532.525

Paper No. 190-11

Flow Characteristics of a Jet Adjacent to a Plane Wall*

(1st Report, Jet Parallel to Adjacent Wall)

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Flow characteristics of a two-dimensional air jet issuing from a nozzle of 5mm in width which is parallel to a horizontal wall with the vertical distance L have been experimentally and analytically investigated.

The adjacent wall jet consists of three regions in the flow direction and the velocity profiles in each region are approximately composed of a free jet profile and a one-fifteenth power law profile.

The flow characteristics are independent of $Re (= u_0 \cdot L / \nu = 2.3 \times 10^4 \sim 4.0 \times 10^4, u_0$; maximum of u at the upper end of the wall, ν ; the kinematic viscosity) and are of similarity.

The bend of the main stream line, that is, the locus of the maximum velocities, was analyzed by using the momentum equations and the energy equations. The results agreed with the experimental results.

1. Introduction

Flow characteristics of jets impinging normally to a wall have been studied by a number of investigators.(1)~(5) However, characteristics of a jet flowing parallel or inclined to an adjacent plane, such as burner jet, exhaust gas jet are remarkably different from those of a free jet, because of existence of curvature on the jet axis, a pressure rise near the wall and occurrence of a kinetic energy loss in the vicinity of the adjacent wall.

In this series of studies, the influences of the existence of a plane wall on characteristics of two-dimensional and axisymmetrical jets were experimentally and theoretically investigated. The investigation of a two-dimensional jet issuing from a nozzle parallel to an adjacent plane has been presented in this first report. There are two cases for the jets as shown in Fig.1(a) and (b). In the case of Fig.1(a) Coanda effect takes place owing

to a dead zone enclosed by the jet and the plane, and the axis is curved toward the plane.

In the case of Fig.1(b), a part of the adjacent plane is lacking and Coanda effect does not occur. The characteristics of curvature of the main stream line owing to Coanda effect have been clarified by a number of investigators, but in the latter case the main stream line is also curved not by the Coanda effect, and this phenomenon has not been clarified.

In this report, the fundamental flow characteristics of the jet such as the bending of the main stream line, the decay of maximum velocity, the increase of the jet width, and the change of the wall boundary layer along the flow direction have been analyzed theoretically and experimentally.

2. Experimental apparatus and procedures

The experimental apparatus is shown schematically in Fig.2. Air from a blower

Fig.2 Experimental apparatus

* Received 10th July, 1979.

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is discharged (with a uniform velocity u_s) from a rectangular nozzle (2) of 5mm height, 300mm width installed in a pressure chamber (1) into a two-dimensional passage. This passage is constructed of a couple of vertical endplates (3), spaced with 300mm distance. A horizontal wall is installed in the passage at a location where the boundary of the jet reaches the front of the horizontal wall. Thus, there is not a wall between the front section of the nozzle and the front end of the horizontal plate and no upper plate in the passage either. Air velocity was measured by a traversable pitot tube (6) of 3mm in dia. as shown Fig.1 in most of cases, and by an I-type hotwire anemometer in the vicinity of the wall transversing it perpendicular to the wall. The position was measured by a microscope, and the minimum distance between the sensor and the surface of the wall was 0.15mm.

3. Flow pattern

The experiment was carried out under the conditions of the width of the nozzle exist $t=5\text{mm}$, the vertical distance between the center of the nozzle exist and the horizontal wall surface $L=2.5\text{mm}\sim 33.5\text{mm}$, $L/t=0.5\sim 6.7$, and the discharge velocity at the nozzle $u_s=44.5\text{m/s}$. Reynolds number ($=u_0L/\nu$) is from 4.2×10^5 to 4.0×10^4 (where, u_0 : the maximum velocity at the section of the front end of the wall, ν : kinematic viscosity).

