532.542:532.527

Paper No. 193-6

Vortex Breakdown Phenomena in a Circular Pipe*

(1st Report, Modes of Stationary Breakdown)

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The purpose of this investigation is to give a theoretical explanation for the vortex breakdown phenomena of a swirling flow in a pipe. In the proposed mathematical model, the mean flow is assumed to be a rigidly rotating one (constant angular velocity) with uniform axial velocity. It is found that the internal wave (inertial wave), which may occur in the swirling flow, plays a significant role in the breakdown phenomena. An integer parameter in the formula describing the internal wave, has a close relation with the structural mode of breakdown. Thus, the first three values of the parameter, 0, 1, 2, correspond to the typical three observed types of breakdown, axisymmetric, spiral, and double helix, respectively.

The present study concerns only time-independent types of these phenomena. The time-dependent ones will be discussed in another paper.

1. Introduction

An abrupt change in flow structure can occur at some place along the axis in a swirling flow when the azimuthal velocity is relatively large as compared with the axial velocity. Such phenomena, called the vortex breakdown, were first observed by Pekham and Atkinson⁽¹⁾ on delta wings at large incidences. Since Pekham and Atkinson, many experimental investigations have been made, early ones by Lambourne⁽²⁾ and Hummel⁽³⁾ on breakdown over wings, and later ones on the phenomena in a pipe. Theoretical investigations have also been made by Squire⁽⁴⁾, Benjamine⁽⁵⁾, Ludwig⁽⁶⁾, Hall⁽⁷⁾, Bossel⁽⁸⁾, Leibovich⁽⁹⁾, Randall⁽¹⁰⁾, Grabowski⁽¹¹⁾ and others.

Sarpkaya⁽¹²⁾ found in detail experiments on the phenomena in a slightly divergent pipe, that breakdowns occur in three different types: (1) axisymmetric (bubble type), (2) spiral and (3) double helix (triangular sheet type). In addition, Leibovich⁽¹³⁾ observed six distinct types of breakdown occurring in a pipe similar to Sarpkaya's.

As mentioned above, there are many reports on the vortex breakdown, but due to complexity and variety of flow structure of the breakdown, most of theoretical investigations are limited mainly to the axisymmetric type. Nor does any model give a systematic explanation for the occurrence of various types of breakdown. There remain a lot of peculiar features of the phenomena

to be clarified.

The aim of the present study is to give a theoretical explanation for the vortex breakdown phenomena of the swirling flow in a circular pipe. It is found that the internal wave (inertial wave), which may occur in the swirling flow, plays a significant role in the breakdown phenomena, and that the azimuthal modes of the wave correspond directly to distinct types of flow structures which include the three types of Sarpkaya and also six types of Leibovich.

This investigation is limited to time-independent (stationary) types of the phenomena, and the time-dependent (unsteady or periodic in time) types will be discussed in another paper.

2. Wave disturbance (internal wave) in swirling flow within pipe.

Since Kelvin's report,⁽¹⁴⁾ it is well known that an internal wave can occur in a swirling flow field. Sorberg-Bjerknes,⁽¹⁵⁾ Long,⁽¹⁶⁾ and Fultz⁽¹⁷⁾ treated the wave in particular cases of rotating flow. In the following the wave disturbance is investigated to relate it with the vortex breakdown phenomena in a circular pipe.

2.1 Basic equation for wave disturbance

For analyzing the wave disturbance occurring in the swirling flow within a circular pipe, cylindrical polar coordinates (r, θ, z) are introduced as shown in Fig. 1. In the analysis the following is

Fig. 1 Coordinates system

* Received 10th April, 1979.

