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Improvement of the Method of Measuring In-plane, Normal Strain Using Interferometry of Diffraction Beams*

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Methods of measurement using interference patterns of diffraction beams are effective in measurement of in-plane strains of plane surface of objects. This paper deals with a new method of measuring the first order derivatives of displacement. In this method an interferometer with a triangular, looped path is used and two beams producing an interference pattern pass the same, looped path clockwise and counter-clockwise respectively. Merits of this method compared with usual methods are as follows: (i) errors in interference fringes caused by unevenness of surface of mirrors, distortion of wavefront due to aberration of a collimator lens and non-uniformity of thickness of film (or specimen gratings) are compensated sufficiently for the practical purpose, (ii) diffraction beams of high order can be used and high sensitivity in measurement is achieved, (iii) the optical arrangement is simple and adjustment is easy, and (iv) the interference pattern is stable against shock or disturbance.

1. Introduction

Moiré methods by use of interferometry of diffraction beams at specimen gratings are classified into two methods: one is the multiplication method which is characterized by high sensitivity in measurement and the other is the shifting method which is effective for the measurement of derivatives of displacement(1). As for the latter, the authors reported three modified methods(2). In the first method an interferometer with rectangular, looped path is used, in which adjustment of the optical apparatus is rather complicated, though errors due to non-uniformity of thickness of film (specimen gratings), unevenness of surface of mirrors, etc. are compensated completely. In the second method using Sciammarella's optical system (1967), compensation of errors is not complete. In the third method double exposures are adopted, but two step processing, i.e. photographing and reconstruction, is rather troublesome. So, a further improvement of the method is needed, in which errors are compensated and processing of measurement is simplified.

In this paper a new method is proposed. It can satisfy the above requirements. Contours of the first order derivatives of displacement, or strain, are obtained as interference fringes in this method.

The principle will be explained at first, then characteristics, i.e. completeness of compensation of errors,

sensitivity in measurement, etc. will be discussed.

2. Interferometer with tri-angular, looped path

A new optical system shown in Fig.1 will be proposed. It is composed of an interferometer with a triangular, looped path $M_1M_2M_3M_1$ and a system for separation of diffraction beams (lenses L_1 , L_2 and an aperture in Fig.1). The grating on an object is photographed on film or plate and it is used as a specimen gratings.

Let us project a mono-chromatic, collimated beam on a specimen grating and pay attention to a diffraction beam of order m . The diffraction beam is split into two beams by the mirror M_1 : one beam passing the looped path $M_1M_2M_3M_1$ clockwise and the other counter-clockwise. They are assembled again by M_1 . When two beams are shifted laterally from each other, interference fringes are produced.

Shifting between two beams is given in the following way. First, the mirrors M_1 , M_2 , M_3 are so arranged that the diffraction

Fig.1 Interferometer with tri-angular, looped path

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beam of the m -th order may pass along the dot-and-dash line in Fig.2(a), where no shifting is given between two beams. The mirror M_3 is translated by a small amount $\Delta s'$. Then a shifting occurs. The amount of shift between two beams Δs becomes

$$\Delta s = 2\Delta s' \sin \theta \quad (1)$$

$$\theta = \angle M_1 M_3 M_2$$

When $\theta = 45^\circ$

$$\Delta s = \sqrt{2} \Delta s'$$

Each beam produces an image and the amount of shift between two images is the same as that in Eq.(1), where lenses of equal focal length are used for the sake of simplicity. The interference pattern obtained here corresponds to the contour of the gradient of a displacement component, since distortion of the wavefront of each beam corresponds to a displacement component, normal to grating lines. The difference of the gradient between neighboring contour lines, say a step of contour lines, is equal to $p/(m\Delta s)$ (p : pitch of a grating) (refer to Appendix 1).

Now characteristics of the interferometer will be examined. Let the mirror M_3 be rotated by a small amount. As is shown in Fig.2(b), directions of two beams remain parallel in spite of this disturbance, though directions of each beam varies. Therefore, an interference pattern is not affected by the rotation of the mirror M_3 . Here, as for the shift due to the rotation of the mirror M_3 , variation of it is negligible, if the rotation is not so great. It can be shown in a similar way that rotation of mirrors M_1 and M_2 gives also only a little influence on an interference pattern.

