Balancing for a Flexible Rotor Considering the Vibration Mode*

(1st Report, Balancing in which Modal Influence Coefficients Contain Amplitude Errors)

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As regards a balancing method for a flexible rotor, a modal-least squares balancing method is derived, which has both features of the modal balancing and least squares balancing methods. The effect of each correction mass, which is obtained by the above method, on the vibration is physically clarified. Even if the influence coefficients contain errors initially, they are easily corrected by applying the above method, and a good balancing result is then obtained.

It is described by some numerical simulations how the above method is effective for the balancing of the flexible rotor.

1. Introduction

With an increase in size and capacity of steam turbines and generators, rotors have been growing larger and becoming multi-span in type. However, enlarging a rotor reduces the rigidity of its shaft, which tends to produce vibrations. In order to produce reliable, high-performance rotors, axial vibrations should be reduced as much as possible. Therefore, balancing is becoming increasingly important in the production process. From the viewpoint of productivity, it should be made possible to balance a rotor quickly.

The least squares balancing method has been reported to be an effective way of balancing flexible rotors1)-7). The least squares balancing method is advantageous, because a large number of vibration data can be taken into account simultaneously. However, since many items of data are statistically processed, the physical meanings of the plurality of correction mass are not definite. Accordingly, there are sometimes errors in the influence coefficients that are used to calculate the correction masses, which means that more than one balancing is usually necessary in this method.

In the modal balancing method, which was first reported many years ago, balancing is performed considering the physical characteristics of the unbalanced vibration mode. This technique, however, has a disadvantage in that a large number of vibration data cannot be processed simultaneously, as can be done with the least squares balancing method. The authors have been examining a "modal-least squares balancing method", which incorporates the advantages of both the least squares and the modal balancing methods. They have also examined applicabilities of this method to balancing when there are errors in the influence coefficients.

In this paper, the adequacy of the Modal-least Squares Balancing Method and the results of corrections of influence coefficients that contain errors in amplitude are studied in balancing simulations and the results obtained are reported.

2. Definition of Symbols

The main symbols used in this paper are as follows:

- N : number of correction planes in each mode.
- L1 : number of positions at which vibrations are detected.
- L2 : number of directions from which vibrations are detected.
- \( a_{\text{mm}}(i) \) : influence coefficient of the i-th mode. \( (n = 1, 2, \ldots, N; m = 1, 2, \ldots, M; M = L1 \times L2) \)
- \( a'_m(i) \) : influence coefficient of the i-th corrected mode.
- Am(1) : initial vibration of the i-th.
- \( \delta_m(1) \) : residual vibration of the i-th.
- \( \delta_m(1) \) : initial vibration of the i-th.
- \( \delta_m(1) \) : evaluated residual vibration of the i-th.
- \( \delta_m(1) \) : correction mass at the center of a rotor.
- Cw : anti-phase correction masses at both ends of a rotor.
- \( \delta \) : reference value of correction for influence coefficients.

3. Balancing Calculations

3.1 Influence Coefficient

When a rotating shaft system is assumed to be linear, a linear relation holds between a mass added to the rotor...
and the vibration produced by the mass. The influence coefficients are generally defined, using the above linear relation, as the vibration produced by adding a unit mass at a position on the rotor.

In defining the influence coefficients used in the present paper, the positions on a rotor at which unit masses are to be added are distributed so as to excite a vibration mode. The vibration mode of a flexible rotor and the distributed mass exciting the mode are shown in Fig. 1. This distributed mass is used to calculate the influence coefficients, which are treated as an additional unit mass. Accordingly, the influence coefficients that are obtained by adding a mass at the center of a rotor and those that are obtained by adding masses at both ends of a rotor, are influential in the first and the second vibration modes, respectively. In the second vibration mode, this is true only when the two masses are out of phase with each other. The influence coefficient is referred to as "modal influence coefficient" hereinafter in the present paper.

3.2 Balancing Calculation Method
Correction masses are added to a rotor to reduce vibrations. The points at which correction masses are added so as to excite vibration modes are designated "modal correction planes". The correction masses that are added to a rotor at those points are designated as "modal correction masses".

