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Compressibility Effects on Cavitation in High Speed Liquid Flow

(First Report-Subsonic Liquid Flow)

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Firstly the freestream Mach number for subsonic liquid flow is defined as being smaller than the lower critical Mach number in which the critical condition can be found on the cavity streamline. The governing equation of elliptic type for velocity potential is solved by local linearization technique and then the corresponding relation are derived between subsonic and incompressible liquid flows. Compressibility effects are minutely discussed on the cavity characteristics of symmetrical wedge through the Mach number range and also comparisons are made with the former linearized analysis.

1. Introduction

The recent researches on cavity flows have particularly drawn attention to the speed increase of blade elements of rocket pumps in liquid e.g. water. The question of compressibility effects on flow pattern and cavity characteristics of supercavitating hydrofoil elements at high speed are urgently calling for clarification for the achievement of a high performance of hydraulic machinery. In order to attain this goal, the simplest and most versatile approximate method is one based on a uniform linearization[1] of governing equations which stem from the pioneering work of Prandtl-Glauert. Although this linearized theory of compressible cavity flows has been developed in recent applications, it has two limitations that are of significance in the following discussion: Firstly, the linearized theory gives only a first order approximation which cannot ascertain the local features peculiar to a compressible perturbed flow field. Secondly, the most significant hindrance is that the linearized theory yields infinite perturbed velocity and fluid forces at Mach number 1 and in general requires that the freestream Mach number be sufficiently removed from unity such that the flow is either purely subsonic or purely supersonic. Therefore, corrections are necessary and a higher order theory is desirably developed in cavity flows to fill the needs.

The difficulty of solving the nonlinear differential equation of motion in two-dimensional compressible inviscid liquid is not a new one. The use of hodograph technique can transform this nonlinear equation to a linear form and can provide useful tool for solving the equation of nonlinear type. But the resulting boundary value problem is

still more difficult to solve. Very recently, Nishiyama and Higashino[2] have proposed a method based on this technique with a pressure correction formula given by Karman-Tsien[3]. But their technique has shown great complexity in computation and therefore appears not to be feasible.

Against the above stated backgrounds, this paper is timely presented which is found simple and suitable for practical application. The analysis is based on a novel method capable of yielding useful results throughout that Mach number range which may fall outside of subsonic region which is of more accuracy than any one proposed up to date. The analytical technique is based on local linearization by Spreiter[4] in transonic gas flow.

The basic general relations between velocity, pressure and sound speed in compressible liquid flows are shown within the framework of linearization technique. The range of freestream Mach numbers is definitely defined for the subsonic liquid flow corresponding to the cavitation number. Finally, numerical examples with a symmetrical wedge are illustrated to compare the present theory with the presently available linearized and hodograph theories[1,2] and also to examine its accuracies.

2. Isentropic Relation in Liquid Flow

2.1 Energy Equation

The conventional equation of state for water proposed by Tait[5] is

$$\frac{P+B}{\rho^\kappa} = \text{constant}, B = 3000 \text{ kg/cm}^2; \kappa = 7.15 \quad (1)$$

The local sound speed, the equation of continuity and Bernoulli's equation for compressible liquid are respectively

$$a^2 = \frac{\partial P}{\partial \rho} = \kappa \frac{P+B}{\rho} \quad \dots \dots \dots (2)$$

$$\text{div}(\rho \mathbf{q}) = 0; \mathbf{q} = \mathbf{q}(u, v) \quad \dots \dots \dots (3)$$

$$d(q^2/2) + dP/\rho = \text{constant} \quad \dots \dots \dots (4)$$

By using Eqs. (1), (2) and (4), an energy equa-

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tion for water can readily be derived as

$$\kappa \frac{P+B}{\rho} + \frac{\kappa-1}{2} q^2 = \text{constant} \dots\dots\dots (5)$$

Also,

$$\frac{(P+B)}{(P_0+B)} = \left(1 + \frac{\kappa-1}{2} M^2\right)^{-\kappa/(\kappa-1)}; M = \frac{q}{a} \dots\dots (6)$$