These results suggest useful characteristics of the optical system. First, an interference pattern does not vary even if the mirror M_3 is rotated by a small amount during its translation. Second, the interference pattern is not affected by shock or disturbance. Third, unevenness of surface of mirrors causes no error to the interference pattern, since each portion of mirrors can be considered to be displaced from precise planes. Rigorously, finite difference of unevenness of mirrors relating to the amount of shift causes an error on the interference pattern, but it is usually very small. Details will be discussed in Appendix 2.

The optical arrangement is adjusted in the following way. Mirrors M_1 , M_2 , M_3 are so placed that optical paths of two beams, passing the interferometer clockwise and counter-clockwise respectively, may coincide with each other. An aperture is set at the focussed point of the diffraction beam of the 0-th order. Then the mirror M_0 in Fig.1 is rotated and the diffraction beam to be used is made to pass the aperture mentioned above. Precise adjustment becomes possible in this way even when a diffraction beam is so faint and can hardly be observed. Further, diffraction order can be changed easily.

3. Applications

Contours of various gradients of displacement components can be obtained by using one-way gratings, which are composed of vertical or horizontal lines. An application to measurement of deformation of uniform bending of a rectangular bar will be shown at first. Cartesian coordinates x, y are set as in Fig.3(a). x - and y -way gratings are printed on front and back surfaces of a specimen respectively. Density of a specimen grating is

(a) Translation

(b) Rotation

Fig.2 Effect of motion of mirror

(a) Coordinates

(b) $m=1$

(c) $m=2$

(d) $m=3$

(e) $m=4$

(f) $m=5$

Fig.3 ϵ_x in uniform bending
(Specimen grating: 500 lpi,
 $\Delta x = 1.4$ mm, $\alpha = 1$, $h = 20$ mm,
Al, Rectangular bar)

500 lpi (lines/in) and magnification factor in photographing gratings $\alpha = 1$, so density of a specimen grating is 500 lpi. Fax Nikkor ($F=5.6$, $f=210$ mm, Nippon Kogaku Co.) is used in photographing gratings and KPR (Eastman Kodak Co.) is used in printing gratings.

The purpose of this paper is to propose a method using a new interferometer. So only displacement gradients, such as $\partial u/\partial x$, etc. are obtained, though magnitude of strain is not infinitesimal. Finite strains can be obtained numerically from displacement gradients(4).

The contours of $\epsilon_x (= \partial u/\partial x)$ obtained by using a y -way grating and by shifting in x direction are shown in Figs.3(b)~(f). Diffraction orders m are 1, 2, ... 5. Normal strain ϵ_x distributes uniformly in x direction and varies monotonously in y direction, as is known from solid mechanics. It is also found that density of fringes is proportional to the diffraction order m . Undulation of fringes is caused by non-uniformity of thickness of film. It can be neglected when a comparatively small amount of shift is used (refer to Fig.11).

Other interference patterns corresponding to gradients of displacement, $\partial u/\partial y$, $\partial v/\partial x$ and $\epsilon_y (= \partial v/\partial y)$ are shown in Figs.4(a)~(c). Amounts of shift Δx and Δy are 1.4 mm and differences of strains at neighboring contour lines $[\Delta\epsilon]$ are 0.018 and 0.009 for diffraction order $m=2$ and 4, respectively. Shear strain γ_{xy} is found to vanish if we notice that sign of $\partial u/\partial y$ is minus in Figs.4(a) and (b).

Next application is about simple tension of a perforated sheet. Density of lines of a grating on an object is 200 lpi and magnification factor in photographing $\alpha = 1/2.5$, so density of a specimen grating is 500 lpi. One-way gratings, whose lines are parallel and orthogonal to tensile direction respectively, are used. They are prepared by using gratings on front and back surfaces of an object as in the previous example. Diffraction order that can be used depends on the contrast of a grating printed on an object, so surface of an object is finished deliberately by buffing and a uniform, dull surface is obtained.