In the condition of $L/t=0.6\sim 0.8$, when the potential core of jet impinges on the horizontal wall surface, the velocity profile is still developing, therefore, the velocity profile in the vertical section at the front of the horizontal plate changes with the magnitude of L/t and similarity of the flow characteristics does not hold.

On the contrary, the velocity profile of the jet which impinges on the flat plate in a fully developed region shows similarity regardless of L . The fully developed region is $L/t>2$ and $x_0'/t\geq 10$. In this report are presented the flow characteristics of the latter case which is called adjacent wall jet. By the preliminary experiments, the conditions of the adjacent wall jet were confirmed to be $x_0'/L=5$ and $x_0'/t\geq 10$.

The flow pattern of the "adjacent wall jet" is shown in Fig.3. In the ini-

Fig.3 Flow pattern and coordinates

tial region, $x/L=0\sim 1.75$, the jet axis is a straight horizontal line and the velocity profile is approximately equal to an ordinary free jet profile and the boundary layer δ_w is negligibly small. The location of the end of this region, x/L , was examined by the experiments, and confirmed to be 1.75 independently of $Re(=u_0L/\nu)$. In general, the development of a boundary layer on a flat plate in a stream with a uniform velocity profile at the entrance depends on the transition Reynolds number, but in this experiment the structure of the jet is so different from the wall jet that its transient point has to be determined by the experiments.

In the transient region, $x/L=1.75\sim 9$, the thickness of the wall boundary layer, δ_w , increases with the distance x , and the inner boundary layer δ_i which is defined by the distance between δ_w and the maximum velocity location decreases with the distance x , becoming zero at the end of the transient region, and also the outer layer increases with x similarly to a free jet. The velocity profiles of the inner and the outer layer are similar to that of a free jet.

In the wall jet region downstream of the transient region, $x/L\geq 9$, there are two layers; the wall boundary layer δ_c and the outer jet layer δ_o .

The locus of the maximum velocity point δ'/L (main stream line) approaches gradually the wall surface in the transient region, the magnitude of δ_c/L becomes minimum at $x/L=20$ and further downstream in the wall region it increases gradually like the ordinary wall jet.

4. Velocity distribution

An example of the time averaged velocity profiles at cross sections perpendicular to the horizontal wall is shown in Fig.4. At the distance, $X=0$, the jet is

Fig.4 An example of the time averaged velocity profile

an ordinary free jet with a velocity profile which is symmetric to the axis and in the downstream region its profile becomes nonsymmetric owing to the influence of the wall. The magnitudes of jet width δ_o , the width of each layer δ_c (δ_c in the wall jet region), δ_w , etc. are defined as shown

in Fig.5.

Fig.5 Definition of δ_c , U_c , δ_i , δ_w and δ_o

The point of the maximum velocity, ξ , is determined from a line $\xi-\xi'$ which is drawn connecting the center points $a_0, b_0, c_0, \dots$ etc., where $a_1 a_0 = a_0 a_2, b_1 b_0 = b_0 b_2, c_1 c_0 = c_0 c_2, \dots$. Thus, δ_c is determined as $\delta_c = 0.5\xi'$. Velocity u at this point δ_c is defined as the maximum velocity u_c . The width of symmetric part of the velocity profile is defined as the inner jet layer δ_i and the width of the wall side of nonsymmetric part is defined as the wall boundary layer δ_w . And the width of the outer layer δ_o is defined as the value of $2.27y'_{0.5}$, where $y'_{0.5}$ is the width at $u=0.5u_c$.

Each dimensionless width of layer $S_0 = \delta_o/L$, $S_c = \delta_c/L$ ($S_c = \delta_c'/L$ in the wall jet region), $S_w = \delta_w/L$, and the dimensionless maximum velocity $U_c = u_c/u_0$ which are determined by this method are shown in Fig.6. In the initial region; $X=0 \sim 1.75$,

Fig.6 Calculated result

the profile of u is approximately equal to that of the ordinary free jet and S_w is equal to zero, and S_c is equal to unity. S_0 increases with X and is expressed by an empirical formula $S_0 = 0.2X + 1$.