2.2 Lower Critical Mach Number

From Eq.(6), we can write

$$\frac{(P_c+B)}{(P_\infty+B)} = \left[\frac{1 + (\kappa-1)/2 M_c^2}{1 + (\kappa-1)/2 M_\infty^2} \right]^{-\kappa/(\kappa-1)} \dots\dots\dots (7)$$

Now defining the cavitation number as

$$\sigma \equiv \frac{(P_\infty - P_c)}{(1/2)\rho_\infty U^2} \dots\dots\dots (8)$$

we can get from Eq.(7),

$$M_c^2 = \frac{2}{(\kappa-1)} \left[\frac{1 + (\kappa-1)/2 M_\infty^2}{(1-\kappa\sigma M_\infty^2/2)^{(\kappa-1)/\kappa}} - 1 \right]; \quad \sigma < 2/\kappa M_\infty^2 \dots\dots\dots (9)$$

The lower critical Mach number is defined as the lowest subsonic freestream Mach number at which the local sonic velocity first occurs on the surface of the body. For a symmetrical wedge in subsonic cavity flows, as the freestream Mach number is increased, sonic velocity first occurs on the cavity. Thus, when the cavity Mach number M_c becomes just unity, Eq.(9) then provides a lower critical Mach number from the isentropic relation and the whole flow field can be considered subsonic throughout until it attains this lower critical speed. Hence, the following equation is to be solved for the lower critical Mach number:

$$\left(1 + \frac{1}{2} \sigma \kappa (M_\infty^2)_{cr, l.}^2\right)^{\frac{(\kappa-1)}{\kappa}} - \frac{1}{(\kappa+1)} \left(2 + (\kappa-1) (M_\infty^2)_{cr, l.}^2\right) = 0 \dots\dots (10)$$

3. Governing Equations

3.1 Steady Nonlinear Potential Equation

A two-dimensional rectangular Cartesian coordinate (x, y) is taken as shown in Fig.1 in an inviscid compressible liquid flow past an arbitrary symmetrical wedge of half the apex angle θ , the x -axis being parallel to the freestream velocity. Let the perturbation potential be ϕ and the perturbation velocity components parallel to x and y axes be $u = \partial\phi/\partial x$ and $v = \partial\phi/\partial y$ respectively. Now assume that these perturbation velocities are so small that we can neglect the terms $M_\infty^2 (u/U)^2 \ll 1$, $M_\infty^2 uv/U^2 \ll 1$, $M_\infty^2 (v/U)^2 \ll 1$ and its higher orders. Then by using Eqs.(3) and (4), we can get

$$(1-M_\infty^2) \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = (\kappa+1) M_\infty^2 \frac{u}{U} \frac{\partial u}{\partial x} + (\kappa-1) M_\infty^2 \frac{u \partial v}{U \partial y} + M_\infty^2 \frac{v}{U} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \dots\dots (11)$$

The left-hand side of Eq.(11) is linear, but on the right it is nonlinear and contains κ

Fig.1 Correspondence between two-dimensional compressible and incompressible flows for a symmetrical supercavitating wedge.

in the first two terms. Though $\kappa=7.15$, further simplifications are possible if and only if the perturbation velocities compared with the freestream Velocity U and velocity gradients of u and v in y -direction are sufficiently small. Thus we may neglect all the terms on right-hand side, in comparison to those on the left, to give the linearized perturbation potential equation [1]. But if we impose the condition $M_\infty \rightarrow 1$, then the coefficient of $\partial u/\partial x$ on the left-hand side becomes negligibly small and in that case we cannot neglect the first term on the right-hand side of Eq.(11). However, the condition $M_\infty \rightarrow 1$ does not affect the term $\partial v/\partial y$ and so all the other terms on the right-hand side of Eq.(11) may still be neglected in a linearized manner to give a two-dimensional steady nonlinear potential equation valid for subsonic, transonic and supersonic liquid flows as follows:

$$(1-M_\infty^2) \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = (\kappa+1) M_\infty^2 \frac{u}{U} \frac{\partial u}{\partial x} \dots\dots\dots (12)$$

Eq.(12) is in formal agreement with the transonic gas flow equation [8].

