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Study on Welded Structure Gears*

(2nd Report, Web Structures and Root Stress Distributions)

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This paper presents a study on the effects of web structure on the root stress distributions in welded structure gears. The root stresses in welded gears with different web structures are measured by means of strain gages and calculated by the true stress formula and the two dimensional finite element method (FEM).

Stress conditions in the root fillet of thin rim gears under normal running conditions become those of partly reversed stresses. The root fillet stress shows the highest value above the web in the case of single web gears. A symmetric type of single web gear is more advantageous for maldistribution of load along the face width because of its relaxing effect, while a double web gear is disadvantageous for maldistribution of load. The points which should be taken into consideration in the design of the web structures of welded gears are suggested.

1. Introduction

Materials which have much higher strength than cast iron or cast steel can be used for welded structure gears, and a welded structure makes it possible to reduce the size and the weight of gears and therefore to lower the manufacturing cost of gearsets. On the other hand, there are few published data on welded gears and the design standards for this type of gears are not yet established. Cast gears, therefore, are still in considerable use, though welded gears have much more merits than comparable cast gears.

In the previous paper⁽¹⁾, the weldability and the bending fatigue strength of rim materials and the effects of rim and web thickness on the root stress distributions and the bending fatigue strength of gear teeth were clarified to a considerable extent.

In the present paper, the authors investigated the effects of web structures on the root stresses of welded gears by carrying out a static loading test of welded gears with different web structures. Furthermore the root stresses of a solid gear and a gear with a fan-shaped hole were calculated by the true stress formula

and the finite element method and compared with the measured results for welded gears with thin ribs and webs. Thus the relationship between the root stresses and the gear structures has been clarified considerably.

2. Calculation of root stresses

2.1 Calculation of root stresses by true stress formula

Root stresses of a solid gear are calculated by a true stress formula to compare with those of welded gears. The load applied to a tooth must be calculated by the use of the load sharing factor⁽²⁾, which is obtained from the stiffness of gear tooth, to calculate the root stress waveforms. The deflections of gear tooth are calculated by the formula proposed by Ishikawa⁽³⁾ to obtain the stiffness of gear tooth and the true root stresses are calculated by the formula proposed by Aida and Terauchi⁽⁴⁾.

2.2 Calculation of root stresses by finite element method⁽⁵⁾⁽⁶⁾

The root stresses and the deflections of gear tooth are calculated for a solid gear and a gear with a fan-shaped hole by the two dimensional finite element method (FEM), for the same purpose as in sec. 2.1.

3. Experimental methods and apparatus

3.1 Test gears

Dimensions of test gears are shown in Table 1 together with their materials and working and heat treatment conditions.

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Table 1 Test gears

Gear sign	Gear structures	Materials	Treatment conditions	Gear dimensions					
				m	α	z	b mm	l_w mm	b_w mm
G.P	Symmetric	S35C	Hobbed Normalized (Ribbed)	4.5	20°	108	50	2m*	8
G.Q	Symmetric (Ribbed)								4 × 2
G.R	Double webs								8
G.S	Asymmetric (Ribbed)								8
G.T	Symmetric	SCM21	Casehardened	4.5	20°	36	50	∞ **	50
Remarks	* m : Module, ** ∞ : refers to solid gear								

Table 2 Materials of each part of test gears

Parts	Materials
Rim	S35C
Web	SS41
Boss	S25C
Rib	SS41

Table 2 shows the materials of each part of test gears. Figure 1 shows the structures and the dimensions of test gears (G.P, G.Q, G.R, G.S). The accuracy of these gears is of 4th class in JIS. Gear G.T is a solid gear and meshes with one of test gears (welded gears). G.T is of SCM21 steel with teeth surfaces ground after casehardening, and the accuracy is of 1st class in JIS.

3.2 Static loading test

The root stresses of test gears are measured by carrying out a static loading

test to investigate the effects of web structures on the root stress distributions. Strain gages (gage length : 1 mm) were pasted on tooth T_0 just above a rib and T_{54} between ribs (the 54th tooth from T_0). Figure 2 shows the pasted positions of strain gages. Strain gages 1~5 were pasted on the tensile fillet determined by Hofer's method in tooth height direction. Strain gages 1 and 5 were pasted at the points 5 mm inside from ends of tooth trace. Strain gages 6~8 were pasted on the center line of the tooth bottom in tooth trace direction on the compressive side. Strain gages 9 and 10 were pasted at the side of rim in circumferential direction. Stress values are obtained by multiplying strain readings by modulus of elasticity $E = 21000 \text{ kg/mm}^2$.

In the static loading apparatus used in this experiment, a test gear can be fixed at arbitrary meshing positions and loaded with a manual pump and a loading bar as shown in Fig.3. The center distance is variable and a backlash is set by changing the center distance. The backlash in this experiment was selected as $c_0 = 210 \mu$ in circumferential backlash, which is equal to the minimum value specified in JIS.