In the transient region $X=1.75 \sim 9$, the wall boundary layer S_w increases with X from 0 to 0.87 as shown in this figure and reaches the value of S_c at $X=9$. And the outer layer, S_0 , increase linearly as shown in the figure. The velocity profiles of the outer jet layer and the inner layer are similar to that of the ordinary free jet and that of the wall boundary layer is expressed by that of the $1/n$ -power law. That is, the relation between the dimensionless velocity u/u_1 which is a normalized u by the velocity u_1 (the maximum velocity of u in the wall boundary

layer δ_w) and the dimensionless distance, y/δ_w , at each location X is the same independently of L as shown in Fig.7.

Fig.7 Dimensionless velocity profile in the wall boundary layer

And its profile is approximately expressed by the curve of $u/u_1 = (y/\delta_w)^{1/5}$, in the range of $Re (=u_0 L/\nu)$ from 2.3×10^4 to 4.0×10^4 .

In the wall jet region, $X=9 \sim 40$, the value of S_c decreases gradually from $S_c = 0.87$ at $X=9$ to the minimum value $S_c = 0.6$ at $X=20$, and then increases with X . The increase rate of the width S_0 in this region is a little lower than that in the transient region. The dimensionless velocity profile at the wall boundary layer is shown in Fig.8. As shown in this

Fig.8 Dimensionless velocity profile in the wall jet region

figure the dimensionless velocity profile between u/u_c and y/δ_c at every location of X is expressed by an empirical formula $u/u_c = (y/\delta_c)^{1/5}$. This result agrees with the experimental results of a two-dimensional wall jet by Schwarz⁽⁴⁾.

The velocity profile of the outer layer in the initial, the transient, and the wall jet regions agrees with that of a free jet owing to the free diffusion outward as shown in Fig.9 and is expressed

Fig.9 Velocity profile of the outer layer

by the expression $u/u_c = \{1 - (y'/y'_{\max})^{1.5}\}^2$.

5. Decay of maximum velocity

The decay of the maximum velocities u_c along the axial distance x' from the nozzle is shown in a dimensionless form in Fig.10. In the case that the dimensionless vertical distance L/t is equal to 0.5 (L is equal to 2.5mm), the jet is similar to an ordinary wall jet on the horizontal wall. Therefore the experimental points fall just on the theoretical curve of Mayers and others' work of two-dimensional wall jet on the horizontal wall shown by a dotted line in the figure. However, in the cases of $L/t \geq 2$ ($L=10\sim 30\text{mm}$), the rate of decay is equal to that of the free jet till the end of the transient region shown by a bold solid line, and in the downstream region the decay rate decreases and approaches that of the wall jet.

Fig.10 Decay of the maximum velocities along the axial distance x'

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Furthermore, the decay of the dimensionless velocity u_c/u_0 (u_0 is maximum velocity at $X=0$) along the axial distance X is shown in Fig.11. In cases of $L/t=0.6$

Fig.11 Decay of the dimensionless velocity u_c/u_0 along the axial distance X

($u_0 \sim 0.8$, $L=3\sim 4\text{mm}$), the larger the L is, the larger the decay of velocity at the same dimensionless distance X is, owing to the non-similarity of the velocity profile at $X=0$. In these cases the velocity distributions are not fully developed and the potential core remains at $X=0$, therefore the shape of velocity profiles depends on the magnitude of L . On the contrary in cases of $L/t \geq 2$ the rate of decay of u_c/u_0 is the same as shown in the figure. Also, the inner jet layer δ_i/L , the wall boundary layer δ_w/L , and the outer jet layer δ_o/L in these cases are functions of $X=x/L$ independently of L as shown in Fig.6.