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assumed: (1) Fluid is inviscid and incompressible. (2) Mean flow (original flow before disturbance occurs) is a cylindrical flow and the velocity components are $\bar{W}(r)$ and $\bar{V}(r)$, where the azimuthal velocity $\bar{V}(r)$ and the axial velocity $\bar{W}(r)$ are arbitrary functions of radius r . (3) The velocity variations are small amounts in comparison with $\bar{V}(r)$ and $\bar{W}(r)$ of mean flow. From the assumption (1), Euler equations and continuity equation are applied to fundamental equations governing the fluid motion. From the assumption (2), the mean flow is expressed as follows:

$$\bar{u}=0, \quad \bar{v}=\bar{V}(r), \quad \bar{w}=\bar{W}(r), \quad \bar{p}/\rho=(\bar{p}_0/\rho)+\int_0^r (\bar{V}(r)^2/r)dr \dots\dots\dots (1)$$

where u, v and w are the velocity components on coordinates r, θ and z ; p is the pressure and p_0 is the pressure at the axis of a pipe (over-bar denotes the mean flow component). The velocity components of the flow field involving the disturbance are expressed as the sum of both the mean flow and the disturbance components, and can be described in the following form.

$$\left. \begin{aligned} u &= \bar{u} + \tilde{u} = U(r)e^{-i(s\theta+mz)+\lambda t}, & v &= \bar{v} + \tilde{v} = \bar{V}(r) + V(r)e^{-i(s\theta+mz)+\lambda t} \\ w &= \bar{w} + \tilde{w} = \bar{W}(r) + W(r)e^{-i(s\theta+mz)+\lambda t}, & p &= \bar{p} + \tilde{p} = \bar{p}_0 + \rho \int_0^r (\bar{V}^2/r)dr + P(r)e^{-i(s\theta+mz)+\lambda t} \end{aligned} \right\} \dots\dots\dots (2)$$

where $U(r), V(r), \dots$ are complex functions of r (hereafter denoted simply by $U, V, \dots$), λ a complex number, m a real number, and s a positive integer (including zero). The tilde $\sim$ indicates components of the disturbance.

$$U\{\lambda - (is\bar{V}/r) - im\bar{W}\} - 2(\bar{V}/r)V = -(1/\rho)(dP/dr) \dots\dots\dots (3 \cdot a)$$

$$V\{\lambda - (is\bar{V}/r) - im\bar{W}\} + \left\{ \left(\frac{d\bar{V}}{dr} \right) + (\bar{V}/r) \right\} U = (1/\rho)(is/r)P \dots\dots\dots (3 \cdot b)$$

$$W\{\lambda - (is\bar{V}/r) - im\bar{W}\} + \left(\frac{d\bar{W}}{dr} \right) U = (1/\rho)imP \dots\dots\dots (3 \cdot c)$$

$$\left(\frac{dU}{dr} \right) + (1/r)U - (is/r)V - imW = 0 \dots\dots\dots (3 \cdot d)$$

The solution to the above equations which satisfies a boundary condition at the pipe wall, describes a wave disturbance that occurs in the swirling flow within a pipe.

2.2 Neutral stable disturbance in rigidly rotating flow with uniform axial velocity and the perturbed flow field

It is difficult to solve Eqs. (3) analytically. To facilitate the analytical solution, two assumptions are added: [1] The mean flow is rigidly rotating with uniform axial velocity, that is,

$$\bar{V}(r) = \Omega r, \quad \bar{W}(r) = W_0 = \text{constant} \dots\dots\dots (4)$$

Indeed, it is seen in many experimental studies⁽¹⁾⁽²⁾⁽³⁾⁽¹⁰⁾ that the azimuthal velocity $V(r)$ in the vicinity of the pipe axis, is close to a rigid rotation profile (constant angular velocity Ω : $\Omega > 0$). [2] The disturbance is limited to a neutral stable wave, i.e., when λ is written as

$$\lambda = \alpha + in \dots\dots\dots (5)$$

only the case of $\lambda = in$ ($\alpha = 0$, n : real) is considered. Under the assumptions [1] and [2], Eqs. (3) can be simplified and, the equation with respect to W alone is derived after elimination of other variables in the following Bessel equation form.

$$\frac{d^2 W}{dr^2} + \frac{1}{r} \frac{dW}{dr} + W \left[\frac{m^2 \{4\Omega^2 - (n - s\Omega - mW_0)^2\}}{(n - s\Omega - mW_0)^2} - \frac{s^2}{r^2} \right] = 0 \dots\dots\dots (6)$$

W is generally a complex function, but in this case the real function $\bar{W}$ ($W = \bar{W} + iW_i$) satisfies the same equation as Eq. (6). Then, hereafter W indicates a real function. (U, V and P also represent real functions.) Now, from the solution of Eq. (6), after several calculations the real solution of the neutral stable disturbance can be expressed in the following form.