The boundary conditions require that the perturbation velocity vanish at infinity and the flow be tangential to the surface of the wedge. For a symmetrical wedge the second condition may be approximated to

$$\left(\frac{\partial \phi}{\partial y}\right)_{y=0} = (v)_{y=0} = U \frac{dy}{dx} \dots\dots\dots (13)$$

3.2 Reduction to Incompressible Flow by Local Linearization

The freestream Mach number for subsonic liquid flow is hereafter defined as being smaller than the critical lower Mach number. The subsonic flows represented by the equation of elliptic type are recognized by the sign of total coefficient of $\partial u/\partial x$ in Eq.(12). For subsonic approximation we rewrite this equation in the following form:

$$\beta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad \dots\dots\dots (14)$$

where ,

$$\left. \begin{aligned} \beta^2 = (1-M^2) = 1-M_\infty^2 - (\kappa+1)M_\infty^2 \frac{u}{U} > 0 ; \\ 0 \leq M_\infty \leq (M_\infty)_{cr,l} \end{aligned} \right\} \dots\dots (15)$$

Now assuming that β is finite and varies so slowly that the derivative can be locally disregarded [4] . Then by using the following transformations:

$$\begin{aligned} x = x_i, \quad \beta y = y_i; \quad \phi(x,y) = n\phi_i(x_i, y_i); \\ n = \text{constant} \dots\dots (16) \end{aligned}$$

we have,

$$\frac{\partial^2 \phi_i}{\partial x_i^2} + \frac{\partial^2 \phi_i}{\partial y_i^2} = 0 \quad \dots\dots\dots (17)$$

Here (x_i, y_i) and ϕ_i are the corresponding coordinate and perturbation potential in a two dimensional incompressible flow field . The perturbation velocities in two-dimensional subsonic liquid flows are

$$u = nu_i; \quad v = n\beta v_i \quad \dots\dots\dots (18)$$

Under the condition of keeping the form of wedge constant in compressible and incompressible liquid flows , the tangential flow condition on the surface of a super-cavitating wedge yields

$$n = \frac{1}{\beta} \quad \dots\dots\dots (19)$$

therefore ,

$$u = \frac{1}{\beta} u_i \quad \dots\dots\dots (20)$$

On differentiation, we have

$$\frac{\partial u}{\partial x} = \frac{1}{\beta} \frac{\partial u_i}{\partial x_i} \quad \dots\dots\dots (21)$$

Now inserting the nonconstant expression for β at this stage and performing integration we can obtain from Eq.(21)

$$-\frac{2}{3} \frac{U}{(\kappa+1)} \frac{1}{M_\infty^2} \left[1-M_\infty^2 - (\kappa+1)M_\infty^2 \frac{u(x)}{U} \right]^{\frac{3}{2}} = u_i(x_i) + \text{constant} \dots\dots (22)$$

The constant of integration is determined from the fact that for any β , Eq.(20) implies that $u = 0$ when $u_i = 0$. Thus ,

$$\text{constant} = -\frac{2}{3} \frac{U(1-M_\infty^2)^{\frac{3}{2}}}{(\kappa+1)M_\infty^2} \quad \dots\dots\dots (23)$$

Therefore the correspondence between the perturbation velocities in compressible and incompressible liquid flows can be put in the final form as

$$\frac{u(x)}{U} = \frac{(1-M_\infty^2)}{(\kappa+1)M_\infty^2} \left\{ 1 - \left[1 - \frac{3(\kappa+1)}{2} \cdot \frac{M_\infty^2}{(1-M_\infty^2)^{\frac{3}{2}}} \cdot \frac{u_i(x_i)}{U} \right]^{\frac{2}{3}} \right\} \dots\dots\dots (24)$$

In order to insure that the basic differential equation is of elliptic type in subsonic flow, we have initially considered $\beta > 0$ which implies that Eq.(24) is applicable only when the term in the second bracket is positive i.e. when

$$\frac{u_i(x_i)}{U} \leq \frac{2}{3} \frac{(1-M_\infty^2)^{\frac{3}{2}}}{(\kappa+1)M_\infty^2} \quad \dots\dots\dots (25)$$