6. Approximate calculation of the width of jet layers and the maximum velocities

As described above, the flow characteristics of velocity decay and the boundary layer are similar to those of the

adjacent wall jets of $L/t \geq 2$. In this section, on the basis of this similarity, the fundamental flow characteristics such as the development of each layer, the locations of the maximum velocities, the bending of the main stream line toward the horizontal wall, and the decay of the maximum velocities have been analyzed. The analysis is not a purely theoretical one and it was conducted using experimental results, that is the characteristics values mentioned above were calculated based on the momentum balance and energy balance in each layer using the velocity profiles which were obtained by the experiment. In order to make a semi-theoretical analysis, the following assumptions were used.

6.1 Assumption

- (1) The velocity profile of the jet at $X=0$ ($x/L=0$) is expressed by the ordinary dimensionless free jet velocity profile.
- (2) The velocity profiles at the cross sections perpendicular to the wall in each boundary layer are expressed respectively as follows:

(a) The wall boundary layer δ_w : $u/u_1 = (y/\delta_w)^{1/5}$ (in the transient region)

(b) The wall boundary layer δ_c : $u/u_c = (y/\delta_c)^{1/5}$ (in the wall jet region)

(c) The outer jet layer δ_o ($y'_{\max}$): $u/u_c = \{1 - ((y - \delta_c)/\delta_o)^{1.5}\}^2 = \{1 - (y'/y'_{\max})^{1.5}\}^2$

(d) The inner jet layer δ_i : $u/u_c = \{1 - ((\delta_c - y)/\delta_o)^{1.5}\}^2 = \{1 - ((\delta_c - y)/y'_{\max})^{1.5}\}^2$

- (3) The turbulent shear stress τ_t in the inner and the outer jet layer, and the wall shear stress τ_w are expressed as follows.

$$\tau_t = \rho k \delta_o u_c (du/dy), \quad \tau_w = \rho u_1^2 \zeta c_0 (\nu/u_1 \delta_w)^{1/4}$$

(in the transient region), $\tau_w = \rho u_c^2 \zeta c_0$.

($\nu/u_c \delta_c$)^{1/4} (in the wall jet region), where c_0 : constant=0.0225, ζ : the correction factor, k : a constant for the turbulent shear stress, ρ : the density of the fluid (that is, c_0 is equal to 0.0225 and is a constant which is deduced from Blasius' relation in a uniform flow). The definitions of the representative velocity and Reynolds number differ from those used in Blasius' equation in the uniform flow, therefore, the correction factor ζ is used in order take into account the difference in the definition of the representative velocity.

- (4) The spread angle of jet boundary is constant in the initial region, and $\delta_c=L$, $\delta_w=0$, and $\tau_w=0$.

6.2 Momentum balance and kinetic energy equation

6.2.1 Initial region

The momentum equation along x in the initial region is given by the following

equation on the assumption (4), $\tau_w = 0$;

$$\rho \frac{d}{dx} \left[\int_0^b u^2 dy \right] = 0 \dots\dots\dots (1)$$

By substituting the velocity profile of the jet layer (presented in the assumption (2)) into the above equation, the following equation is given;

$$0.6312u_0^2 L = u_c^2 \left\{ \delta_0 \left(\frac{L}{\delta_0} \right) - 1.6 \left(\frac{L}{\delta_0} \right)^{2.5} + 1.5 \left(\frac{L}{\delta_0} \right)^4 - 0.7273 \left(\frac{L}{\delta_0} \right)^{5.5} + 0.1429 \left(\frac{L}{\delta_0} \right)^7 + 0.3156 \right\} \dots\dots (2)$$

Furthermore, the spread angle of the jet is obtained from the experimental results, and the relation is given as follows;

$$\frac{\delta_0}{L} = C \left(\frac{x}{L} \right) + 1 \dots\dots\dots (3)$$

where C is the experimental spread coefficient, and C is equal to 0.2⁽⁵⁾. By substitution of the value of C into eq. (2) the variation of the maximum value of the velocity u_c under the initial condition of $x/L=0$, $b=2L$, and $\delta_0=L$ along x axis is obtained.