$$\tilde{u} = U \sin(nt - s\theta - mz) = A \frac{m}{\sigma} \left[J_{s-1}(\sigma r) - \left\{ \frac{2\Omega + (n - s\Omega - mW_0)}{(n - s\Omega - mW_0)} \right\} \left(\frac{s}{\sigma r} \right) J_s(\sigma r) \right] \sin(nt - s\theta - mz) \dots\dots\dots (7 \cdot a)$$

$$\begin{aligned} \tilde{v} &= V \cos(nt - s\theta - mz) = \frac{A}{\sigma} \frac{m}{(n - s\Omega - mW_0)} \left[2\Omega J_{s-1}(\sigma r) - \left(\frac{s}{\sigma r} \right) \{2\Omega + (n - s\Omega - mW_0)\} J_s(\sigma r) \right] \\ &\quad \times \cos(nt - s\theta - mz) \dots\dots\dots (7 \cdot b) \end{aligned}$$

$$\tilde{w} = W \cos(nt - s\theta - mz) = A J_s(\sigma r) \cos(nt - s\theta - mz) \dots\dots\dots (7 \cdot c)$$

$$\frac{\tilde{p}}{\rho} = \frac{P}{\rho} \cos(nt - s\theta - mz) = A \frac{(n - s\Omega - mW_0)}{m} J_s(\sigma r) \cos(nt - s\theta - mz) \dots\dots\dots (7 \cdot d)$$

where A is an arbitrary constant indicating the amplitude of disturbance, J_s is an s -order Bessel function of the first kind; and σ is thus defined:

$$\sigma = [m^2 \{4\Omega^2 - (n - s\Omega - mW_0)^2\} / (n - s\Omega - mW_0)^2]^{1/2} \dots\dots\dots (8)$$

The value of σ is determined from the following eigenvalue equation, which is introduced with the boundary condition $u = 0$ at the pipe wall.

$$J_{l-1}(\sigma a)/J_l(\sigma a) = (s/\sigma a) \{1 \pm \sqrt{1 + (\sigma/m)^2}\} \dots\dots\dots (9)$$

where a is the pipe radius. For any double sign in the above equation, it is found that there exists an infinite number of positive eigenvalue $\sigma_{\pm g}$ ($g=1, 2, 3, \dots$) [$+g$ or $-g$ indicates an eigenvalue of the g th order respectively for positive or negative sign in Eq. (9)].

Expressions (7.a) ~ (7.d) indicate the wave motion propagating in both the axial and azimuthal directions, and signify the internal wave occurring in the mean swirling flow field. The wave propagating downstream (in the direction of positive z) or upstream is referred to as a forward or a backward progressive wave, respectively. According to the positive or negative values of m and n , the propagating modes of the wave are determined as follows. (1) $m > 0, n > 0$: forward progressive wave with a positive azimuthal phase velocity. (2) $m > 0, n < 0$: backward progressive wave with a negative azimuthal phase velocity. (3) $m < 0, n > 0$: backward progressive wave with a positive azimuthal phase velocity. (4) $m < 0, n < 0$: forward progressive wave with a negative azimuthal phase velocity. These waves have an azimuthal phase velocity equal to n/s in the z section. When $s = 0$, the wave becomes an axisymmetric disturbance which propagates only in the z direction. The flow field involving the wave disturbance is expressed as follows.

$$u = \bar{u}, \quad v = Qr + \bar{v}, \quad w = W_0 + \bar{w}, \quad (p/\rho) = (\bar{p}/\rho) + (1/2)Q^2 r^2 + (\bar{p}/\rho) \dots\dots\dots (10)$$

2.3 Wave disturbance with no phase velocity (standing wave)

The disturbance expressed by Eqs. (7) signifies a progressive wave (in general $n/m \neq 0$, $n/s \neq 0$). However, if $n = 0$, it becomes a disturbance resting in the coordinates or fixed in the space (referred to as a standing wave). The expression of standing wave is obtained as follows [c.f., Eqs. (7)].

