicles. A cylinder block of 4 cylinders is divided into 7 subs. of thin shells as shown in Fig.13. Meanwhile the cylinder block of 6 cylinders is divided into 9 subs. of thin shells as shown in Fig.14. Each sub. is divided again into rectangular finite elements of 18 degrees of freedom. The maximum degree of freedom in one sub. is 468. The solid lines of Figs.15 and 16 show the calculated results of mutual compliances between the points F and A of the blocks of 4 and 6 cylinders respectively. The vertical dotted lines in Figs.15 and 16 show the experimental results of natural frequencies. The maximum error in the calculated results of the natural frequency is 7%.

Fig.12 Compliance of box model between F and B

Solid line: Calculated result (undamped value).
Dotted line: Experimental result.

Fig.13 Modeling of cylinder block of 4 cylinders

Sub.1: Right wall.
Sub.2: 4: Partitions.
Sub.5: Cam wall.
Sub.6: Light wall.
Sub.7: Middle wall.

Fig.14 Modeling of cylinder block of 6 cylinders

Sub.1: Right wall.
Sub.2: 6: Partitions.
Sub.7: Cam wall.
Sub.8: Left wall.
Sub.9: Middle wall.

Fig.15 Undamped compliance of cylinder block of 4 cylinders (calculated result).

Dotted line: Experimental result of natural frequency.

Fig.16 Undamped compliance of cylinder block of 6 cylinders (calculated result).

Dotted line: Experimental result of natural frequency.

5. Conclusions

The basic procedure of analysis of the forced vibration by the reduced impedance method (RIM) the authors proposed is explained in the present report. Calculations with RIM and experiments are conducted using three kinds of model specimens, and both results about the natural frequencies and the compliances are compared with each other. The forced vibration of actual cylinder blocks of 4 and 6 cylinders is calculated by RIM. As the result, it is proved that RIM is an efficient method to analyze the forced vibration of mechanical structure so complex that its vibration can hardly be analyzed with direct application of FEM.

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