6.2.2 Transient region

The variation of the width of each layer δ_w , δ_i and δ_o , and the velocity profiles along x were, obtained by the following method. The whole jet layer is divided into three control volumes: EBCF, AEFG, and ABCD as shown in Fig.3. The momentum balance equation on each control volume is given as follows;

$$\rho \frac{d}{dx} \left[\int_{\delta_c}^b u^2 dy \right] + \rho u_c \frac{d}{dx} \left[\int_0^{\delta_c} u dy \right] = 0 \dots\dots (4)$$

$$\rho \frac{d}{dx} \left[\int_{\delta_w}^{\delta_c} u^2 dy \right] - \rho u_c \frac{d}{dx} \left[\int_0^{\delta_c} u dy \right] + \rho u_1 \frac{d}{dx} \left[\int_0^{\delta_w} u dy \right] = - (\tau_t)_{y=\delta_w} \dots\dots\dots (5)$$

$$\rho \frac{d}{dx} \left[\int_0^b u^2 dy \right] = - \tau_w \dots\dots\dots (6)$$

Furthermore, the energy balance equation on the control volume EBCF is given by the following equation;

$$\frac{1}{2} \rho \frac{d}{dx} \left[\int_{\delta_c}^b u^3 dy \right] + \frac{1}{2} \rho u_c^2 \frac{d}{dx} \left[\int_0^{\delta_c} u dy \right] + \int_{\delta_c}^b \tau_t \frac{du}{dy} dy = 0 \dots\dots\dots (7)$$

The substitutions of the velocity profile described in the section 6.1 (2), the shear stress τ_w and τ_t into eqs. (4) ~ (7) yield eqs. (4) ~ (7)'. Here the following dimensionless numbers: $U_c = u_c/u_0$, $U_1 = u_1/u_0$, $S_0 = \delta_0/L$, $S_w = \delta_w/L$, $S_i = \delta_i/L$, and $\nu' = \nu/u_0 L$ are used.

$$(0.6312S_0 + 0.9375S_w r + S_0 q) \frac{dU_c}{dX} + U_c (0.3156 + q) \cdot \frac{dS_0}{dX} + 0.9375U_c r \frac{dS_w}{dX} + U_c (0.813S_w Z + S_0 r) \frac{dS_i}{dX} = 0 \dots\dots\dots (4)'$$

$$\{ S_0 (2p - q) + 0.9375S_w r (r - 1) \} \frac{dU_c}{dX} + U_c (p - q) \frac{dS_0}{dX} + 0.9375U_c r (r - 1) \frac{dS_w}{dX} + \{ U_c (r - 1) (2.813S_w Z + S_0 r) \} \frac{dS_i}{dX} = 3\kappa U_c Z \dots\dots\dots (5)'$$

$$(0.6312S_0 + 1.765S_w r^2 + 2S_0 p) \frac{dU_c}{dX} + U_c (0.3156 +$$

$$p) \frac{dS_0}{dX} + 0.882U_c r^2 \frac{dS_w}{dX} + U_c r (5.294S_w Z + S_0 r)$$

$$\frac{dS_i}{dX} = -\zeta C_0 \cdot U_c r^2 \left(\frac{\nu'}{U_c \cdot r S_w} \right)^{1/4} \dots\dots\dots (6)'$$

$$\{ S_0 (0.753 + q) + 0.9375S_w r \} \frac{dU_c}{dX} + U_c (0.251 + q) \cdot$$

$$\frac{dS_0}{dX} + 0.9375U_c r \frac{dS_w}{dX} + U_c (2.813S_w Z + S_0 r) \frac{dS_i}{dX} = -2.322\kappa U_c \dots\dots\dots (7)'$$

$$\text{where, } p = \int_0^K (1 - \xi^{1.5})^4 d\xi, \quad q = \int_0^K (1 - \xi^{1.5})^2 d\xi \\ , \quad r = (1 - K^{1.5})^2, \quad Z = (K^2 - K^{0.5}), \quad K = S_i / S_0.$$

