533.6.08:533.6.07

Paper No. 193-10

The Error in Measuring an Accelerated Flow Velocity by a Spark Tracer Method*

By Kazuyasu MATSUO**, Toshiaki SETOYUCHI***,
and Yoshihisa YAMAMOTO****

An ionized gas particle created by spark discharge between electrodes has a density different from that of the surrounding gas. This difference of densities causes errors in measuring the velocity when a spark tracer method is applied to an accelerated flow.

In this paper, assuming a simple flow model, the error associated with the difference of densities of tracer and surrounding gas has been calculated for an unsteady accelerated flow induced by an expansion fan in a shock tube and for a steady accelerated flow in a subsonic nozzle. The error increases with time in the latter case, but in the former case it becomes maximum at a certain time. The calculated results qualitatively agree well with the experimental values measured in a driver section of a shock tube. Furthermore, the relation between the error and the magnitude of acceleration has been discussed.

1. Introduction

A spark tracer method⁽¹⁾, in which a gas particle is ionized and thereby illuminated by discharging a high frequency series of high voltage pulses between electrodes with a spatial distance in a flow field, is not only a technique for flow visualization using the ionized gas particle as a tracing element but also a method of quantitative measurement of flow velocity. As this method has various merits compared to the previous methods of velocity measurements⁽²⁾, it has been employed to measure velocities not only in steady flows but also in unsteady flows such as a flow around an oscillating aerofoil⁽³⁾ or flows in internal combustion engines^{(4) (5)}. However, when the spark tracer method is applied to an accelerated flow or an unsteady flow in which there exist various kinds of external forces, errors in measuring velocities arise because of the difference of densities of the ionized gas particle and the surrounding gas. Bomelburg et al.⁽⁶⁾ proposed a simple relation that the error increases linearly with the magnitude of acceleration. Nagao et al.⁽⁴⁾ investigated the error using an accelerated flow field in a rotating vessel and showed an experimental formula in

which the error was proportional to the square root of the magnitude of acceleration. Further, Asanuma et al.⁽³⁾ examined the error in a steady accelerated flow using a subsonic nozzle, and proposed an experimental formula. However, the above-mentioned relations between the error and the magnitude of acceleration are very different from each other, and there are many unknown aspects about the error.

In this paper, the error caused by the difference of densities of a tracer and the surrounding gas has been theoretically analysed. On the other hand, using an unsteady accelerated flow induced by an expansion fan in a driver section of a shock tube, the above-mentioned error has been experimentally investigated and the experimental values have been compared with calculated results.

2. Measurement errors in an accelerated flow

When flow velocity is measured by a spark tracer method, the following measurement errors will arise : (1) Personal error (2) Error caused by distortion and diffusion of an ionized path (3) Error caused by buoyancy of an ionized path (4) Error due to aerodynamic disturbance generated by electric spark discharge (5) Error associated with the averaging of velocity, and (6) Error caused by the difference of densities of an ionized gas particle and the surrounding gas. The errors (1) to (4) described above appear in measuring the velocities of a steady and uniform flow, and they were discussed in Ref.(6). In the case of an accelerated flow, the errors (5) and (6) appear in addition to the errors (1) to (4). In this Chapter, the errors (4), (5), and (6) will be discussed.

* Received 28th July, 1978.

** Professor, Graduate School of Engineering Sciences, Kyushu University, Fukuoka.

*** Post-Graduate Student of Mechanical Engineering, Kyushu University, Fukuoka.

**** Faculty of Engineering, Kyushu University, Fukuoka.

2.1 Error due to electric spark discharge

When an electric spark is struck between a pair of electrodes, the flow near the electrodes will be disturbed by spark energy. In order to eliminate this effect, the energy released at each discharge should be as small as possible. It is said that an electric energy transferred to a flowing gas is about 0.5 percent of that supplied between electrodes⁽⁴⁾, but the effect of discharge energy on flow field has not yet been well clarified.