Interference patterns of ϵ_x are shown in Figs.5(a)~(d), where diffraction orders $m=6\sim 9$. It is found from these results that interference patterns of diffraction order $m=7$ have sufficient contrast and that they can be used practically for measurement of strain. An amount of shift $\Delta s = 0.7$ mm and difference of gradient of displacement at neighboring contour lines $[\Delta\epsilon] = 0.012\sim 0.008$ for $m=6\sim 9$. Gradients of displacement, $\partial u/\partial y$, $\partial v/\partial x$ and $\epsilon_y (= \partial v/\partial y)$ are shown in Fig.6. Conditions in photographing are the same as those in Fig.5.

As is known from examples of Fig.5(b) and Figs.6(a)~(c), the range of magnitudes of strain that can be measured by this method is about 0.05~0.1. It varies to some extent

Fig.4 Gradients of displacement in uniform bending ($\Delta x = \Delta y = 1.4$ mm)

Fig.5 ϵ_x in simple tension of perforated sheet (Specimen grating: 500 lpi(200 lpi on object), $\Delta x = 0.7$ mm, $\alpha = 1/2.5$, Width $b = 32$ mm, Thickness $t = 0.5$ mm, Al-O sheet)

Fig.6 Gradients of displacement in simple tension of perforated sheet ($m=7$, $\Delta x=\Delta y=0.7$ mm)

Fig.7 Fundamental interferometer for shifting

Fig.8 Distortion of wavefront after passing slightly convex glass plate (placed at B in Fig.7)

depending on magnitude of strain gradient, because appropriate amount of shift changes (5).

In this method only the first order derivatives of displacement can be obtained directly. So some further processing is needed in order to obtain shear strain γ_{xy} . Drawing curves passing points of intersection of two families of contour lines of $\partial u/\partial y$ and $\partial v/\partial x$ is one way (5).

4. Discussions about compensation of errors

There are three main causes which give errors to interference fringes, that is, unevenness of wavefront of the beam projected on a specimen grating, unevenness of surface of mirrors constituting an interferometer and non-uniformity of thickness of film.

The first and the second causes will be examined experimentally at first. Let us represent the wavefront distorted by the above causes by one passing a slightly convex glass plate. Arbitrary amount of distortion of the wavefront can be given easily in this way. Distortion by the glass plate is expressed by interference fringes shown in Fig.8, which is obtained by placing the plate at B in Fig.7. Let us place the glass plate at left-hand side of the mirror M_0 in Fig.1, while the specimen grating is removed, and let us translate the mirror M_3 by a small amount, which causes some

Fig.9 Fringes by shifting method (Fig.1 is used)

(a) Error of interferometer in Fig.7
(b) Non-uniformity of thickness of film

Fig.10 Fringes due to non-uniformity of thickness of film

shift between two wavefronts, decomposed by the mirror M_1 . Then, distortion of the wavefront which can be regarded as that due to aberration of a collimator lens or that due to unevenness of mirrors can be estimated.

Interference patterns are shown in Figs.9(a)~(c). Fringes do not arise until the amount of shift Δs becomes comparatively large. In usual cases, distortion of wavefront due to a collimator lens and mirrors is small and the amount of shift is also small in comparison with that in Figs.9(b) and (c), so errors in interference patterns can be neglected for the practical purpose (more rigorous discussions are made in Appendix 2).

Now an error due to non-uniformity of thickness of film will be examined. The optical system of Fig.7 is used. Its error due to mirrors is expressed by fringes in Fig.10(a), in which mis-match is given. Let a film be placed at B in Fig.7, then fringes vary as is shown in Fig.10(b). Difference between Figs.10(a) and (b) corresponds to non-uniformity of thickness of film.