Equations (4)' ~ (7)' of U_c , S_0 , S_w and S_i can be solved with boundary conditions at the end of the initial region $X=1.75$. Arranging eqs. (4)' ~ (7)' in the form of 4-pole allied differential equations by Cramer's method, these equations were solved numerically by Runge-Kutta method, and the variation of the maximum velocity and widths of the layers δ_o , δ_w and δ_c were obtained.

6.2.3 Wall jet region

The values of U_c , δ_o and δ_c' in the wall jet region were obtained as follows. The momentum balance equations on the control volume E'B'C'F' and A'B'C'D' are expressed by the following equations;

$$\rho \frac{d}{dx} \left[\int_{\delta_c}^b u^2 dy \right] + \rho u_c \frac{d}{dx} \left[\int_0^{\delta_c} u dy \right] = 0 \dots\dots (8)$$

$$\frac{d}{dx} \left[\int_0^b u^2 dy \right] = - \tau_w \dots\dots\dots (9)$$

And the energy equation on the control volume E'B'C'F' is expressed by the following equation;

$$\frac{1}{2} \rho \frac{d}{dx} \left[\int_{\delta_c}^b u^3 dy \right] + \frac{1}{2} \rho u_c^2 \frac{d}{dx} \left[\int_0^{\delta_c} u dy \right] + \int_{\delta_c}^b \tau_t \frac{du}{dy} dy = 0 \dots\dots\dots (10)$$

$$\frac{du}{dy} dy = 0 \dots\dots\dots (10)$$

The substitutions of the expression of the velocity profiles, τ_w and τ_t described in section 6.1 into eqs. (8) ~ (10) give the following equations;

where $S_c = \delta_c' / L$.

$$(0.6312S_0 + 0.9375S_c) \frac{dU_c}{dX} + 0.3156U_c \frac{dS_0}{dX} + 0.9375U_c \frac{dS_c}{dX} = 0 \dots\dots\dots (8)'$$

$$0.6312S_0 + 1.764S_c \frac{dU_c}{dX} + 0.3156U_c \frac{dS_0}{dX} + 0.882U_c \frac{dS_c}{dX} = -\zeta C_0 U_c \left(\frac{\nu'}{U_c S_c} \right)^{1/4} \dots\dots (9)'$$

$$(0.753S_0 + 0.9375S_c) \frac{dU_c}{dX} - 0.251U_c \frac{dS_0}{dX} - 0.9375U_c \frac{dS_c}{dX} = -2.322 U_c \dots\dots\dots (10)'$$

These equations of U_c , S_0 and S_c vs. X , can be solved using the values which are given at $X=9.23$ as the boundary condition in the wall jet region.

6.3 Calculated results and discussions

The numerical calculation was conducted in the range of κ : 0.007~0.011 and ζ : 1.0~3.48 in the case of Reynolds number 2.3×10^4 . Examples of the calculated result on a case of $\zeta = 3.48$, $\kappa = 0.009$ in the transient region and on five cases of different values of ζ and κ in the wall jet region are shown in Fig.12.

