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Determination of Mathematical Model in Structural Analysis of Machine Tools*

— 1st Report, Model of Basic Structural Component
Using Rigidity Distribution Diagram —

by

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The study on the computer aided design system for machine tool structures has made a startling progress. The present computing methods cannot be applied to the absolute estimation of structural rigidity, only based on the information from engineering drawings, because there are some problems in the determination of the mathematical model. In future, it is, therefore, necessary to estimate absolutely and quantitatively the rigidity of a machine tool as a whole, using the information from engineering drawings. There are, however, two difficult aspects to the problems; one is a reasonable procedure for the determination of the mathematical model to analyse the characteristics of the structural components, and the other is a procedure concerning how to incorporate the joint characteristics into the mathematical model.

In this paper, therefore, a new reasonable procedure to determine the mathematical model using the rigidity distribution diagram for the basic structural component was proposed, and as an example, the static stiffness of the column of a floor type horizontal boring machine was analysed and the propriety of the procedure was certified.

1. Introduction

The study on the computer aided design for machine tool structures has made a startling progress. In the present stage, it may be said that the static, dynamic and thermal characteristics of machine tools can be calculated by means of computers to some extent.⁽¹⁾ From the reports on research works or on practical applications up to the present it may be said that the present computing methods cannot be applied to the absolute and quantitative estimation of the characteristics, only based on the information from engineering drawings, but can be applied only to the relative estimation such as the improvement of rigidity by the modification of the structures. There are, for example, some typical reports; Kark et al.⁽²⁾ investigated the effect of ribs on the vibration characteristics of columns, and Weck et al.⁽³⁾ investigated the static characteristics of various types of beds to attain a complete design data.

At present, although it is required as the next step to estimate absolutely and

quantitatively the rigidity of a machine tool, there are two difficult aspects to this problem; one is a rational procedure for the determination of the mathematical model to analyse the characteristics of the structural components, and the other is a procedure concerning how to incorporate the joint characteristics into a mathematical model.

The mathematical model has drawn much attention; beam and lumped mass models have mainly been used for computation, and it has been stated by the joint work of CIRP⁽⁴⁾ that the different mathematical models give different results. It is, moreover, pointed out that there is the same problem in the computing method using a finite element model, which was expected to be able to solve above problems, because of the capacity of a computer and the exactness of a finite element model. The mathematical model is determined from the structural configuration or from the currently available program, but the model is finally decided by the experience and sense of designers at present stage.

By the way, it will be necessary to rationally the determination method or methodology to make the analysis of structural rigidity by means of computer. With regard to the problem, only Hijink et al.⁽⁵⁾ has made primary investigations. In this paper, a procedure to determine the mathematical model using the rigidity distribution diagram for the basic structural component is proposed, and the propriety of the determination method of mathematical model is confirmed using the column of a floor type horizontal boring machine as a concrete model. For convenience, in this paper, the determination of the finite element model to compute the static stiffness only is

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discussed.

2. Coception for determination of mathematical model

The configuration of structural components is usually complex to attain the desired purpose in machine tools. For example, in the column of a horizontal boring machine, the thin-walled box-beam, which is a basic unribbed structure, is not only reinforced by various ribs, partitions and others, but also has core holes and cutouts to make easy the assembly, overhaul or fabrication. Many mathematical models would, therefore, image for the same structure if the determination were left to the experience and sense of a designer as in the past. Consequently, the computed result needs to be compared with the measured one to confirm the propriety of any one of the mathematical models. This is an obstacle to establishing the analysis of the structural rigidity by means of computer and it needs to be removed. When the designer transforms the actual machine tool structure into a mathematical model, it is desirable to use the design data in addition to the experience and the sense in establishing the rational design procedure.

Therefore, in this paper, a method of determining the mathematical model using the rigidity distribution diagram as the design data is proposed. As shown in Fig.1, if the designer positively uses the rigidity distribution diagram, in designing process the mathematical model can be made rational. The rigidity distribution diagram which plays an important role is a kind of the deformation diagram modified so as to suit this study. An example of the diagram is shown in Fig.2.

Fig.1 Role of rigidity distribution diagram

Fig.2 Conception of rigidity distribution diagram

The actual structure is divided into a basic unribbed structure and other reinforcing elements. The stiffness of the basic unribbed structure is computed and next the stiffness of the structure reinforced by each element is computed. The rigidity distribution can be obtained by arranging these computed results on a diagram. To put it another way, the diagram shows an increment of stiffness due to reinforcing elements.

The mathematical model is determined according to the following procedures. As a first step, the basic structure is decided from the engineering drawing of an actual machine tool. And in the second step, referring to the rigidity distribution diagram which may not play an important part from the viewpoint of rigidity are eliminated. And finally, the modified structure is replaced with the mathematical model.