The pressure coefficient is approximated to first order by

$$C_P(x) \equiv \frac{(P - P_\infty)}{\frac{1}{2}\rho_\infty U^2} = (1 + \sigma)^{\frac{1}{2}} \left(\sigma - 2 \frac{u(x)}{U} \right) \dots\dots (26)$$

$$C_{Pi}(x_i) \equiv \frac{(P_i - P_{i\infty})}{\frac{1}{2}\rho_\infty U^2} = (1 + \sigma_i)^{\frac{1}{2}} \left(\sigma_i - 2 \frac{u_i(x_i)}{U} \right) \dots\dots (27)$$

Now using Eqs.(24),(25),(26) and (27) we can write further the pressure coefficient as follows:

$$\begin{aligned} C_P(x) = \sqrt{(1+\sigma)} \left\{ \sigma - \frac{2(1-M_\infty^2)}{(\kappa+1)M_\infty^2} + \frac{2}{(\kappa+1)M_\infty^2} \left\{ (1-M_\infty^2)^{\frac{3}{2}} \right. \right. \\ \left. \left. - \frac{3(\kappa+1)}{4} \sigma_i M_\infty^2 + \frac{3(\kappa+1)}{4\sqrt{(1+\sigma_i)}} M_\infty^2 C_{Pi}(x_i) \right\}^{\frac{2}{3}} \right\} \dots\dots (28) \end{aligned}$$

provided,

$$C_{Pi}(x_i) \geq \sqrt{(1+\sigma_i)} \left\{ \sigma_i - \frac{4(1-M_\infty^2)^{\frac{3}{2}}}{3(\kappa+1)M_\infty^2} \right\} \dots\dots (29)$$

3.3 Lower Critical Mach Number

As defined earlier in isentropic flow field, the lower critical Mach number is the lowest subsonic freestream Mach number at which the local sonic velocity first occurs on the cavity. This is equivalent mathematically to $\beta=0$ in Eq.(15). Now substituting $\beta=0$ gives the following equation to be solved for the lower critical Mach number:

$$(M_\infty)_{cr,l}^2 = \frac{2}{2+(\kappa+1)\sigma} \quad \dots\dots\dots (30)$$

The variation of this lower critical Mach number with cavitation number is shown in Fig.2 along with that obtained from the

Fig.2 Lower critical Mach number versus cavitation number
isentropic relation. It is found that the lower critical Mach number decreases with an

increase of cavitation number and for a small cavitation number the difference between the two lower critical Mach numbers is very small.

4. Cavity Characteristics

The corresponding cavitation number and thickness in cavity termination in a two-dimensional incompressible flow can be written from Eqs.(24) and (16) respectively as

$$\sigma_i = \frac{4(1-M_\infty^2)}{3(\kappa+1)M_\infty^2} \left\{ 1 - \left\{ 1 - \frac{(\kappa+1)}{2} \frac{M_\infty^2}{(1-M_\infty^2)} \sigma \right\}^{\frac{3}{2}} \right\} \quad (31)$$

provided, $\sigma \leq \frac{2(1-M_\infty^2)}{(\kappa+1)M_\infty^2}$

$$\frac{y_i(L_i, 0)}{y(L, 0)} = \frac{b_i}{b} = \sqrt{1-M_\infty^2 - (\kappa+1)/2 \cdot \sigma M_\infty^2} \quad \dots (32)$$

The cavity drag coefficients are given by

$$C_D \equiv \frac{D}{\frac{1}{2}\rho_\infty U^2 (2C\theta)} = \frac{1}{C\theta} \int_0^C C_{Pi}(x) \frac{dy}{dx} dx \quad \dots (33)$$

$$C_{Di} \equiv \frac{D_i}{\frac{1}{2}\rho_\infty U^2 (2C\theta)} = \frac{1}{C\theta} \int_0^C C_{Pi}(x_i) \frac{dy_i}{dx_i} dx_i \quad \dots (34)$$