When a modal correction mass \( W_n \) exciting up to the \( n \)-th mode is added to a rotor, residual vibration of the \( i \)-th mode \( \varepsilon_m(1) \) is generally expressed as follows, using the initial vibration \( A_m(1) \) and modal influence coefficient \( \omega_m(1) \):

\[
\varepsilon_m(1) = A_m(1) + \sum_{n=1}^{N} \omega_m(1) W_n(1) \quad (1)
\]

\( m = 1, 2, \ldots, M, \quad i = 1, 2, \ldots, I \)

when \( M \times I = N \) in Eq. (1), \( W_n \) is determined by solving the simultaneous equations. When \( M \times I > N \), the Least Squares Method is applied to determine \( W_n \). That is, in order to minimize \( \varepsilon_m(1) \), the following performance function is defined.

\[
J(\varepsilon_m(1), W_n, \ldots, W_M) = \sum_{i=1}^{M} \sum_{m=1}^{I} |\varepsilon_m(1)|^2 \quad (2)
\]

In Eq. (2), \( \varepsilon_m(1) \) is a complex number, and \( J \) is a scalar quantity. In this case, calculating the correction mass becomes a problem of minimizing the performance function \( J \) of Eq. (2).

The modal correction mass \( W_n \), obtained with the Least Squares Method satisfies the condition that Eq. (2) becomes zero when it is partially differentiated with respect to \( W_n \). Namely,

\[
\frac{\partial J}{\partial W_n} = \sum_{i=1}^{M} \sum_{m=1}^{I} \frac{\partial J}{\partial \varepsilon_m(1)} \frac{\partial \varepsilon_m(1)}{\partial W_n} \quad \frac{\partial J}{\partial W_n} = 0 \quad (3)
\]

The modal correction mass which reduces the initial vibration \( A_m(1) \) is obtained by solving the simultaneous Eq. (3).

3.3 Method of Correcting Influence Coefficients
The critical and rated speeds of a rotor are selected as the balancing speeds. When rotor speed is increased through the addition of modal correction masses to a rotor, vibration is not always reduced as much as the residual vibration obtained with Eq. (1) indicates it should be. Accuracies in the measurement of the initial vibration \( A_m(1) \) and in the measurement and calculation of the modal influence coefficient \( \omega_m(1) \) are regarded as the cause of the above problems. Errors in the influence coefficient \( \omega_m(1) \) and a method of correcting them are discussed in the following paragraph.

Vibrations at the first and second critical speeds of the \#1 and \#2 journals shown in Fig. 1(a) and (b) are obtained from Eq. (1) as follows:

\[
\varepsilon_m(1) = A_m(1) + \omega_m(1) W_1 + \omega_m(2) W_2 \quad \text{(first critical speed)} \quad (4)
\]

\[
\varepsilon_m(2) = A_m(2) + \omega_m(1) W_2 + \omega_m(2) W_1 \quad \text{(second critical speed)} \quad (5)
\]

where \( W_1 \) and \( W_2 \) are modal correction masses. If the residual vibration of the rated speed is \( \varepsilon_m(r) \) and the vibration mode is the second one, then

\[
\varepsilon_m(r) = A_m(r) + \omega_m(1) W_1 + \omega_m(2) W_2 \quad \text{(rated speed)} \quad (6)
\]

when \( m = 1, 2, \ldots, M \), and the suffix \( r \) denotes the rated speed.

Since the first vibration mode is dominant at the first critical speed, the modal influence coefficient \( \omega_m(1) \), which is obtained with the modal correction mass \( W_1 \), is approximately \( \omega_m(1) \approx 0 \). On the other hand, the modal influence coefficient \( \omega_m(2) \), which is obtained with the modal correction mass \( W_2 \), is approximately \( \omega_m(2) \approx 0 \), since the second vibration mode is dominant at the second critical speed. In this model rotor, the second and third vibration modes are excited together at the rated speed. However, the third vi-
Fig. 2 Balancing flow chart

Fig. 3 Unbalance distribution
Vibration mode is here regarded as negligibly small and the second vibration mode is considered to be dominant. Therefore, the modal influence coefficient $a_{11}^{(1)}$, which is obtained with the modal correction mass $W_1$, is approximately $a_{11}^{(1)} \approx 0$.