In the present study, a blast wave generated by a spark discharge in the stagnant atmosphere was observed by a schlieren method. An example of the schlieren photographs is shown in Fig.1. Figure 1 (a) shows a sketch of a cylindrical shock wave, and (b) and (c) are schlieren photographs of a shock wave viewed from axial and radial directions respectively. In Fig.1, E_s is a voltage supplied between electrodes, $2h$ a length of spark gap, p pressure, T temperature, and t a time interval between the moment when the spark is struck and the moment when the schlieren photograph is taken. A cylindrical shock wave generated by spark discharge is clearly observed in these figures. Let r be the distance from the electrodes to the shock wave, as shown in Fig.1 (a). An example of the relation between r and t is shown in Fig.2. As is evident from this figure, r increases linearly with an increasing t , except an extremely limited region near the origin, and the propagating speed of the shock obtained from the gradient of this straight line is 343 m/s. This fact means that the propagating Mach number M of the shock is almost equal to unity. In other words, this shock wave may be regarded as a sonic wave, and therefore its effect on flow field may be negligible in the present experiment.

2.2 Error associated with the averaging of velocity

Velocity obtained by a spark tracer method is an average value of spacial intervals between two ionized paths. Therefore if the velocity varies linearly with an increasing time, the measured velocity agrees exactly with the velocity just at the middle time of the two moments when the two sparks are struck, otherwise an error associated with the averaging of velocity would be yielded. In order to estimate this error, let us consider an accelerated flow induced by an expansion fan in a driver section of a shock tube, as an example. Figure 3 shows a wave diagram of a driver section, where x is a distance from a position of a diaphragm and τ is time from the moment of rupture of a diaphragm. The dashed line represents a path of a gas particle which locates at the position x_0 initially, and the path is expressed as follows :

$$x = \frac{2a_4\tau}{\gamma-1} - \frac{(\gamma+1)\tau_0 a_4}{\gamma-1} \left(\frac{\tau}{\tau_0} \right)^{2/(\gamma+1)} \quad (1)$$

where γ is ratio of specific heats, a_4 is initial sound speed in a driver section, and $\tau_0 = |x_0|/a_4$. The velocity u of the particle along this path is obtained by differentiating Eq.(1) as follows.

$$u = \frac{2a_4}{\gamma-1} \left\{ 1 - \left(\frac{\tau_0}{\tau} \right)^{(\gamma-1)/(\gamma+1)} \right\} \quad (2)$$

Assuming that the first and second sparks are struck at points 1 ($\tau = \tau_s$) and 2 ($\tau = \tau_s + 1/f$, f : prescribed spark frequency) in Fig.3, the velocities u_1 and u_2 at these points 1 and 2 can be calculated by putting $\tau = \tau_s$ and $\tau = \tau_s + 1/f$ in Eq.(2). The mean velocity u_s at this time interval is given by $u_s = (u_1 + u_2)/2$. On the other

Fig.1 Cylindrical shock wave generated by spark discharge
($E_s = 16.0$ kV, $2h = 8$ mm, $p = 1.024 \times 10^5$ Pa, $T = 292$ K, $t = 27$ μ s)

Fig.2 Wave diagram showing shock wave behavior

Fig.3 x - τ diagram of driver section in a shock tube

hand, the velocity u_m at the middle point of the first and second sparks (point m in Fig.3) is calculated by putting $\tau = \tau_s + (1/2f)$ in Eq.(2), and this is a true velocity at $\tau = \tau_s + (1/2f)$. Consequently, the error due to the averaging of velocity may be expressed as $u_s - u_m$. The ratio of this error to the true velocity $|u_s - u_m|/u_m$ is shown against the frequency f in Fig.4. In this figure, the position of electrodes x_s (see Fig.3) is assumed to be at 256mm upstream of the diaphragm, and the curves a and b represent the calculated results in the cases of $t_H = 20\mu s$ and $75\mu s$, respectively, where t_H (see Fig.3) is a time interval measured from the moment when the head of the expansion fan arrives at the position of electrodes to the moment when the first spark is struck. The value of α in the figure shows the magnitude of acceleration at the instant when the first spark is struck, and it becomes larger for a smaller t_H . It may be said from this figure that, in the case of an unsteady expansion fan, the error due to the averaging of velocity is less than 1 percent when the spark frequency $f > 20\text{kHz}$. In the present experiment, f was set at a value larger than 20kHz, because otherwise a series of continuous spark paths could not be obtained. Therefore, in this work, this error may be negligible.