Next we examine the relation between uniform and local linearization: Taylor's series expansion of Eqs.(31) and (32) yields respectively

$$\sigma \approx \frac{\sigma_i}{(1-M_\infty^2)^{\frac{1}{2}}} + \frac{(\kappa+1)M_\infty^2}{8(1-M_\infty^2)^2} \sigma_i^2 + \frac{(\kappa+1)^2 M_\infty^4}{24(1-M_\infty^2)^{\frac{7}{2}}} \sigma_i^3 + \dots (35)$$

$$\frac{y(L, 0)}{y_i(L_i, 0)} = \frac{b}{b_i} \approx \frac{1}{(1-M_\infty^2)^{\frac{1}{2}}} + \frac{(\kappa+1)M_\infty^2}{4(1-M_\infty^2)^{\frac{3}{2}}} \sigma + \frac{3(\kappa+1)^2 M_\infty^4}{32(1-M_\infty^2)^{\frac{5}{2}}} \sigma^2 + \dots (36)$$

From uniform linearization we have ,

$$\sigma = \sigma_i / (1-M_\infty^2)^{\frac{1}{2}} ; \frac{y(L, 0)}{y_i(L_i, 0)} = \frac{b}{b_i} = 1 / (1-M_\infty^2)^{\frac{1}{2}} \quad \dots (37)$$

Thus, Eq.(31) may be termed as a cavitation correction formula because the cavitation number σ_i is given in terms of σ and M_∞ with no further dependence on wedge shape. Fig.3 shows this correspondence between the cavi-

tation numbers in compressible and incompressible flows together with the uniform linearization. It turns out that the corresponding cavitation number in an incompressible flow with freestream Mach number obtained from the present theory is smaller than that of uniform linearization and has a lower limit.

Fig.4 shows the variation of thickness in cavity termination in comparison with uniform linearization. The thickness in cavity termination in an incompressible flow with freestream Mach number depends on

Fig.3 Correspondence between two-dimensional compressible and incompressible flows for cavitation number.

Fig.4 Correspondence between two-dimensional compressible and incompressible flows for cavity termination thickness.

cavitation number which is different from uniform linearization. A comparative study has been made in Table 1 to show the essential differences between the results of uniform and local linearization.