$$\bar{u} = C \frac{m}{\sigma} \left[J_{l-1}(\sigma r) + \frac{2Q - (sQ + mW_0)}{(sQ + mW_0)} \left(\frac{s}{\sigma r} \right) J_l(\sigma r) \right] \sin(s\theta + mz) \dots\dots\dots (7 \cdot a)'$$

$$\bar{v} = \frac{C}{\sigma} \frac{m}{(sQ + mW_0)} \left[2Q J_{l-1}(\sigma r) - \left(\frac{s}{\sigma r} \right) \{2Q - (sQ + mW_0)\} J_l(\sigma r) \right] \cos(s\theta + mz) \dots\dots\dots (7 \cdot b)'$$

$$\bar{w} = -C J_l(\sigma r) \cos(s\theta + mz) \dots\dots\dots (7 \cdot c)'$$

$$\frac{\bar{p}}{\rho} = C \frac{(sQ + mW_0)}{m} J_l(\sigma r) \cos(s\theta + mz) \dots\dots\dots (7 \cdot d)'$$

where $C = -A$. As mentioned before, this study concerns the time-independent type of vortex breakdown, so the above standing wave is mainly discussed in the following.

The equations and expressions introduced so far, are nondimensionalized by taking the representative length and time as the pipe radius a and Ω^{-1} . Main nondimensional quantities are: $R_0 = W_0/aQ$, $M = am$, $N = n/Q$, $\sigma' = a\sigma$, $T = Q\Omega$, $C' = C/aQ$. R_0 indicates the Rossby number. Now, from Eq. (8), N is written as

$$N = s \pm MR_0 \pm 2\sqrt{M^2/(M^2 + \sigma'^2)} \dots\dots\dots (8)'$$

where the double sign corresponds to that in Eq. (9). Though there exists an infinite number of values of σ' in the above equation, only the least order of σ' (σ'_1 or σ_1) is here taken into account because it is inferred that most phenomena appear in the mode of the least order.

First, the case of σ'_1 is considered. From Eq. (8)', the inequality $N/M > R_0$ always holds for $M > 0$. Hence the standing wave cannot exist for $M > 0$. However, the standing wave can exist for $M < 0$. Figure 2 shows the above relation, which plots the nondimensional angular frequency N of the disturbance against the nondimensional wave length of the axial direction. Figures (a), (b) and (c) correspond to the parameter $s = 0, 1, 2$, respectively. The parameter signifies the azimuthal mode of the disturbance. Broken lines and chain lines in the figures represent the cases of $M > 0$ and $M < 0$, respectively. The curves are calculated for different values of the Rossby number. The curves for $R_0 = 0$ is shown by a solid line. (In this particular case N does not

depend on the sign of M .) The intersecting point of the calculated curve and the abscissa $N = 0$, signifies the standing wave. As seen in the figure, in case of σ'_1 , the standing wave can exist only for $M < 0$.

When σ'_1 [which corresponds to the negative sign in Eq. (8)'] is considered, the calculated curves assume a form close to the mirror image of those in Figs. (a), (b) and (c) with respect to the lines $N = 0, 1, 2$, respectively. The curves for $s = 1$ are shown in Fig. (d). From the results concerning σ'_1 it is known that there is a range of L_z where the standing wave of $s = 0, 1$ can exist even for $M > 0$, but the standing wave of $s = 2$ cannot exist for $M > 0$.

Several properties of the standing wave are described in the following. Equation (8)' gives

$$(N/M)_{R_0=0} = \frac{1}{M} [s \pm 2\sqrt{M^2/(M^2 + \sigma'^2)}]$$

where $(N/M)_{R_0=0}$ indicates the axial phase velocity of the disturbance in the swirling flow with no mean axial velocity (i.e., $W_0 = 0$). The above equation and Eq. (8)' for $N = 0$ yield

$$(N/M)_{R_0=0} = -R_0 \dots\dots\dots (11)$$

which is rewritten in the dimensional form as

$$(n/m)_{W_0=0} = -W_0 \dots\dots\dots (11)'$$

This means that when the standing wave occurs, the relative upstream velocity of the disturbance is equal to the mean axial velocity, so that the wave remains fixed in the space.