2.3 Error due to the difference of densities

The density of an ionized gas particle ρ_b created by a spark discharge (we call it here "bubble") is smaller than that of surrounding gas ρ_g (we call it here "gas"), because the temperature of bubble may be much higher than that of gas. This difference of densities causes an error in measuring the velocity when a spark tracer method is applied to an accelerated flow.

In order to estimate this error, we assume as follows, in the same way as in Ref.3.

(1) A bubble is a cylinder with a diameter d and a length l and its density ρ_b is constant.

(2) The drag force acting on the bubble is proportional to the square of relative velocity $\Delta u \equiv u_b - u_g$, where u_b and u_g are the velocities of bubble and gas, respec-

tively, and the drag coefficient c_D is constant.

(3) There are no mass and energy transfers between the bubble and gas.

(4) The effect of virtual mass may be negligible.

According to the above assumptions, the equation of motion of the bubble may be expressed as,

$$\frac{\pi l d^2}{4} \rho_b \frac{D_b u_b}{Dt} = -\frac{\pi l d^2}{4} \frac{\partial p}{\partial x} - \frac{c_D \rho_g l d}{2} (\Delta u)^2 \quad (3)$$

where, $D_b/Dt = \partial/\partial t + u_b(\partial/\partial x_b)$, and p represents static pressure of gas flow. For the gas flow, Euler's equation holds:

$$\rho_g (D_g u_g / Dt) = -\partial p / \partial x$$

where $D_g/Dt = \partial/\partial t + u_g(\partial/\partial x_g)$

According to Ref.(7), the convective part of the substantial derivative operators D_b/Dt and D_g/Dt may be negligible compared with the unsteady part if the following relation is satisfied.

$$\left| -\frac{u_b - u_g}{u_g} \right| = \left| \frac{\Delta u}{u_g} \right| \ll 1$$

In this work, the experimental value of $|\Delta u/u_g|$ is in the range from 0 to 0.15, as will be mentioned later. Therefore we neglect the convective part, and, noting $\partial u_g / \partial t = du_g / dt = \alpha$, Eq.(3) can be written as

$$\frac{\pi l d^2}{4} \rho_b \frac{du_b}{dt} = \frac{\pi l d^2}{4} \rho_g \alpha - \frac{c_D \rho_g l d}{2} (\Delta u)^2$$

Subtraction of $\pi l d^2 \rho_b \alpha / 4$ from both sides of the above equation yields

$$\frac{d(\Delta u)}{dt} = \alpha \left(\frac{\rho_g}{\rho_b} - 1 \right) - \frac{2 c_D \rho_g (\Delta u)^2}{\pi \rho_b d} \quad (4)$$

Assume that ρ_g/ρ_b and α are constant, and consider an accelerated flow ($\alpha > 0$). Then, with an initial condition of $\Delta u = 0$ at $t = 0$, the analytical solution of Eq.(4) is

$$\Delta u = \Delta u_\infty \frac{1 - \exp(-2\beta t)}{1 + \exp(-2\beta t)} \quad (5)$$

where

$$\Delta u_\infty \equiv [\pi \alpha (1 - \rho_b/\rho_g) d / 2 c_D]^{1/2} \quad \left. \begin{aligned} \beta &\equiv [2 c_D \alpha (\rho_g/\rho_b) \{ (\rho_g/\rho_b) - 1 \} / \pi d]^{1/2} \end{aligned} \right\} \quad (6)$$