In the method using the optical system of Fig.1, finite difference of thickness of film relating to the amount of shift gives an error to an interference pattern. This will be examined about the film mentioned in Fig.10(b). Let the film be placed at A in Fig.7, while shifting is given by translating the mirror M_1 . Then interference fringes relating to various amounts of shift are obtained. They are shown in Figs.11(a)~(d). In Fig.11(a) $\Delta s = 0$, and it coincides with Fig.10(a), which expresses the error of the optical system of Fig.7. Difference of interference fringes in Figs.11(b)~(d) from those in Fig.11(a) corresponds to the finite difference of thickness of film. In the method by use of Fig.1, this error remains. However, its influence on interference fringes is less than 0.2 fringe when Δs is as much as 1 mm, so it may be neglected in usual cases.

Finally it is concluded that unevenness of wavefront of the beam projected on the specimen grating, unevenness of mirrors constituting the interferometer and non-uniformity of thickness of film give only negligible errors to an interference pattern. Aberration of a collimator lens, however, should be inspected if it is not known to be small. Further, film of uniform thickness should be used if a large amount of shift is needed.

5. Comparison with related interferometries

The method of measurement mentioned above will be compared with the conventional, related methods and their characteristics will be discussed.

The fundamental method shown in Fig.7 is treated at first. A specimen grating is placed at A and shifting is given by translating the mirror M_1 . Interference patterns using the same specimen grating as one in Fig.5 are shown in Figs.12(a) and (b). Diffraction orders are 6 and 8, respectively. Contrast of fringes in Fig.5 and

Fig.12 is nearly equal, so sensitivity in measurement, or diffraction order that can be used, becomes equal in both methods. This is comprehensible from the fact that beams producing an interference pattern pass a specimen grating once in both methods.

Now let us take up the method using an interferometer with a square, looped path, which enables the most complete compensation of errors among the three methods mentioned in the previous paper(2). Interference patterns are shown in Figs.13(a) and (b). Diffraction orders $m=5$ and 6, and

Fig.11 Fringes corresponding to finite difference of thickness of film relating to various amounts of shift

Fig.12 ϵ_x in perforated sheet by fundamental interferometer in Fig.7 ($\Delta x = 0.7$ mm)

Fig.13 ϵ_x in perforated sheet by interferometer with square, looped path(2) ($\Delta x = 0.7$ mm)

other conditions are the same as those in Fig.5. Comparison of the methods used in photographing Fig.5 and Fig.13 leads to the conclusion that the diffraction order that can be used in the former method is higher than that in the latter method by about 2. The reason is that beams producing an interference pattern pass a specimen grating once in the former method, whereas it does twice in the latter method and noise is increased.

Moreover, the former method has the following merits: the interference pattern is intense and the optical apparatus is simple. Compensation of errors is sufficiently complete for the practical purpose, although it is not so rigorously complete as that in the latter method.

Comparison with the other two methods in the previous paper will be omitted, because their characteristics seem to be clear from the reference (2).

6. Conclusions

A method of measuring the first order derivatives of displacement is proposed. In this method an interferometer with a triangular, looped path is used, which possesses various characteristics. Some experiments show that this method is excellent in the following respects: practically sufficient compensation of errors, high sensitivity and simple procedure in measurement.

Appendix 1. Expression of interference fringes

Let Cartesian coordinates x and y be set in the leaf and normal to the leaf in Fig.1, respectively. A specimen grating is composed of lines parallel to y -axis. Its amplitude transmittance is expressed by

$$T = \sum_{m=-\infty}^{\infty} b_m \exp \left[i \frac{2\pi m}{p} (x - u(x, y)) \right] \quad (2)$$

u, v : x -, y - components of displacement

p : pitch of a specimen grating

When plane wave a is projected on the specimen grating, diffraction wave of m -th order is

$$ab_m \exp \left[i \frac{2\pi m}{p} (x - u(x, y)) \right] \quad (3)$$

Decomposing it into two beams and composing them again after shifting by Δx , we obtain interference fringes as follows:

$$I = 2a^2 b_m^2 \left[1 + \cos \left\{ 2\pi \frac{m \Delta x}{p} \frac{\partial u(x, y)}{\partial x} \right\} \right] \quad (4)$$

Consequently, interference fringes correspond to contour lines of $\partial u / \partial x$ and their difference at neighboring lines $\{\Delta \epsilon\}$ becomes $p / (m \Delta x)$.