When the above conditions are introduced in Eqs. (4) to (6), the following equations are obtained for the residual vibrations $e_m^{(1)}$, $e_m^{(2)}$, and $e_m^{(r)}$:

$$e_m^{(1)} = A_m^{(1)} + a_{12}^{(1)} W_1$$
$$e_m^{(2)} = A_m^{(2)} + a_{22}^{(2)} W_2$$
$$e_m^{(r)} = A_m^{(r)} + a_m^{(r)} W_2$$

Since the modal correction masses $W_1$ and $W_2$ are nearly independent, the influence coefficients are calculated again, that is, the approximate modal influence coefficients $a_m^{(1)}$, $a_m^{(2)}$, and $a_m^{(r)}$ are obtained from the following equations:

$$a_m^{(1)} = \frac{a_{11}^{(1)}}{W_1}$$
$$a_m^{(2)} = \frac{a_{22}^{(2)}}{W_2}$$
$$a_m^{(r)} = \frac{a_m^{(r)}}{W_2}$$

When Eqs. (10) to (12) are used the modal influence coefficients for the critical speed can be corrected in only one running test.

As for balancing calculations after correction of the influence coefficients, modal influence coefficients can be obtained for the second time from Eq. (3) by substituting the corrected modal influence coefficients obtained with Eqs. (10) to (12) for the ones that contained errors.

The correction standard for modal influence coefficients is described in the

Fig. 4 Oil film characteristics at #1 bearing

Fig. 5 Balancing result at #1 journal using modal-least squares balancing method
following. First, the modal correction mass \( M_n \), which is used to reduce the initial vibration \( A_m(1) \), is determined by Eq. (3). The modal influence coefficients used in the calculations may be either calculated or measured values. The \( M_n \) that is obtained, is introduced into Eq. (4) and an evaluated the value for the residual vibration \( \varepsilon_m(1) \) is calculated. With \( M_n \) added to the modal correction plane of the rotor, rotor speed is increased and the residual vibrations \( \varepsilon_m(2) \) at the critical and rated speeds of the running rotor are determined. When there are no errors in the influence coefficients, the relation \( \varepsilon_m(1) = \varepsilon_m(1) \) is obtained and when there are errors, the relation \( \varepsilon_m(1) \neq \varepsilon_m(1) \) is obtained. The modal influence coefficients are therefore corrected when the following two conditions are realized:

(i) Residual vibration \( \varepsilon_m(1) \) exceeds the allowable value

(ii) Difference between residual vibrations becomes

\[
|\varepsilon_m(1) - \varepsilon_m(1)| > \delta
\]

In condition (ii), \( \delta \) is an arbitrary scalar quantity.

A flow chart for this balancing method is shown in Fig. 2.

4. Simulation of Balancing

4.1 Distribution of Unbalances

Figure 3 presents a mathematical model and a distribution of unbalances that can be used to examine the accuracy of the balancing calculations analyzed in Section 3 of this paper. The unbalances are distributed on the eleven disks of the rotor. The magnitude and angle of the unbalance added to the \#1 disk are 7.5 g and 0°. The magnitude and angle of the unbalance added to the \#2 disk are 0.5 g and 33° more than those of the unbalance added to the \#1 disk, and so on to the \#11 disk. This distribution of unbalances is not likely to simulate the distribution found in an actual machine, rather it is made more complicated than the actual distribution, in order to allow the effectiveness of this method of calculation to be evaluated.

The rated speed of the model rotor is assumed to be 3600 min⁻¹. The first critical speeds are 1050 min⁻¹ and 1150 min⁻¹ (horizontal and vertical, respectively), and the second critical speed is 3150 min⁻¹ (horizontal). Elliptical type oil film bearings are used to support the rotor. The calculated oil film constants for the \#1 bearing are shown in Fig. 4. A transfer matrix method⁶ is used to calculate the unbalanced vibrations.

4.2 Example of Numerical Calculations

The effectiveness of the modal-least squares balancing method is examined in numerical calculations in which the models described in section 4.1 are used. Calculations are carried out for the case in which there are no errors in the amplitudes of the modal influence coefficients and for the case in which artificial errors are added to the amplitude.

4.2.1 Modal and Least Squares Balancing Calculations

The following balancing conditions are set in order to reduce the unbalanced vibrations caused by the unbalance distribution shown in Fig. 3.