Eqs.(5) and (6) are identical with Eq.(3) in Ref.3. The spark tracer method gives a value of u_b as a flow velocity, while the true velocity is u_g . Therefore the value of Δu in Eq.(5) shows the error due to the difference of densities. According to the present optical observations by a schlieren method, the value of d is 0.3mm. The Reynolds number of a cylinder based on this value of d is about 500, and the drag coefficient of the cylinder $c_D = 1.1$. Assume $\rho_b/\rho_g = 1/3$, as an example. Then using these values of d , c_D , and ρ_b/ρ_g ,

Fig.4 Error associated with the averaging of velocity ($x_s = -256\text{ mm}$)

Eq.(5) can be calculated. The results are shown in Fig.5 as the solid lines. The dashed lines represent the value of Δu_{∞} calculated by Eq.(6), and α is taken as a parameter in the figure. Figure 5 shows that when α becomes larger, the error Δu approaches a constant value of Δu_{∞} faster and the value Δu_{∞} increases. The asymptotic increase in Δu may be due to the fact that the drag force acting on the bubble increases with time, and Δu_{∞} corresponds to a terminal velocity.

In the discussions mentioned above, we assumed constant values of α and ρ_b/ρ_g , but they may not be kept constant in a real flow. Next, as examples of accelerated flows, we consider an unsteady flow induced by an expansion fan and a steady accelerated flow in a subsonic nozzle.

In an unsteady expansion fan, the variations of density ρ_g and acceleration α with time along the particle path of the dashed line shown in Fig.3 are expressed as follows.

$$\rho_g = \rho_4 \left[\frac{2}{\gamma+1} - \frac{\gamma-1}{\gamma+1} \left\{ \frac{x}{a_4(t+\tau_g)} \right\}^2 \right]^{2/(\gamma-1)} \quad (7)$$

$$\alpha = \frac{2a_4}{(\gamma+1)\tau_0} \left(\frac{t+\tau_g}{\tau_0} \right)^{-2\gamma/(\gamma+1)} \quad (8)$$

where ρ_4 is initial density in a driver section, and $t = \tau - \tau_g$, that is, time from the moment when the first electric spark is struck. Substituting ρ_g of Eq.(7) and α of Eq.(8) into Eq.(4), and assuming that $d = 0.3\text{mm}$ and $c_D = 1.1$, as were assumed in Fig.5, the variation of the error Δu with time can be calculated for a prescribed value of ρ_b/ρ_4 .

Next, suppose a flow in a two-dimensional convergent nozzle with a half-convergence angle of 10° , as shown in Fig.6. The angle is identical with that in Ref.3. Let x be the distance along the flow from the nozzle inlet A. Assume a steady and isentropic flow with stagnation density of ρ_0 , and further assume that the flow is choked at the nozzle exit B. In this case, the magnitude of acceleration at $x_s = 563\text{mm}$ becomes $\alpha = u_g(\partial u_g/\partial x) = 2.53 \times 10^5 \text{ m/s}^2$, which corresponds to the value of acceleration at the point of $x_s = -350\text{mm}$ and $t_H = 77.6\mu\text{s}$ in the unsteady expansion fan shown in Fig.3. Therefore, assume

Fig.5 Variation of error Δu with time when α and ρ_b/ρ_g are constant

that a pair of electrodes is located at $x_s = 563\text{mm}$. Then the variations of density ρ_g and acceleration α of a gas particle with x (or with time) which is initially located at x_s can be calculated by using relations of steady isentropic flow. Substituting these values of ρ_g and α into Eq.(4), and assuming that $d = 0.3\text{mm}$ and $c_D = 1.1$ again, the variation of the error Δu with time can be calculated for a prescribed value of ρ_b/ρ_0 .