The following result is obtained with regard to the range of the existence of the standing wave. From Eq. (8)', it follows for $s = 0$ that,

$$R_0 = 2/\sqrt{M^2 + \sigma'^2} \dots\dots\dots (12)$$

and then the eigenvalue equation (9) reduces to $J_1(\sigma')=0$, which gives σ' equal to 3.83 (as the least order of the value). Therefore the range of the existence of the standing wave for $s = 0$ is

$$0 < R_0 (=W_0/a\Omega) < 0.522 \dots\dots\dots (13)$$

Hence the axisymmetric mode of the standing wave is possible for a Rossby number less than a constant value. However, in case of the nonaxisymmetric mode of the standing wave ($s=1,2$), there is no restriction on the Rossby number, because for any Rossby number the value of M can be determined such as to give $N = 0$ [c.f., Eqs. (8)', (9)' and Fig. 2]. It should be noted that the flow field involving the standing wave, itself becomes a steady flow.

3. Standing wave and stationary vortex breakdown

In this chapter it is shown that the swirling flow involving the standing wave as a disturbance, is in a good agreement with the observed pattern of time-independent (stationary) breakdown among various types of reported breakdowns.

As mentioned before, Sarpkaya^(1,2) identified three distinct types of breakdown, i.e., (1) axisymmetric-bubble, (2) spiral and (3) double helix. Moreover, Leibovich^(1,3) reported that 6 different types of breakdown mode exist called type 0 ~ type 6. Type 0, 2 and 5 are almost the same as Sarpkaya's bubble, spiral and double helix types, respectively. In addition, there are new types 3, 4 and 6. (Type 1 is a deformed one of type 0). Type 6 indicates that the central fluid filament moves off axis at a certain position and curves into a gentle spiral with the same sense as the rotating direction of the mean flow; type 4 forms a flattened bubble, and in type 3 the central filament moves abruptly off axis with a sharp kink, oscillating from side to side in a preferential plane. Leibovich pointed out that among these types, only type 5 and type 6 are purely stationary (that is, the streamlines visualized with the dye remain steady and fixed in the space). In this chapter type 0 (approximately stationary) and types 5 and

Fig. 2 Angular frequency N of wave disturbance versus wave length L_z

and 6 are mainly discussed, and type 4 is briefly treated. Type 2, which is unsteady and temporally periodic, and type 3, which seems to be related with type 2, are excluded here. (They are to be discussed in another paper.)

As will be mentioned in Chapter 4, it is considered that the vortex breakdown has a close relationship with the wave disturbance which comes from the instability of the mean flow. This paper does treat not the occurrence modes of the disturbance, but the structural modes of the breakdown, dealing in particular with the stationary breakdown. Hence, a neutral stable wave disturbance is adopted to calculate the flow pattern of the flow field involving the disturbance. In addition, in spite of the assumption of small disturbance, the amount of the amplitude (C') of the disturbance is taken as the order of unity to calculate streamlines or velocity profiles of the flow field. The result can be considered to give an approximate model of the first order for the final state of breakdown, which may be affected by the nonlinearity of fluid motion after a small disturbance develops.

3.1 Axisymmetric mode ($s=0$) of standing wave and bubble type (type 0) of breakdown

The breakdown of the Leibovich type 0, which exhibits an axisymmetric bubble at a position of the swirling axis, is closely related to the occurrence of the $s=0$ type of the standing wave. The velocity components u , v and w are derived from Eqs. (7)' and (10). The equation for the streamline is written in the following dimensional form.

$$dr/u = r d\theta/v = dz/w \dots\dots\dots (14)$$

Figure 3(a) shows the streamlines calculated by computer through integrating the above equation for $s=0$. In the calculation the standing wave is assumed to exist in the range of $\bar{z} > 0$ ($\bar{z} = z/a$). $\bar{z}=0$ means the upstream boundary of the figure. The figure shows 12 streamlines which surround the pipe