3. Preparation of rigidity distribution diagram

Consideration has been made in this proposed method such that the experience and sense of the designer do not play any significant role. The rigidity distribution diagram which is a design data to determine the mathematical model can easily be prepared by the experiment or the theoretical calculation. In this paper, the finite element method was used to prepare the diagram.

The basic unribbed structure and its finite element model used in this paper are shown in Fig.3. This is a scale model of a horizontal boring machine. And the column is reinforced by the longitudinal and the cross ribs, the partitions and the end plate. The stiffness and the rigidity distribution diagram used here can be explained as follows.

(a) Basic structure (b) Mathematical model

Fig.3 Basic structure and its mathematical model

As shown in Fig.4, the deformation of the column of the horizontal boring machine is complex, as the cutting forces and the dead weight of the head stock are acting upon the column in all direction. The basic forces are, however, bending and torsional ones. Then the bending stiffness, K_b , and the torsional stiffness, K_t , are defined as follows;

$$K_b = \frac{P_1}{\delta} \dots \dots \dots (1)$$

$$K_t = \frac{T}{\phi} = \frac{P_2 a}{\phi} \dots \dots \dots (2)$$

If these equations are considered to be independent of the applied force, the basic unribbed structure and the reinforcing elements can be represented by the sum of linear springs as follows;

$$K = K_0 + \sum_i K_i \dots \dots \dots (3)$$

where, K : stiffness of column with reinforcing elements

K_0 : stiffness of basic unribbed structure

K_i : stiffness of i th reinforcing element

Then, Eq.(4) can be obtained from Eq.(3).

$$\frac{K}{K_0} = 1 + \frac{\sum K_i}{K_0} = 1 + \frac{\Delta K}{K_0} \quad (4)$$

In Eq.(4), the second term of right hand side denotes the increment of the stiffness, so the ratio of K_i/K_0 to $\Delta K/K_0$ is defined as "stiffness share rate of i th reinforcing element", and the histogram obtained for all these rates is called "rigidity distribution diagram". Then, some examples of the diagrams are shown in the following sections.

Fig.4 Definition of applied force and stiffness

3.1 Effect of cross and longitudinal ribs

Cross and longitudinal ribs of cross-section $10 \times 10 \text{ mm}^2$ were replaced by beam elements, and were attached to the basic finite element model shown in Fig.3(b), and rigidity distribution diagrams presenting the effects of these ribs were made as shown in Figs.5 and 6. In these diagrams, both bending and torsional forces are applied to the position corresponding to the maximum height of the head stock. In the position, the deformation of the column is maximum. The values in the figures stand for 100% of $\Delta K/K_0$, and the values in parentheses are for 100% of K_0 . The necessary and unnecessary ribs can be easily distinguished in a rigidity distribution diagram. For example, in a column subjected to bending and torsional forces, the cross rib between the loading point and the fixed side of the column can be disregarded.

(a) Bending

(b) Torsion

Fig.5 Rigidity distribution diagram for cross ribs

3.2 Effect of partitions

Next, the effect of partitions with core holes as shown in Fig.7 was investigated. It is found that the partition shows the same characteristics as a cross rib, because a partition with a large cutout is close to a taller cross rib. However, in this section a partition was replaced by a rectangular element when the rigidity distribution diagram was prepared.

On the other hand, a column with longitudinal partitions can often be encountered and these partitions can not be disregarded when the mathematical model is determined; however, holes and cutouts of the partitions shown in Fig.8 create problems. Then the effect was investigated as shown in Fig.9. The rigidity distribution diagram can not be applied to such examples, but Fig.9 is one of the design data which is useful for the determination of the mathematical model. From the

figure it can be seen that the longitudinal partition with cutouts such as Fig.8(b) may be regarded as a plate with no cutouts.

As shown in some examples, the rigidity distribution diagrams are useful data for the determination of the mathematical model. And if the effects of reinforcing elements can not be expressed on the rigidity distribution diagram, the diagram such as Fig.9 becomes more useful. Moreover, the diagram which was arranged to make above results more useful is shown in Fig.10. Because the most important reinforcing element can be detected at the first sight in the figure.

Here, it can be said that for determining the mathematical model the longitudinal rib is more important under bending force, and the partition is important under torsional one. So, it becomes necessary to change the model according to the type of the acting force; thus, it is difficult to determine a generalized mathematical model. However, if

Fig.6 Rigidity distribution diagram for longitudinal ribs

Fig.7 Rigidity distribution diagram for partitions

the rigidity distribution diagram is prepared as shown in this chapter, it may be said that the problem mentioned above can easily be solved.