Fig.12 Effects of ζ and κ on U_c , S_c and S_o

As can be seen from the figure, the values of S_o , U_c are not affected very much by the value of ζ , but the value of S_c is affected significantly by the constant value of κ . This means that the wall shear stress dominates the bending of the main stream line toward the wall surface. It also can be seen from each curve of $\zeta = \text{const} = 3.48$ with a different value of κ , that S_c and U_c are not affected very much but S_o is affected significantly by the value of κ . This proves that the larger the diffusivity of turbulent flow κ is, the larger the spread of the jet S_o is. Comparison of the calculated results with the experiment shows that the calculated results at $\zeta = 3.48$, $\kappa = 0.009$ agree well with the experimental results both in the transient and in the wall jet region as shown in Fig.6. The reason why the value of $\zeta = 3.48$ is larger than the ordinary value $\zeta = 1$ is that τ_w was expressed conventionally by the assumption (3), although the inner structure of the adjacent wall jet was different from that of the ordinary boundary layer of the uniform flow. The fittest value of the proportional constant κ in the turbulent shear stress $\tau_t = \rho \kappa \delta_o \cdot u_c (du/dy) = \rho \cdot \kappa y'_{\max} u_c \cdot (du/dy)$, is 0.009 as described above. The reason why the value of κ , 0.009, in the adjacent wall jet is larger than in the wall jet and smaller than in the free jet⁽²⁾ is that the strength of turbulent diffusivity of the adjacent wall jet comes between the other two jets.

Furthermore, the influence of Reynolds number on the experimental and calculated results shown in Fig.6 is explained as follows. Experimental points for different Reynolds numbers fall approximately on a curve. It appears that Reynolds number little affects the flow characteristics described in this paper.

Also, the results of theoretical calculation in the range of $2.3 \times 10^4 \sim 4.0 \times 10^4$ of $1/\nu' = \text{Re}$ in the transient region show

that the difference for each Reynolds number is so small that these curves can be expressed approximately by a single curve. Furthermore, in the wall jet region the difference of the calculated result of S_c is $\pm 5\%$ and those of U_c , S_o and S_w are much smaller than this value.

Next, the characteristics of the bending of the main stream line of the adjacent wall jet are discussed.

In the transient region, the turbulent diffusivity of the outer jet layer is relatively large, and the turbulent shear stress acting on the layer is larger than either that of inner jet layer or that of wall boundary layer, therefore the decay of the velocity of the outer layer is larger than that of inner layer, because the representative velocity u_1 of wall shear stress is low and the stress is very small. Therefore, the decelerations of the wall boundary layer and inner jet layer are smaller than that of the outer jet layer. Thus the location of the maximum velocity shifts from the center axis toward the wall, as can be seen in Figs. 6 and 12.

In the upper stream part of the wall jet region the turbulent shear stress of the outer layer is larger than that of the wall boundary layer of the wall side. Therefore the deceleration of the outer layer is larger and the point of the maximum velocity, δ'_c , also shifts toward the wall. However, in the further downstream part the wall shear stress increases gradually owing to a decrease of δ'_c and then the wall shear stress becomes comparable with the turbulent shear stress of the outer layer δ_o . The magnitude of δ'_c reaches the minimum value at the cross section where both stresses are equal. Downstream of this section, as the deceleration caused by the wall boundary stress is superior to the deceleration caused by the turbulent shear stress of the outer jet layer, δ'_c increases again gradually and the location of the maximum velocity departs from the wall. Thus, the bending of the jet axis can be explained by a difference in the diffusion phenomenon and an interaction with the wall shear stress.

6.4 Distributions of flow rates, momentum and energy

As illustrated in the above section the velocity profiles of inner jet layer (in the layer of δ_w and δ'_c) and the characteristics of the decay of the maximum velocity are different from those of a free jet owing to existence of an adjacent wall. The distributions of the volume flow rates (Q), momentum fluxes (J), and energy fluxes (E) (subscript s means location of the exist section) along the axis are discussed comparing with the free jet.