(a) Streamlines of $s=0$ mode

(b) Sectioned streamlines of $s=0$ mode

Fig.3 $s=0$ mode of stationary breakdown ($R_0=0.496$, $M=-1.26$, $\sigma'=3.83$, $C'=1.0$)

axis at the upstream end with a small circle of the radius $R=r/a=0.01$, and a streamline off axis by $R=0.3$. Figure 3(b) shows the intersecting lines of the axisymmetric stream-surfaces and a plane including the pipe axis, that is, the curves (referred to as sectioned streamlines) obtained by integrating the following equation with $\theta = \text{const.}$

$$dr/u = dz/w \dots\dots\dots (14)'$$

Both figures (a) and (b) indicate the occurrence of an axisymmetric bubble. Figure (a) explains qualitatively the results of the views by Sarpkaya^(1,2) and Leibovich^(1,3) those which visualized the flow with dye. Figure (b) agrees well with the streamlines given by Nakamura^(1,8) measuring with a laser doppler anemometer. Figure 4 gives the velocity components $\bar{U}$, $\bar{V}$ and $\bar{W}$ ($\bar{U} = u/a\Omega$, $\bar{V} = v/a\Omega$, $\bar{W} = w/a\Omega$) at several $\bar{z}$ sections [from Eqs. (7)', (10)]. The position $\bar{z} = 2.5$ corresponds to the center of the vortex bubble. Figure 4, especially Fig. (c), explains well an abrupt deceleration near the front edge of the bubble, the stagnation and the reversal flow in the bubble. Thus the features of the phenomena, which have been experimentally observed, are obtained in spite of the fact that the assumptions under which Eqs. (7)', (10) are derived conflict to some extent with the actual flow. Moreover, the variation of $\bar{W}$ profile in the $\bar{z}$ direction also agrees well with the experimental result by Nakamura^(1,8), except in the vicinity of the pipe wall. Therefore, it is seen that the occurrence of the standing wave for $s=0$ adequately represents the axisymmetric bubble type (type 0) of

Fig.4 Velocity component profiles of perturbed flow field given in Fig.3

the breakdown phenomena. It is noted that the vortex bubble shown in Fig.3, appears to be periodic along the axis, which is seen from Eqs.(7)'. The successive bubble does not occur in the actual swirling flow, because once a vortex bubble develops, the flow field is largely disturbed in the wake of the bubble. Thus only one bubble is shown in the figure. However, a few observers^(1,2) have reported a second bubble which occurs after the first one.

3.2 Nonaxisymmetric modes of breakdown

In this section it is shown that the flow field in which the $s=1$ or $s=2$ mode of standing wave occurs, exhibits the stationary, spiral type (type 6) or double helix type (type 5) of vortex breakdown, respectively. Figure 5 shows a calculated result of the streamlines for $s=1$ [from Eqs.(7)', (10), (14)]. It draws 6 streamlines surrounding the axis at $\bar{Z}=0$ (i.e., at the upstream end of the standing wave region). Figure (a) shows a view perpendicular to the axis, and Fig. (b) one from the downstream side. The central streamlines at the upstream deflect off the axis, and form a gentle spiral, which shows good agreement

Fig.5 Streamlines of stationary breakdown of $s=1$ mode

Fig.6 Sectioned streamlines of $s=1$ mode (same conditions as Fig.5)

with the flow pattern of the type 6 breakdown of Leibovich. Figure 6 presents the sectioned streamlines [from Eq.(14)' for $\theta = 0, (\pi/4), (\pi/2)$], and Fig.7 shows the velocity profile in the same manner as Fig.4 for $s=0$.

Figure 8 shows the calculated streamlines for $s=2$. As seen in the figure, the central streamlines (12 lines) expand into a slightly curved sheet, both edges of which envelope each other and form a double helix. This agrees well with the double helix type of breakdown observed by Sarpkaya^(1,2), Ikeda^(4,5) and Leibovich.^(1,3) In the figure the 12 fluid elements which passed through the upstream end ($\bar{Z}=0$) at the same time, are denoted by $\blacksquare$ at time t_1 and by $\square$ at time t_2 ($t_2 > t_1$). These indications show more clearly that the streamlines expand into a sheet and form a successive double helix. The sectioned streamlines are shown in Fig.9, and the velocity component profiles are shown in Fig.10. From comparison of Fig.7 (c) for $s=1$ and Fig.10(c) for $s=2$ with Fig.4 (c) for $s=0$, it is seen that the stationary breakdown of the nonaxisymmetric type does not exhibit an abrupt deceleration or a stagnation. This differs from the case of the axisymmetric type (which agrees with the result of Leibovich). In addition, the nonaxisymmetric type of breakdown does not show periodic character in the axial direction. (Although the corresponding sectioned streamlines show periodic variation in the axial direction, the streamlines themselves do not show the periodicity due to the simultaneous variation of the azimuthal velocity.) This also differs from the case of

Fig.7 Velocity component profiles of $s=1$ mode (same conditions as Fig.5)

the axisymmetric type.