The calculated results are shown in Fig.7(a), where the solid lines represent Δu , and the dashed lines represent Δu_{∞} . In the calculation of Δu_{∞} , the values of acceleration and density at the corresponding time of the horizontal axis were used in Eq.(6). Figure 7(b) shows the variations of density ratio ρ_b/ρ_g and flow acceleration α with time. When $\rho_b/\rho_0 = 1/5$ in the nozzle flow (curve ②), the density ratio of bubble to gas ρ_b/ρ_g at the position of electrodes ($x_s = 563\text{mm}$) is equal to $1/4.43$. As mentioned before, the value of acceleration $\alpha = 2.53 \times 10^5 \text{ m/s}^2$ at this point is equal to that at the point of $x_s = -350\text{mm}$ and $t_H = 77.6\mu\text{s}$ in the unsteady expansion fan in Fig.3. Therefore, if we

Fig.6 Nozzle contour

(a) Variation of error Δu with time

(b) Variation of density ratio ρ_b/ρ_g and acceleration α with time

Fig.7 Variation of error Δu with time when α and ρ_b/ρ_g vary with time

assume that $\rho_b/\rho_g = 1/4.43$ at the point of $x_s = -350\text{mm}$ and $t_H = 77.6\mu\text{s}$, then ρ_b/ρ_g in the case of the unsteady expansion fan becomes $1/4.69$. Curve ① in Fig. 7 represents the calculated results in this case. Curve ③ calculated by Eq. (5) is also shown, for reference, where α and ρ_b/ρ_g are assumed to be constant values of $2.53 \times 10^5 \text{ m/s}^2$ and $1/4.43$, respectively. As is evident from the figure, the error Δu increases at first with time in any case, and then decreases slightly in the case of curve ①. On the other hand, Δu in the case of curve ② continues to increase. The behaviors of Δu with time may be explained by considering the variation of acceleration α with time shown in Fig. 7(b) as follows. From the figure, although an unsteady expansion fan induces an accelerated flow, the flow acceleration decreases with time, and so the error Δu decreases after becoming maximum. On the other hand, in the case of a steady accelerated flow in a subsonic nozzle, the acceleration increases with time, and Δu continues to increase with time. It should be noted that all of the solid curves ①, ②, and ③ agree perfectly with the corresponding dashed lines after a rising period. This means that the error Δu at any time after rising agrees with Δu_∞ . Therefore it may be concluded that the error after rising is independent of time and depends only on the magnitude of acceleration and density ratio. Also, as is evident from Eq. (4), the error Δu is independent of gas velocity u_g , therefore relative error $\Delta u/u_g$ becomes smaller when u_g increases.

3. Experimental apparatus

The experimental apparatus used in this work is shown schematically in Fig. 8. The shock tube, which consists of seven tube elements, has a cross-section of 38mm square and a length of 4.97m. The test section with a length of 245mm is located upstream of the diaphragm and one of its side-walls is made of glass. At the test section, the pressure transducers (Kistler 202A5, resonant frequency 250 kHz, rise time $2\mu\text{s}$) are mounted flush on the upper wall, and a pair of electrodes with a di-

Fig. 8 Shock tube facility and its measuring system

ameter of 0.76mm and a length of spark gap of $2h \approx 6\text{mm}$ are inserted.

Experimental procedures are as follows. First, the frequency of the oscillator and the number of pulses of the preset counter are prescribed. The frequency of the oscillator is monitored by a frequency counter. When a diaphragm is ruptured and the head of expansion fan arrives at the pressure transducer for trigger, the signal from the transducer triggers the preset counter and wave memory after a prescribed time delay. Then the prescribed number of pulses is boosted by both a trigger pulse generator and a pulse transducer, and supplied to the electrodes. The outputs of pressure transducers are memorized in a wave memory, together with the pulse signals of electric spark discharge. The specifications of the pulse generator used in this work are shown in Table 1. Air is used as working gas and a sheet of cellophane is used as a diaphragm.

In the present experiment, it was confirmed that the errors (1) to (5) mentioned in Chapter 2 were extremely smaller than the error (6), and they could be neglected.