(i) Correction plane: A total of three planes including the \#6 disk and the

Fig.6 Responses at \#1 journal in which modal influence coefficients contain amplitude errors at 1st critical speed
(ii) Balancing speed: A total of four speeds, including the rated speed and three critical speeds.

(iii) Balancing position and direction: Vertical and horizontal directions at the #1 and #2 journals.

The influence coefficients used in the calculations should not include errors. The results obtained in calculations of the vibration response at the #1 journal, before and after balancing, are shown in Fig. 5. Excessive initial vibrations appear at the critical and rated speeds, but such vibrations are reduced after balancing. This verifies the effectiveness of the modal and least squares balancing methods, which are used to calculate the first and second modal correction masses in order to reduce the unbalanced vibrations caused by the unbalance distribution shown in Fig. 3.

4.2.2 Balancing calculations with errors in the amplitude of the influence coefficient

The unbalance distribution and balancing conditions used in the following calculations agree with those given in Fig. 3 and Section 4.2.1, respectively. The following relation holds between the modal influence coefficients $a_{mne}(i)$, which contain errors and are used in the first balancing trial and the correct modal influence coefficients $a_{mn}(i)$, which do not contain errors.

$$a_{mne}(i) = K \cdot a_{mn}(i)$$

(13)

Fig. 7 Correction masses in which modal influence coefficients contain amplitude errors at 1st critical speed

Fig. 8 Rms values in which modal influence coefficients contain amplitude errors at 1st critical speed
where $K$ is an error ratio factor of scalar quantity, and errors in the influence coefficient are limited to the amplitude. The conditions under which errors are introduced into the balancing calculations are set as follows:

(i) I-0.4: Add errors only to the amplitude at the first critical speed and put $K = 0.4$. Put $K = 1.0$ for the other amplitudes.

(ii) II-0.4: Add errors only to the amplitudes at the first and second critical speeds and put $K = 0.4$. Put $K = 1.0$ for the other amplitudes.

(iii) III-0.4: Add errors to the amplitudes at the first and second critical speeds and to the amplitude at the rated speed and put $K = 0.4$.

The effectiveness of balancing calculations and the correction of the errors produced in the partial and total amplitudes when modal influence coefficients are underestimated will now be examined.

(a) The case with condition (I): The results obtained in the balancing calculations for the #1 journal under the condition I-0.4 are shown in Fig. 6. In the first balancing trial, vibrations which exceed the initial vibrations are caused at the first critical speed and the balancing produced poor results. On the other hand, an adequate balancing was obtained at speeds other than the first critical speed. After correction of the first modal influence coefficients, a second balancing trial was carried out and adequate balancing effects were obtained at all the speeds, as shown in Fig. 6. The first and second modal correction masses were calculated when $K = 0.2 \sim 3.0$ under condition (I). The results of the calculations are shown in Fig. 7. The value at $K = 1$ represents a correction mass without errors. In the first balancing trial, only the modal correction mass $S_{00}(\omega_{0})$, which varies according to $K$, changes greatly. This is explained by the balancing effect, which is shown in Fig. 6. In the second balancing trial, the modal correction masses $S_{00}$ and $C_{00}$, when $K = 0.2 \sim 3.0$, are nearly equal to the same masses when $K = 1.0$. The root mean squared value (Rms) of the residual vibration that occurs when the modal correction mass shown in Fig. 7 is added to a rotor is shown in Fig. 6. In the first balancing trial, Rms is at a minimum when $K = 1$ and increases at $K$ increases or decreases. In the second balancing trial, the Rms that is obtained when $K = 0.2 \sim 3.0$ is nearly equal to the Rms that is obtained when $K = 1.0$.

The response curve, modal correction mass curves and Rms value curve in Figs. 6 to 8 prove that an adequate balancing can be obtained in a second balancing trial when the amplitude of the first critical speed of the modal influence coefficient contains errors.

(b) The case with condition (II): The results obtained in balancing calculations at the #1 journal under condition II-0.4 are shown in Fig. 9. In the first balancing trial, there is an inadequate reduction of vibrations that exceed the level of the initial vibrations at the first critical speed and of those that exceed the level of the initial vibrations at the second critical speed. After correction of the first and second modal influence coefficients, a second balancing trial was carried out. Adequate balancing is observed.