Table 1 Comparative study between the results of uniform and local linearization

Uniform Linearization	Local Linearization
$\frac{u}{U} = \frac{1}{(1-M_\infty^2)^{\frac{1}{2}}} \frac{u_i}{U}$	$\frac{u}{U} = \frac{(1-M_\infty^2)}{(\kappa+1)M_\infty^2} \left[1 - \left\{ 1 - \frac{3(\kappa+1)M_\infty^2}{2(1-M_\infty^2)^{\frac{3}{2}}} \left(\frac{u_i}{U} \right)^{\frac{2}{3}} \right\}^{\frac{3}{2}} \right] \approx \frac{1}{(1-M_\infty^2)^{\frac{1}{2}}} \left(\frac{u_i}{U} \right) + \frac{(\kappa+1)M_\infty^2}{4(1-M_\infty^2)^2} \left(\frac{u_i}{U} \right)^2 + \frac{(\kappa+1)^2 M_\infty^4}{6(1-M_\infty^2)^{\frac{7}{2}}} \left(\frac{u_i}{U} \right)^3 + \dots$
$\sigma = \frac{\sigma_i}{(1-M_\infty^2)^{\frac{1}{2}}}$	$\sigma = \frac{2(1-M_\infty^2)}{(\kappa+1)M_\infty^2} \left[1 - \left\{ 1 - \frac{3(\kappa+1)M_\infty^2}{4(1-M_\infty^2)^{\frac{3}{2}}} \sigma_i \right\}^{\frac{2}{3}} \right] \approx \frac{\sigma_i}{(1-M_\infty^2)^{\frac{1}{2}}} + \frac{(\kappa+1)M_\infty^2}{8(1-M_\infty^2)^2} \sigma_i^2 + \frac{(\kappa+1)^2 M_\infty^4}{24(1-M_\infty^2)^{\frac{7}{2}}} \sigma_i^3 + \dots$
$\frac{y(L,0)}{y_i(L_i,0)} = \frac{b}{b_i} = \frac{1}{(1-M_\infty^2)^{\frac{1}{2}}}$	$\frac{y(L,0)}{y_i(L_i,0)} = \frac{b}{b_i} = \frac{1}{(1-M_\infty^2 - \frac{(\kappa+1)}{2} \sigma M_\infty^2)^{\frac{1}{2}}} \approx \frac{1}{(1-M_\infty^2)^{\frac{1}{2}}} + \frac{(\kappa+1)M_\infty^2}{4(1-M_\infty^2)^{\frac{3}{2}}} \sigma + \frac{3(\kappa+1)^2 M_\infty^4}{32(1-M_\infty^2)^{\frac{5}{2}}} \sigma^2 + \dots$
$C_P = \frac{C_{Pi}}{(1-M_\infty^2)^{\frac{1}{2}}}$	$C_P = \frac{2(1-M_\infty^2)}{(\kappa+1)M_\infty^2} \left[1 - \left\{ 1 - \frac{3(\kappa+1)M_\infty^2}{4(1-M_\infty^2)^{\frac{3}{2}}} C_{Pi} \right\}^{\frac{2}{3}} \right] \approx \frac{C_{Pi}}{(1-M_\infty^2)^{\frac{1}{2}}} + \frac{(\kappa+1)M_\infty^2}{8(1-M_\infty^2)^2} C_{Pi}^2 + \frac{(\kappa+1)^2 M_\infty^4}{24(1-M_\infty^2)^{\frac{7}{2}}} C_{Pi}^3 + \dots$
$C_D = \frac{C_{Di}}{(1-M_\infty^2)^{\frac{1}{2}}}$	$C_D = \frac{2(1-M_\infty^2)}{(\kappa+1)M_\infty^2} \int_0^1 \left[1 + \frac{(1-M_\infty^2)}{(\kappa+1)M_\infty^2} \left\{ 1 - \left\{ 1 - \frac{3(\kappa+1)M_\infty^2}{4(1-M_\infty^2)^{\frac{3}{2}}} \sigma_i \right\}^{\frac{2}{3}} \right\} \right] \left[\left\{ 1 - \frac{3(\kappa+1)M_\infty^2}{2(1-M_\infty^2)^{\frac{3}{2}}} \frac{u_i(x_i)}{U} \right\}^{\frac{2}{3}} - \left\{ 1 - \frac{3(\kappa+1)M_\infty^2}{4(1-M_\infty^2)^{\frac{3}{2}}} \sigma_i \right\}^{\frac{2}{3}} \right] d\left(\frac{x}{C}\right)$

5. Numerical Examples and Discussion

Numerical calculations are made for the supercavitating wedge of apex angle 15 degrees in subsonic liquid flows. In order to carry out the calculation, the following set of equations are used in an incompressible flow field. The equations are derived from their integral form given in references [6, 7]

$$\frac{u_i(x_i)}{U} = \frac{\sigma_i}{2} \left[1 - \left\{ \frac{(x_i/L_i)}{(x_i/L_i - 1)} \right\}^{\frac{1}{2}} \right] + \frac{\theta}{\pi} \sqrt{1+\sigma_i} \left[\left\{ \frac{(x_i/L_i)}{(x_i/L_i - 1)} \right\}^{\frac{1}{2}} \right] \left[\log \left\{ \frac{(1+C/L_i)^{\frac{1}{2}} + (C/L_i)^{\frac{1}{2}}}{(1+C/L_i)^{\frac{1}{2}} - (C/L_i)^{\frac{1}{2}}} \right\} - \frac{\theta}{\pi} \sqrt{1+\sigma_i} \left[\log \left| 1 + \left\{ \frac{(1+C/L_i)}{(C/L_i)} \right\} \right| \right] \right]$$

$$\left. \frac{1+C/L_i}{(x_i/L_i)} \right\}^{\frac{1}{2}} \left/ \left[1 - \left\{ \frac{(1+C/L_i)}{(x_i/L_i - 1)} \right\}^{\frac{1}{2}} \right] \right] \dots \dots \dots (38)$$