Similar expressions can be obtained for other derivatives of displacement.

Appendix 2. Discussion about errors

Cartesian coordinates is shown by two dots-and-dash lines in various locations in Fig.14 will be used (y : normal to the leaf). Let the wavefront of the diffraction beam of m -th order after passing a specimen grating be designated by $f(x)$ and deviation

Fig.14 Distortion of wavefront due to unevenness of mirrors

of actual surfaces of mirrors M_1 , M_2 and M_3 from ideal planes be designated by $\delta_1(x)$, $\delta_2(x)$, $\delta_3(x)$ where x_1 , x_2 , x_3 are coordinates in surfaces of mirrors. f corresponds to displacement. Coordinate y , normal to the leaf in Fig.14, is omitted in δ_1 , δ_2 , δ_3 and f , because it is evident that variation of these variables along y -axis does not affect the resulting interference fringes.

The situation that thickness of the mirror M_1 is zero will be considered at first. Wavefront of a diffraction beam $f(x)$ is decomposed into two beams, which pass the triangular looped path clockwise and counter-clockwise respectively. Let us give attention to the former beam. It is shifted by the mirror M_3 and the error due to M_3 is added. Wavefront after reflecting at M_3 is

$$f\left(x - \frac{\Delta x}{2}\right) + \delta_3\left(x - \frac{\Delta x}{2}\right)$$

where

$$\delta_3(x) = \delta_3\left(\frac{x}{\cos 22.5^\circ}\right) \frac{1 + \cos 45^\circ}{\cos 22.5^\circ}$$

$$x_3 = \frac{x}{\cos 22.5^\circ}$$

The amount of shift $\Delta x = \sqrt{2} \Delta s'$ in which $\Delta s'$ denotes the amount of translation of the mirror M_3 to the direction $\vec{M}_3 \vec{M}_1$. This is reflected at M_2 , then wavefront becomes

$$f\left(x - \frac{\Delta x}{2}\right) + \delta_3\left(x - \frac{\Delta x}{2}\right) + \delta_2(x) \quad (5)$$

where

$$\delta_2(x) = \delta_2\left(\frac{x}{\cos 22.5^\circ}\right) \frac{1 + \cos 45^\circ}{\cos 22.5^\circ}$$

$$x_2 = \frac{x}{\cos 22.5^\circ}$$

On the other hand the latter beam passing the looped path counter-clockwise is, after reflection at M_1 and M_2

$$f(x) + \delta_1(x) + \delta_2(x)$$

where

$$\delta_1(x) = \delta_1\left(\frac{x}{\cos 45^\circ}\right)$$

$$x_1 = \frac{x}{\cos 45^\circ}$$

It is shifted by M_3 and is reflected at the back surface of the mirror M_1 . It becomes

$$f\left(x + \frac{\Delta x}{2}\right) + \delta_1\left(x + \frac{\Delta x}{2}\right) + \delta_2\left(x + \frac{\Delta x}{2}\right) + \delta_3(x) - \delta_1(x) \quad (6)$$

Difference of two beams expressed by Eqs.(5) and (6) becomes

$$\frac{df}{dx} \Delta x + \frac{d\delta_1}{dx} \frac{\Delta x}{2} + \frac{d\delta_2}{dx} \frac{\Delta x}{2} + \frac{d\delta_3}{dx} \frac{\Delta x}{2} \quad (7)$$

Consequently, interference fringes correspond to contour lines of gradient of displacement, or distortion of wavefront f , and they contain errors due to finite difference of unevenness of mirrors relating to an amount of shift. However, they are negligible in usual cases because gradient of surface of mirrors is very small.

The same consequence is also obtained in the situation that thickness of the mirror M_1 is not zero or that a prism is used instead of the mirror M_1 . This is understandable from the fact that paths of two beams passing the interferometer clockwise and counter-clockwise coincide with

each other, when $\Delta s = 0$.

As for non-uniformity of thickness of film, finite difference of thickness relating to an amount of shift affects interference fringes.

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