It has been mentioned that typical modes of the observed stationary breakdown correspond well to the flow field involving the standing wave of $s = 0, 1$ and 2 . From this fact, it can be inferred that the fundamental modes of observed breakdowns, including the unsteady case, can be explained as three cases of the flow field perturbed by the wave disturbances with the parameter $s = 0, 1$ and 2 . From this viewpoint, characteristic flow patterns for these three cases are shown in Fig.11. The figure indicates the streamlines sectioned by the plane $\bar{Z}=0$ [calculated from $\bar{d}r/u = r d\theta/v$ c.f., Eq.(14)]. As seen from the figure, $s = 0$ shows the axisymmetric flow pattern, $s = 1$ shows the flow deflection in the radial direction, and $s = 2$ gives the center-symmetric and flattened flow pattern. In addition the case of $s = 3$ is shown for reference. The breakdown of $s = 3$ mode is expected to be a triple helix type, which has not been observed yet.

3.3 Other modes of stationary breakdown

As mentioned before, Leibovich

reported on new types of the breakdown, namely, the flattened bubble type 4, and type 3, which exhibits the combined properties of types 2 and 4. Type 4 is explained here (unsteady types 2 and 3 are discussed in another paper). Leibovich said that this type is not always stationary, but the inclination of the flattened bubble remains almost constant in the azimuthal direction. This type of breakdown can be explained in terms of the coexistence of $s = 0$ and $s = 2$ types of the wave disturbance. Figure 12 shows the calculated streamlines in the case of such coexistence, showing that the central streamlines expand into a flattened bubble inclined in the azimuthal direction. This corresponds well to the observed flow pattern of the type 4 breakdown. The result suggests that various types of breakdown will possibly occur due to the coexistence of two waves with different modes.

4. Some inferences on the occurrence of stationary vortex breakdown

With respect to the mode of the vortex breakdown, it is found that the stationary

Fig.11 Sectioned streaklines by $\bar{Z}=0$ plane

vortex breakdown is subject to a wave disturbance with zero phase velocity. i.e., the standing wave. This is a special case of the perturbed flow. As already mentioned, the unsteady types of breakdown are supposed as flow fields perturbed by a wave disturbance with a certain phase velocity ($n/m = 0$).

In regard to the occurrence mechanism of vortex breakdown, we propose the following concept. A breakdown can take place when the mean flow becomes unstable and a wave disturbance grows up. This instability of the mean flow comes from an axial variation of velocity profile due to the fluid viscosity or a gradual change of the sectional area of pipe (as Sarpkaya's or Leibovich's apparatus). Thus, from the viewpoint of stability problem, the following statement will be possible as to the stationary breakdown. It is known that in the flow field which is influenced by an external force (centrifugal force, buoyancy and so on), there can exist an unstable disturbance with zero phase velocity. Taylor vortex in the Couette flow between coaxial cylinders and Benard cell in the convection flow are such cases. In these cases the growth of a disturbance causes a new steady flow pattern. From these facts, the stationary breakdown is recognized as a final form of the flow field perturbed by an unstable wave disturbance with zero phase velocity. The swirling pipe flow is subject to the influence of the centrifugal and Coriolis force.

5. Conclusions

This paper concerns the vortex breakdown phenomena, especially the stationary ones, which occur in a swirling flow within a circular pipe. The form of the flow fields which involve a neutrally stable wave disturbance is examined. It is made clear that the stationary vortex breakdown has close relation to the occurrence of a wave

(a) Streamlines viewed from the vertical direction

(b) Streamlines viewed from the horizontal direction

Fig.12 Flow pattern involving two wave disturbances of different modes ($s = 0$ and $s = 2$)

disturbance with zero phase velocity. It is also found that various types of breakdown, which have been observed, correspond to the values of a parameter s which defines the azimuthal mode of the wave disturbance.

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