Furthermore in order to investigate the effect of maldistribution of load along the face width on root stress distributions, static loading tests were carried out under conditions of maldistribution of load for welded gears with different web structures. Maldistribution factor c_r of load is defined as the ratio

Fig.1 Structures and dimensions of welded test gears

Fig.2 Positions of strain gages

of contact length to full contact length in the meshing position of pitch point under a torque $T = 10 \text{ kgm}$. The root stresses under loading conditions of $c_r = 1, 0.8, 0.6$ are measured. Conditions of maldistribution of load are obtained by adjusting the parallelism of shafts.

4. Calculated and experimental results and discussions

4.1 Stress waveforms of welded structure gear and solid gear

Figure 4 shows the measured root stress waveforms at the middle of tooth trace of G.P meshing with G.T and the calculated root stresses of a solid gear (G.U) and a gear with a fan-shaped hole (G.V, Fig.6 in (1)), whose dimensions are the same as those of G.P. In this figure, the angle of rotation of a driven gear is taken as abscissa, and the position of beginning of engagement as the origin. The calculated root stress waveform shows that for G.U. Figure 5 shows the calculated result of stiffness of gear teeth for G.U meshing with G.T. The stiffnesses k_1, k_2 are given as the ratio to stiffness k_0 at the pitch point. The θ in this figure shows the time scale⁽²⁾. The applied load is calculated by using the load

Fig.3 Static loading apparatus

Fig.4 Calculated and measured results of root stress waveform

sharing factor from this result. The true root stresses are calculated by the formula proposed by Aida and Terauchi. The points $\bullet, \circ$ in Fig.4 indicate the root stresses calculated by FEM for G.U and G.V respectively. The position of load application corresponds to the highest point of single tooth meshing when these gears mesh with G.T.

Figure 6 shows the mesh pattern and the boundary constraint for G.V. Figures 7 and 8 show the root stress distribution (loading point: highest point of single tooth meshing) and the deflection of gear tooth (loading point: tip) for G.V respectively, which were calculated by FEM. It is seen from Figs.4 and 7 that the meshing position of the maximum root stress is delayed considerably from the highest point of single tooth meshing in geometrical engagement and that tensile stresses occur at the root fillet of loaded side before the beginning of engagement and compressive stresses after the end of engagement in thin rim gears. This might be attributed to the fact that the gear tooth deflects and that the amount of deformation of rim part is much larger in the case of such thin rim gears as G.P and G.V than in a solid gear. Thus the stress conditions in the root fillet of thin rim gears under normal running conditions become those of partly reversed stresses, therefore the effects of these stress

c : Time for progress of meshing by one normal pitch on line of action

t : Time from beginning of engagement

Fig.5 Stiffness of gear teeth

Fig.6 Mesh pattern and boundary constraint

conditions must be taken into consideration in the bending strength design of these gears.

4.2 Effects of web positions on root stress waveform

Figure 9(a) and (b) show the measured results of root stresses for G.P-T and

Fig.7 Calculated results of root stresses by FEM (G.V)

Fig.8 Calculated results of deflections of teeth by FEM (G.V)

G.S-T₅₄ respectively under loading condition of $c_r = 1$ and applied torque $T = 50, 30$ kgm. In these figures, the values of root stresses at No. 1 and No. 5 show the mean values of those measured by gages 1 and 2 and by gages 4 and 5 respectively in consideration of the accuracy of gears. The values of root stresses at No. 3 show those measured by gage 3. Gage 1 of G.S was pasted on the web side. For both symmetric and asymmetric types of single web gears, the meshing positions of maximum root stresses are delayed considerably from the highest point of single tooth meshing in geometrical engagement, and tensile stresses occur at the root fillet of loaded side before the beginning of engagement and compressive stresses after the end of engagement. The root fillet stress shows the highest value above the web in the case of single web gears (G.P, G.S). The maximum root stress of G.S is larger than that of G.P for the same torque.

4.3 Effects of maldistribution of load on root stress distributions of different web structure gears

Figure 10 shows the measured results of root stresses for gears G.P, G.R, G.S under the loading conditions of $c_r = 1$ and 0.6. Figure 10(d) G.S-T₅₄' shows the measured root stress waveforms of the middle tooth T₅₄ between ribs in the asymmetric type of single web gear G.S for the case where the web side is heavily loaded and (c) G.S-T₅₄ for the case where another side is heavily loaded.

Fig.9 Effects of web positions on root stress waveforms

From Fig.10(a) and (b) it is seen that the rate of increase in maximum root stress for symmetric type of single web gear G.P due to maldistribution of load is much smaller than that for double web gear G.R. It may be considered from this fact that the increase in maximum root stress due to maldistribution of load is reduced considerably in the symmetric type of single web gear such as G.P by adopting a thin web. The maximum root stress of G.S occurs just above the web even in the case of $c_r = 1$. When the web side is heavily loaded, the difference between stress values of both ends of the tooth trace becomes larger. On the contrary when

another side is heavily loaded, the stress variation along tooth trace becomes smaller and the effect of maldistribution of load on the maximum root stress is expected to be decreased considerably. The meshing positions of maximum root stress differ for $c_r = 1$ and 0.6 in some cases in Fig.10. This might be mainly due to the effect of change of the parallelism of shafts.