Output voltage	8 ~ 20 kV
Spark frequency	10 Hz ~ 100 kHz
Duration of spark	About 0.5 μs
Energy per pulse	About 15 mJ ($f \leq 50 \text{ kHz}$) About 7.5 mJ ($50 \text{ kHz} < f \leq 100 \text{ kHz}$)
Number of pulse	1 ~ 99

Table 1 Specifications of pulse generator

4. Experimental results and discussions

4.1 Experimental results

The error Δu due to the difference of densities of bubble and gas was experimentally investigated using an unsteady accelerated flow induced by an expansion fan. In these experiments, the initial pressure in a driver section p_4 was the atmospheric pressure, and that in a driven section p_1 was made vacuum. A typical pressure variation with time at 850mm upstream of the diaphragm in the case of $p_4/p_1 = p_{41} = 2.09$ is shown in Fig. 9, where τ is the time measured from the moment when the head of the expansion fan arrives at

Fig. 9 Pressure variation with time at 850mm upstream of diaphragm ($p_{41} = 2.09$, $T = 291 \text{ K}$)

$x = -850\text{mm}$ (position of electrodes). From the measured values of pressure in the figure, the velocity of gas u_g can be calculated by using relations of an unsteady centered expansion fan. In carrying out the calculations, we took the effect of a finite opening time of a diaphragm into consideration, by the method presented in Ref.(8). The measurement error of pressure in the present experiments was about 0.1 percent and the error of u_g calculated from the measured values of pressure was at most about 3 percent, which is much smaller than the experimental values of Δu , as will be shown later.

The pulse signals of electric spark discharge in Fig.9 show the times when a series of high voltage pulses are struck between electrodes, and the corresponding spark photograph is shown in Fig.10, where f is a spark frequency, E_s is a voltage supplied between electrodes, and n is the number of pulses. This figure shows that the flow is accelerated with time.

The gas velocity u_g and the bubble velocity u_b are calculated from Figs.9 and 10, respectively. A typical example of the error $\Delta u \equiv u_b - u_g$ thus obtained is shown against $t + t_H$ in Fig.11, where $p_{41} = 1.96$ and $x_s = -210\text{mm}$, and $t + t_H$ represents the time measured from the moment when the head of expansion fan arrives at the position of electrodes. In the figure, t_H (see Fig.3) is taken as a parameter. A series of open and closed symbols connected with lines denote the experimental results when t_H (see Fig.3) = $18\mu\text{s}$ and $75\mu\text{s}$, respectively, and the experimental points on the axis of abscissa denote the

Fig.10 Example of spark photograph in unsteady expansion fan ($f = 30\text{ kHz}$, $p_{41} = 2.09$, $E_s = 14.4\text{ kV}$, $n = 8$)

Fig.11 Variation of error Δu with time in unsteady expansion fan ($x_s = -210\text{mm}$, $p_{41} = 1.96$, $T = 284\text{ K}$)

time when the first spark is struck. Since the opening time of a diaphragm is finite, as mentioned before, calculation of gas velocity from a measured pressure is limited to within a certain time interval. For this reason, the experimental results in Fig.11 are limited to within a certain time interval. The dashed lines in Fig.11 represent the calculated results when it is assumed that $d = 0.3\text{mm}$, $c_D = 1.1$, and $\rho_b/\rho_g = 1/3$, and the calculated results of u_g and a ratio of flow acceleration α to acceleration of gravity g , α/g , are also indicated, for reference. The two curves of α/g when $t_H = 18\mu\text{s}$ and $75\mu\text{s}$ overlap each other. From this figure, it is seen that Δu increases rapidly at first, and then decreases gradually with time, and the experimental results agree well with the calculated values. Also, Fig.11 shows that the maximum value of Δu becomes larger when the magnitude of flow acceleration increases.