For a closed cavity model: $(y_i(L_i) = b_i, C\theta = 0)$

$$\left\{ \frac{(1+C/L_i)}{(C/L_i)} \right\}^{\frac{1}{2}} + \frac{1}{2} \log \left[\frac{(1+C/L_i)^{\frac{1}{2}} + (C/L_i)^{\frac{1}{2}}}{(1+C/L_i)^{\frac{1}{2}} - (C/L_i)^{\frac{1}{2}}} \right] - \frac{\pi \sigma_i}{4\theta(1+\sigma_i)^{\frac{1}{2}}} = 0 \dots \dots \dots (39)$$

For a semiclosed cavity model: $(y_i(L_i) = b_i, C\theta)$

$$\left\{ \frac{(1+C/L_i)}{(C/L_i)} \right\}^{\frac{1}{2}} + \frac{1}{2} \log \left[\frac{(1+C/L_i)^{\frac{1}{2}} + (C/L_i)^{\frac{1}{2}}}{(1+C/L_i)^{\frac{1}{2}} - (C/L_i)^{\frac{1}{2}}} \right] - b_i \left(\frac{C}{L_i} \right) -$$

$$\frac{\pi\sigma_i}{4\theta(1+\sigma_i)^{\frac{1}{2}}} = 0 \dots\dots\dots (40)$$

For an open cavity model:

$$\log\left\{\frac{(1+C/L_i)^{\frac{1}{2}} + (C/L_i)^{\frac{1}{2}}}{(1+C/L_i)^{\frac{1}{2}} - (C/L_i)^{\frac{1}{2}}}\right\} - \frac{\pi\sigma_i}{2\theta(1+\sigma_i)^{\frac{1}{2}}} = 0 \dots\dots\dots (41)$$

Variation of the cavity length with freestream Mach number is shown in Fig.5 with a comparative study for various flow

Fig.5 Dependence of cavity length on freestream Mach numbers and flow models of cavity termination.

models of cavity termination. It is seen that for any particular cavitation number the cavity length increases with the increase of freestream Mach numbers. A minor change in a flow model causes a significant variation in cavity length.

The pressure distributions on the wedge surface for various freestream Mach numbers and cavity models are shown in Fig.6. Fig.7 shows the compressibility effect on cavity drag which depends on the cavity flow models and cavitation numbers. For a small cavitation number the dependence of the compressibility correction to cavity drag on flow models is very small and becomes gradually large in higher cavitation numbers.

A comparative study of the present, uniformly linearized and hodograph theories on cavity drag is shown in Fig.8. For a small cavitation number the present theory yields less effect of compressibility on cavity drag than the uniformly linearized and hodograph theories. Further it shows that the differences in the results between the theories gradually diminish in higher cavitation numbers. Fig.9 shows the comparison of cavity length and drag coefficients with the experimental results [2]. The cavity drag and length by a semiclosed model are at least in agreement with the experimental results.

Fig.6 Pressure distribution on a symmetrical wedge surface.

Fig.7 Dependence of compressibility effect to cavity drag on free-stream Mach numbers and flow models of cavity termination.

Fig.8 Comparison of cavity drag with other theories.

Fig.9 Comparison of cavity drag and length with experimental results.

6. Conclusions

The compressibility effects on the liquid flow past a supercavitating symmetrical wedge at subsonic speed are clarified by a local linearization technique. The main results may be summarized as follows:

i) The correspondences between the steady two-dimensional supercavitating wedge in a compressible and in an incompressible liquid flow are explained within the frame-

work of local linearization technique.

ii) For a small cavitation number the present theory yields less effect of compressibility on cavity drag and length than the former uniformly linearized and hodograph theories.

iii) The dependence of compressibility correction to cavity drag on cavity flow models is shown to be minute. Also comparisons of the proposed theoretical results with experimental one appear to indicate the appropriateness of the semiclosed model in a subsonic cavity flow.

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