Figures 11 and 12 show the measured root stress distributions for symmetric types of gears G.P, G.Q, G.R and for an asymmetric type of gear G.S respectively. The maximum root stresses occur at the middle of tooth trace for all symmetric

Fig.10 Effects of maldistribution of load on root stress waveforms

types of gears. These stress values for G.P-T and G.Q-T₅₄ are almost equal and those for G.Q-T₀ and G.R-T become smaller in the order mentioned. Stress values at the end of tooth trace become smaller in the order of G.Q-T₅₄, G.P-T, G.Q-T₀ and G.R-T. The double web gear shows the smallest root stresses over the face width of all symmetric types of gears.

In order to investigate the effects

Fig.11 Root stress distributions of symmetric types of gears

Fig.12 Root stress distributions of asymmetric type of gear

$m = 4.5$, $\alpha_0 = 20^\circ$, $z = 72$, $b = 30$ mm,
 b_w : web thickness mm, l_w : rim thickness mm,
 $P = 200$ kg

Fig.13 Effect of rim thickness on root stresses of a gear with a fan-shaped hole and of a webbed gear

of the rigidity of gear body on root stresses, the root stresses on tensile side were measured for gear with a fan-shaped hole (G.O) and webbed gear (G.N) with various rim thicknesses, the results being shown in Fig.13. The root stresses of G.O show a tendency to increase with an increasing rim thickness. The reasons may be as follows. The normal tooth load P_N is divided into the components P_1 and P_2 normal and parallel to the center line of the tooth respectively. Both tensile stress due to P_1 and compressive stress

Fig.14 Root stress distribution (G.P-T)

Fig.15 Root stress distribution (G.Q-T₅₄)

Fig.16 Root stress distribution (G.Q-T₀)

(absolute value) due to P_2 decrease with an increasing rim thickness, but the composite stress increases with an increasing rim thickness because of the difference in the decreasing rates of each stress component. On the other hand the effect of compressive stress due to P_2 in G.N is smaller, because G.N has a web, and the root stresses of G.N decrease with an increasing rim thickness (with an increasing rigidity of body). Therefore the root stresses of double web gear (G.R), whose rigidity of the gear body is considered to be comparatively high, are smaller compared with those of symmetric types of single web gears (G.P, G.Q). For double web gear G.R the root stress at the middle of the tooth trace is larger than those at the ends, being similar to that of a solid gear, since the face width of G.R is comparatively small. But when the distance between webs in the double web gear is larger, the stress value at the middle of the tooth trace might be considered to become smaller than those at the ends.

For a symmetric type of gear, root stresses increase in proportion to the torque applied. In the case of an asymmetric type of gear, however, root stresses are not always proportional to the applied torque. This fact suggests that the deformation of rim due to the application of load differs with web structures.

Figures 14 ~ 19 show the measured results of root stresses for each gear under the conditions of maldistribution of load. It is seen from these figures that a symmetric type of single web gear is more advantageous for the maldistribution of load because of the lightening effect, while a double web gear is disadvantageous for the maldistribution of load.

4.4 Root stresses in circumferential direction at side of rim and in tooth trace direction

Figure 20(a) and (b) show the effects of the applied torque and the maldistribution of load on the root stress-

Fig.17 Root stress distribution (G.R-T)

es in circumferential direction at the positions of gages 9, 10 (Fig.2) for

Fig.18 Root stress distribution (G.S- T_0 , T_0')

Fig.19 Root stress distribution (G.S- T_{54} , T_{54}')

Fig.20 Effects of torque and maldistribution of load on stresses in circumferential direction at side of rim (G.P)

G.P. In the loading condition of $c_r = 0.6$, the side of gage 5 is heavily loaded. The root stresses in tooth trace direction at the positions of gages 6, 7, 8 for G.P are compressive and tensile before and after the beginning of engagement respectively. And their absolute values are merely several percent of the maximum values of the root stresses in tooth height direction in all test gears used in this experiment.

5. Conclusions

The main results obtained from this investigation are summarized as follows.

(1) For thin rim gears, a tensile stress occurs at the root fillet of loaded side before the beginning of engagement and a compressive stress after the end of engagement, due to the effects of elastic deformation of rim part. Hence the stress conditions in the root fillet of thin rim gears under normal running conditions become those of partly reversed stresses.

(2) The root fillet stresses of a symmetric type of gear show the highest values at the middle of tooth trace. The root fillet stresses of a double web gear are smaller than those of a symmetric type of single web gear over the face width. For an asymmetric type of single web gear, the root fillet stress at the web side is larger than that at another side.

(3) For a symmetric type of gear, root fillet stresses increase in proportion to the torque applied. In the case of an asymmetric type of gear, however, they are not always proportional to the applied torque.

(4) A symmetric type of single web gear is more advantageous for maldistribution

of load because of its relaxing effect, while a double web gear is disadvantageous for maldistribution of load.

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