4.2 Discussions

According to the present calculated results, the error Δu depends not only on acceleration α but also on time during its rising period, but after the period, it is equal to $\Delta u_{\max}$ in Eq.(6) and is independent of time. Therefore it may be said that the maximum value of the error $\Delta u_{\max}$ in Fig.11 depends only on α/g . The relation is shown in Fig.12. Three solid lines represent the calculated results by Eq.(6) assuming that $d = 0.3\text{mm}$, $c_D = 1.1$, and $\rho_b/\rho_g = 1/2$, $1/3$, and $1/10$. In both the experimental and calculated results, $\Delta u_{\max}$ increases with an increasing α/g . The experimental points are a little different from the calculated results, but considering some bold assumptions in the calculation, it may be said that the calculated values agree considerably well with the experimental results. The experimental results show that when $\alpha/g < 3 \times 10^3$, $\Delta u_{\max}$ is almost zero and the effect of the difference of densities may be negligible. On the other hand, when $\alpha/g > 3 \times 10^3$, the error due to the difference of densities may be experimentally expressed as follows.

$$\Delta u_{\max} = 2.2 \times 10^{-4} (\alpha/g)^{1.02} \quad \text{m/s} \quad (9)$$

Fig.12 Relation between maximum error $\Delta u_{\max}$ and acceleration of flow α

The above equation is also plotted in Fig.12.

Previously, three experimental equations showing the relation between Δu and α have been proposed.

$$\Delta u = 0.016 (\alpha/g)^{0.94} \quad \text{m/s} \quad (10)$$

$$\Delta u = 0.18 (\alpha/g)^{0.5} \quad \text{m/s} \quad (11)$$

$$\Delta u = 0.136 (\alpha/g) \quad \text{m/s} \quad (12)$$

Equations (10) and (11) were obtained for a steady accelerated flow in a subsonic nozzle⁽³⁾ and for a flow in a rotating vessel⁽⁴⁾, respectively. Equation (12) was obtained by comparing buoyancy effect in an accelerated air flow with gravitational effect⁽⁶⁾. The errors calculated by Eqs.(10) to (12) in the present experimental range ($3 \times 10^3 < \alpha/g < 4 \times 10^4$) are 30 ~ 340 m/s, 10 ~ 36 m/s, and 410 ~ 5400 m/s, respectively. These are extremely larger than the present experimental and calculated values shown in Fig.12. Consequently, Eqs.(10) to (12) do not seem to be valid for the present experiments.

In general, the magnitude of acceleration is not known, when the spark tracer method is applied to some accelerated flows. In such a case, application limit for the error Δu to be negligible may be obtained as follows. As was mentioned previously, the error Δu can be neglected when $\alpha/g < 3 \times 10^3$. Let u_i be the velocity calculated from the i th and $(i+1)$ th spark paths in a spark photograph, then local acceleration α of the flow becomes $(u_{i+1} - u_i)/(1/f)$. Therefore, the error due to the difference of densities can be negligible if the following equation is satisfied.

$$(u_{i+1} - u_i) \cdot f < 3 \times 10^3 \cdot g \quad (13)$$

The region satisfying Eq.(13) is hatched in Fig.13. For example, when $f = 20$ kHz and a velocity difference $(u_{i+1} - u_i)$ is less than 1.5 m/s, then the error due to the difference of densities may be negligible in experimental results. It should be noted that when f becomes smaller, there appears another error associated with the averaging of velocity, as was described in section 2.2.

Fig.13 Relation between $u_{i+1} - u_i$ and f where error Δu may be negligible

5. Conclusions

The errors in measuring an accelerated flow velocity by a spark tracer method have been investigated. The conclusions are summarized as follows.

1. The error Δu associated with the difference of densities of surrounding gas ρ_g and an ionized gas ρ_b created by a spark discharge has been analyzed, and some numerical examples are given. The results are summarized as follows.

(1) When the acceleration of a gas particle decreases with time as in an unsteady accelerated flow induced by an expansion fan in a driver section of a shock tube, Δu increases rapidly with time at first, and, after its rising period, it decreases gradually. On the other hand, when the acceleration increases with time as in a steady accelerated flow in a subsonic nozzle, Δu increases rapidly during its rising period, and it continues to increase even after its rising period.