4. Application to actual machine tool structure

The fundamentals for the determination of the mathematical model using the rigidity distribution diagram were explained in previous chapters. Then, to confirm its propriety the proposed method was applied to the column of an actual machine tool structure.

4.1 Investigated structure and its mathematical model

The actual structure investigated here is a column of a floor type horizontal boring machine of 130mm spindle diameter. The details of the structure are shown in Fig.11. As shown in this figure, the column is divided by a longitudinal partition with eight core holes, and four longitudinal ribs and 15 cross ribs

Fig.8 Configuration of longitudinal partition

are set throughout the column length. AA and BB cross-sections shown in the figure are arranged alternately. The width and thickness of longitudinal and cross ribs are the same as wall thickness in the longitudinal rib ① shown in Fig.11.

The mathematical model was determined to compute the deformation of the column subjected to static forces, and four models were prepared to confirm the effectiveness of the method proposed here. The bending and torsional forces of 1000 kgf (9.8 kN) were applied to the points L_1 and L_2 .

[Model 1] The considered model is almost similar to the column shown in Fig.11, but the fillet between the rib and the wall, and the minute details of the structure are not considered. Cross ribs are numbered from 1 to 15. The odd-numbered and the even-numbered

Fig.9 Effect of longitudinal partition

Fig.10 Rigidity distribution diagram for each reinforcing element

bered are AA and BB cross-section respectively. The taller longitudinal and cross ribs in the side of slideway, the longitudinal partition and the basic unribbed structure were taken to be rectangular elements, and other ribs and guideway were taken to beam elements. Thus, the column was divided as shown in Figs. 12 and 13.

[Model 2] In this model, the cross ribs from 8 to 15 and the longitudinal rib ① were

considered, and other ribs were not considered

[Model 3] In this model, the cross ribs from 8 to 13 and the longitudinal rib ① were considered.

[Model 4] In this model, the cross ribs from 6 to 13 and the longitudinal rib ① were considered.

Fig.11 The column of a floor type horizontal boring machine

Fig.12 Finite element model of column shown in Fig.11

Table 1 Comparison of bending deflection between Model 1 and others at loading point

Model	L ₁				L ₂			
	X		Y		X		Y	
	Deflection μm	Ratio	Deflection μm	Ratio	Deflection μm	Ratio	Deflection μm	Ratio
1	-34.50	1	-4.972	1	-40.59	1	-4.868	1
2	-35.00	1.014	-5.127	1.031	-41.36	1.020	-5.137	1.055
3	-34.99	1.014	-5.130	1.032	-41.38	1.020	-5.319	1.056
4	-34.91	1.012	-5.093	1.024	-41.14	1.014	-5.033	1.034

Table 2 Comparison of torsional deflection between Model 1 and others at loading point

Model	L ₁				L ₂				Torsional angle	
	X		Y		X		Y			
	Deflec- tion μm	Ratio	Deflec- tion μm	Ratio	Deflec- tion μm	Ratio	Deflec- tion μm	Ratio	θ mrad	Ratio
1	-19.02	1	3.329	1	25.11	1	3.317	1	6.442	1
2	-19.48	1.024	3.804	1.143	25.84	1.029	3.810	1.149	6.616	1.027
3	-19.48	1.029	3.891	1.169	25.98	1.035	3.897	1.175	6.651	1.032
4	-19.20	1.010	3.474	1.044	25.43	1.013	3.473	1.047	6.515	1.010

Fig.13 Division of cross-section

4.2 Computed results and discussion

Four types of mathematical model were computed by means of the finite element method. The results are summarized in Tables 1 and 2, where the ratios of Model 1 to other models are given in each direction. It can be seen from the tables that the bending deflection is not different, but the torsional deflection is different for each other, and Model 4 gives good computed results in both bending and torsional forces. And it can be said that the consideration of taller longitudinal ribs and the cross ribs near the loading points gives enough accuracy to the computed results. From above results it may be concluded that the Model 4 is more rational than Models 2 and 3 when the rigidity distribution diagram is referred to, and the static stiffness of the column can be calculated with sufficient accuracy even if all of the reinforcing elements are not considered. Therefore, the procedure for the determination of

mathematical model using the rigidity distribution diagram proposed here is one of the useful method to proceed with structural design

5. Conclusions

In this study, a procedure to determine the mathematical model for presently available structures has been investigated, and the following conclusions are drawn.

A method using the rigidity distribution diagram is proposed here to determine rationally the mathematical models for the actual structure. And the static stiffness of the column of an actual horizontal boring machine was computed by using the proposed method. It was confirmed from the results that the method is proper one.

In this paper, since the presentation of the methodology was main object, only one example was investigated. It may be said that the method proposed here can be developed further by investigating many examples by it.

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