An example of the variation of the dimensionless flow rate Q/Q_s of the adjacent wall jet ($L/t=4$, $L=20\text{mm}$) is shown in Fig.13. The one-dotted chain line in this figure represents the theoretical value of

Fig.13 Distributions of volumetric flow rates, momentum, and energy

the free jet obtained by Simson's⁽⁶⁾ model, and the solid line and the key (O) are the theoretical value (calculated result for $L/t=4$) and the experimental result of the adjacent wall jet respectively. The flow rate of the adjacent wall jet up to the cross section of $x'/t=20$, is equal to that of the free jet owing to a lack of the horizontal wall. Downstream of the cross section $x'/t=20$, the adjacent wall jet is restricted by the wall, and the absolute value of flow rate and the increasing rate are both lower than that of the free jet, but the flow rate is larger than that of the wall jet shown by a double dotted chain line. This line is based on Myers and others' theoretical value for the wall jet.

Furthermore, J/J_s , the theoretical result and the experimental result of the momentum flux normalized by the exit momentum flux $J_s = \rho u_s^2 t$ are shown by a solid line and a key ($\oplus$) respectively in Fig.13. The momentum decreases in the downstream region from the cross section $x'/t=20$ owing to the wall shear stress. Thereby the experimental result of the momentum flux is calculated from the velocity profile by the experiment as follows.

$$J = \rho \int_0^b u^2 dy + \int_0^b p dy$$

where p is pressure. This dimensionless value is, as shown in the figure, lower than the theoretical value of the wall jet by Myers, et al.⁽³⁾ at the same cross section (shown by the double dotted chain line).

And, the normalized energy flux E/E_s of the adjacent wall jet is shown by a key ($\ominus$) for the experiment and by a solid line for the calculated result in this figure, and the energy flux is defined as follows:

$$E = \frac{1}{2} \rho \int_0^b u^3 dy + \int_0^b p \cdot |u| dy$$

In the upstream region of $x'/t \leq 20$ the theoretical value of the energy of the adjacent wall jet is equal to the value of the free jet owing to the absence of the influence of the wall. Downstream of $x'/t=20$, the kinetic energy is larger than that of a free jet, because a turbulent diffusion occurs on one side whereas it does on both sides in the case of a free jet. But the energy is smaller than that of the wall jet which is shown by a double dotted chain line according to Myers, and others'⁽³⁾ theoretical work on the wall jet.

7. Conclusions

From the semi-theoretical analysis and experimental results of the bending of the main stream and the flow characteristics of a two-dimensional adjacent wall jet parallel to a wall, the following conclusions were obtained.

(1) The adjacent wall jet consists of the initial, the transient, and the wall jet region along the flow axis. The transient region consists of the wall jet layer which has a $1/15$ powered velocity profile; the inner jet layer, and the outer jet layer which have an ordinary free jet profile. And the wall jet region, $X \geq 9$, consists of the wall boundary layer which is a $1/15$ powered profile and the outer jet layer of the free jet profile.

(2) The flow characteristics of the adjacent wall jet normalized by the vertical distance L , and the maximum velocity at the front edge cross section are similar, regardless of L , u_c , and Reynolds No. $= u_c L / \nu$ (ν : kinematic viscosity). In the wall jet region the point of the maximum velocity of the jet shifts toward the wall side in the region of $X \leq 20$ and departs from the wall in $X \geq 20$. The decay rate of the maximum velocity is the same as that of the free jet in the transient region, and downstream of the region, the rate is low.

(3) The development of each jet layer, the bending of the main stream line, and the decay of the maximum velocity at each cross section were predicted by using the momentum flux and the kinetic energy flux equations.

(4) The flow rate and the momentum flux of the adjacent wall jet are smaller than those of the free jet at the same cross section owing to the influence of the wall, and the energy flux is larger than that of the free jet. The wall shear stress in the adjacent wall jet was assumed to be expressed by the following equation.

$$\zeta C_0 Re^{-0.25} = \frac{\tau_w}{\rho u_c^2}$$

(ρ : density, u_c : maximum velocity, τ_w : wall shear stress, C_0 : constant (0.0225), $Re = u_c \delta'_c / \nu$) and the correction factor ζ for the drag coefficient was evaluated to be 3.48 through comparison between the calculated results and the experimental results.

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