(2) During the rising period, the error Δu depends on time as well as the magnitude of acceleration α and the ratio of densities ρ_b/ρ_g . After the rising period, Δu is independent of time.

2. The error Δu mentioned above has been experimentally investigated by using an unsteady accelerated flow induced by an expansion fan in a shock tube. The results are summarized as follows.

(1) The experimental variations of Δu with time are consistent with the variations estimated in the calculation.

(2) The maximum value $\Delta u_{\max}$ of the error increases with the magnitude of acceleration α . The experimental results of $\Delta u_{\max}$ agree roughly with the calculated values.

(3) In this experiment, the error Δu is negligibly small when the ratio of α to acceleration of gravity g , α/g , is less than 3×10^3 . An experimental equation between $\Delta u_{\max}$ and α/g has been presented for the case that $\alpha/g > 3 \times 10^3$.

[Appendix]

A method to measure variation of velocity with time at a position of electrodes by a spark tracer method.

When flow velocity varies with time and place, as in an unsteady accelerated flow induced by an expansion fan, the velocities obtained from a spark photograph are not velocities at the position of electrodes. However, if the locus of a gas particle is identified, the velocity variation with time at the position of electrodes may be calculated from a spark photograph. In this appendix, we will propose an approximate method to measure the variation of velocity with time at the position of electrodes, when the particle path can not be identified.

As an ionized path created by a spark discharge between electrodes moves downstream, the electric resistance of this path increases with time. For this reason, if the distance from the position of elec-

trodes to the ionized path in flow direction becomes larger than a certain value, then spark discharge takes place again near the first spark path. An example of such a case is shown in Fig.14 which was taken in an expansion fan. The number in this figure indicates the order in a series of electric spark discharges. The velocity u_i obtained from the distance between i th and $(i+1)$ th ionized paths is marked by open circular symbols in Fig.15, where the horizontal axis t is time measured from the moment when the head of the expansion fan arrives at the position of electrodes. The solid line represents the velocity calculated from measured pressure at the position of electrodes. Therefore, the solid line represents real velocity variation with time at that position. As is evident from the figure, the difference between u_i (open circular symbols) and the value of the solid line at the same t increases with an increasing number i , but when spark discharge occurs again near the first spark path ($i=7$), the difference becomes very small. In this way, by re-discharging near the first spark path, the velocity at the position of electrodes after a certain time may be measured accurately. Open square symbols in the figure represent the velocities calculated from u_1 and u_7 by a method of interpolation, and these points agree well with the solid line. Therefore, we can also obtain an approximate velocity at intermediate period by the method mentioned above.

Fig.14 Example of photograph showing spark path re-discharged near the first spark path
($f=30$ kHz, $P_{41}=2.28$, $x_8=-180$ mm)

Fig.15 Variation of velocity with time at the position of electrodes obtained from Fig.14

References

- (1) Merzkirch, W., Flow Visualization, (1974), p.195, Academic Press.
- (2) Asanuma, T., edited, Nagare no kashika handbook (in Japanese), (1977 - 3), p.291, Asakura Press.
- (3) Asanuma, T., et al., Proc. of International Symposium on Flow Visualization, Tokyo, (1977), p.151.
- (4) Nagao, F., et al., Bull. JSME, Vol.14, No.78 (1971 - 12), p.1340.
- (5) Kaziya, S., Toyota - Gijutsu (in Japanese), Vol.18, No.3 (1966 - 12), p.207.
- (6) Bomelburg, H. J., et al., Z. Flugwiss., Bd.7, Nr.11 (1959), S.332.
- (7) Hinze, J. O., Turbulence, (1975), p.464, McGraw - Hill.
- (8) Hall, J. G., et al., AIAA J., Vol.12, No.5 (1974 - 5), p.724.