**Fig. 9** Responses at #1 journal in which modal influence coefficients contain amplitude errors at 1st and 2nd critical speeds
at all speeds after the second balancing trial, as shown in Fig. 9. The first and second modal correction masses are calculated when \( K = 0.2 \sim 3.0 \), under condition (ii). The results of the calculations are shown in Fig. 10. In the first balancing trial, the values for the modal correction masses \( S_{w} (\omega_{w}) \) and \( C_{w} (\omega_{w}) \) which vary according to \( K \), change greatly. On the other hand, in the second balancing trial, the values for the correction masses \( S_{w} \) and \( C_{w} \), when \( K = 0.2 \sim 3.0 \), are nearly equal to the same masses when \( K = 1.0 \). The Rms of the residual vibration is found by adding the modal correction masses \( S_{w} \) and \( C_{w} \) shown in Fig. 10 to the rotor. The results of the calculations are shown in Fig. 11. A minimum Rms is obtained in the first balancing trial when \( K = 1.0 \) under condition 1-0.4. In the second balancing trial, the Rms that is obtained when \( K = 0.2 \sim 3.0 \) is nearly equal to the Rms that is obtained when \( K = 1.0 \).

The response curves, modal correction mass curves and Rms value curves in Figs. 9 to 11 prove that an adequate balancing can be obtained in a second balancing trial when errors are included in the amplitudes of the first and second cri-

![Graph](image)

**Fig. 10** Correction masses in which modal influence coefficients contain amplitude errors at 1st and 2nd critical speeds

![Graph](image)

**Fig. 11** Rms values in which modal influence coefficients contain amplitude errors at 1st and 2nd critical speeds
Fig. 12 Responses at #1 journal in which modal influence coefficients contain amplitude errors at 1st and 2nd critical speeds and rated speed.

(c) The case with condition (iii): The results obtained in balancing calculations at the #1 journal under condition III-0.4 are shown in Fig. 12. In the case of the first balancing trial, vibrations that exceeded the level of the initial vibrations occurred at the first and second critical speeds and at the rated speed. After correction of the modal influence coefficients of the first and second critical speeds and of the modal influence coefficient of the rated speed, a second balancing trial was carried out. As a result of the second balancing trial, vibrations were reduced greatly and good balancing conditions were obtained at all speeds. The first and second modal correction masses are calculated when $K = 0.2 \sim 3.0$ under condition (iii). The results of the calculations are shown in Fig. 13. In the first balancing trial both of modal correction masses $S_w$ and $C_w$ which vary according to $K$, change.

Fig. 13 Correction masses in which modal influence coefficients contain amplitude errors at 1st and 2nd critical speeds and rated speed.
greatly. On the other hand, in the second balancing trial, the both correction masses $S_2$ and $C_2$, when $K = 0.2 \sim 3.0$, are nearly equal to the same masses when $K = 1.0$. The Rms of the residual vibrations is found by adding the modal correction masses $S_2$ and $C_2$ shown in Fig. 13 to the rotor. The results of the calculations are shown in Fig. 14. Under conditions I-0.5 and II-0.4, a minimum value for Rms is obtained in the first balancing trial when $K = 1.0$. In the second balancing trial, the Rms that is obtained when $K = 0.2 \sim 3.0$ is nearly equal to the Rms that is obtained when $K = 1.0$.

The response curves, modal correction mass curves and Rms value curves in Fig. 12 to 14 prove that an adequate balancing can be obtained in a second balancing trial when the amplitudes of the first and second critical speeds of the modal influence coefficients contain errors.

As mentioned above, the present method has been proved to be an effective balancing method when the amplitudes of the modal influence coefficients contain errors.

5. Conclusions

A model-least squares balancing method has been introduced to allow a balancing of flexible rotating shafts. Correction masses are determined by the least squares method using modal influence coefficients determined from a combination of plural masses. In order to allow a balancing in two trials when the modal influence coefficients contain errors, approximate equations have been introduced. The influence coefficients in the equations are corrected in accordance with data from the first balancing trial.

After examining the present method in a number of numerical simulations, the following conclusions were obtained:

1. When modal influence coefficients are correct, a correction masses that will effectively reduce vibrations can be obtained from the first trial calculations.

2. When modal influence coefficients contain errors, a satisfactory balancing can be obtained in a second balancing trial.

The results of applications of the present method when the influence coefficients contain phase angle errors will be reported later.

References