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Hydroelastic Characteristics of Flexible Supercavitating Hydrofoils

Report 1. Two-dimensional Theory at Zero Cavitation

Number when Clamped at Trailing Edge*

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Since a typical supercavitating hydrofoil has a sharp leading edge, its chordwise bending deformation and oscillation become a serious problem. Expressions are presented for the pressure distribution, lift and drag of a thin hydrofoil performing chordwise bending oscillation when clamped at the trailing edge at zero cavitation number. By using these expressions, chordwise bending flutter, chordwise bending divergence, strength near the leading edge and cavitation inception on the pressure side are investigated numerically. From the results have been deduced the following;

- (1) The critical speed of cavitation inception on the pressure side is lower than the critical speed of chordwise bending divergence.
- (2) To give a slight thickness near the leading edge is very effective for increasing the critical speed of chordwise bending flutter and cavitation inception on the pressure side or decreasing the stress due to chordwise bending.

1. Introduction

With an increasing speed of hydrofoil craft, of marine propeller, and of pumps for liquid rocket propulsion, many supercavitating foil sections with low drag-lift ratio have been proposed^(1,2). Since typical supercavitating foils have a sharp leading edge and the hydrodynamic force on most of them is concentrated near the leading edge, the inclusion of chordwise bending deformation in analysis might be required. The stress near the leading edge due to the chordwise bending deformation was studied by Grumman Corp.⁽³⁾. Recently Oey⁽⁴⁾ showed the existence of chordwise bending flutter experimentally. But there has been made no theoretical research on the hydroelastic characteristics of chordwise flexible supercavitating hydrofoil, as far as the authors are aware.

In the present paper, a linearized theory for analysing the two-dimensional flow around, and the deformation of a chordwise flexible supercavitation foil when clamped at the trailing edge at zero cavitation number is developed and expressions are presented for the chordwise deformation and the stress distribution along the foil, the pressure distribution on the foil, and steady and unsteady forces acting on the foil. By using these expressions, chordwise bending flutter, chordwise bending divergence, strength near the leading edge, and cavitation inception on the pressure side

are investigated numerically.

2. Nomenclature

D	; flexural rigidity of foil
D_0	; flexural rigidity of foil at the trailing edge
E	; Young's modulus
F	; complex acceleration potential
P	; pressure
P_I	; pressure due to the initial foil shape
$P_{\bar{w}}$	; pressure due to the steady foil deformation
$P_{\tilde{w}}$	; pressure due to the unsteady foil deformation
T	; foil thickness
T_0	; foil thickness at the trailing edge
V	; uniform fluid velocity in X-direction at $X=-\infty$
c	; chord length
t	; time
w	; chordwise bending deformation of foil
Λ	; non-dimensionalized flexural rigidity, $(D_0/\rho V^2 c^3)$
α	; angle of attack
δ	; thickness-chord ratio, (T_0/c)
κ	; reduced frequency, $(\omega c/V)$
λ	; density ratio, $(\gamma T_0/\rho c)$
ν	; Poisson's ratio
ρ	; fluid density
ϕ	; acceleration potential
ψ	; imaginary part of the complex acceleration potential
ω	; angular speed of oscillation
γ	; density of foil material

3. Theory

3.1 Equation of motion for chordwise bending

The equation of chordwise bending motion for a chordwise flexible supercavitating hydrofoil as shown in Fig.1 is

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written as follows;

$$\frac{\partial^2}{\partial X^2} \left(D \frac{\partial^2 w}{\partial X^2} \right) + \gamma T \frac{\partial^2 w}{\partial t^2} = P(X, t) \quad \text{.....(1)}$$

By assuming the chordwise deformation w , and the ordinate of foil wetted surface Y to be small, they can be written as follows;

$$w(X, t) = \bar{w}(X) + \tilde{w}(X)e^{j\omega t} \quad \text{.....(2)}$$

$$Y = y_1(X) + \bar{w}(X) + \tilde{w}(X)e^{j\omega t} \quad \text{.....(3)}$$

And the hydrodynamic force acting on the wetted surface can be described as follows, according to a two-dimensional linearized theory;

$$P(x, t) = P_I(X) + P_{\bar{w}}(X) + P_{\tilde{w}}(X)e^{j\omega t} \quad \text{.....(4)}$$

where P_I , $P_{\bar{w}}$, and $P_{\tilde{w}}$ are hydrodynamic forces due to the initial foil shape, and the steady and unsteady chordwise foil deformations respectively. By substituting Eqs.(2) and (4), Eq.(1) is reduced to the following two equations:

$$\frac{d^2}{dX^2} \left(D \frac{d^2 \bar{w}}{dX^2} \right) = P_I(X) + P_{\bar{w}}(X) \quad \text{.....(5)}$$

and

$$\frac{d^2}{dX^2} \left(D \frac{d^2 \tilde{w}}{dX^2} \right) - \gamma \omega^2 T = P_{\tilde{w}}(X) \quad \text{.....(6)}$$

The boundary conditions for w (for simplicity $\bar{w}$ and $\tilde{w}$ are represented by w) at the clamped trailing edge and the free leading edge are expressed by

$$\left. \begin{aligned} w_{X=c} &= 0 \\ \frac{dw}{dX} \Big|_{X=c} &= 0 \end{aligned} \right\} \quad \text{.....(7)}$$

$$\left. \begin{aligned} D \frac{d^2 w}{dX^2} \Big|_{X=0} &= 0 \\ \frac{d}{dX} \left(D \frac{d^2 w}{dX^2} \right) \Big|_{X=0} &= 0 \end{aligned} \right\} \quad \text{.....(8)}$$

3.2 Hydrodynamic force

The flow around a two-dimensional supercavitating foil of chord c inclined at a small angle to the uniform flow as shown in Figure 1 is analysed. It is assumed that (1)

Fig.1 Physical Plane

the separation occurs at the leading and trailing edges. (2) The amplitude of oscillation is so small that the separation points are fixed and there is no reattachment of flow to the foil surface after separation. (3) The cavitation number is zero. (4) The flow is incompressible and irrotational outside the body-cavity region. As the perturbation velocity can be regarded as small, according to the above assumption, the Euler equation of motion can be linearized and acceleration potential ϕ is defined as

$$\phi = - \frac{P(X, Y, t)}{\rho} \quad \text{.....(9)}$$

From the equation of continuity, it follows that ϕ satisfies the Laplace equation and thus implies the existence of a pair of conjugate harmonic functions such that

$$F(X, Y, t) = \phi(X, Y, t) + i\psi(X, Y, t) \quad \text{.....(10)}$$

In keeping with the linearized formulation of the problem the boundary conditions on the foil and cavity boundaries are to be applied on the real axis, and can be expressed as follows;

- (1) The disturbance pressure P due to the presence of the foil is zero on the cavity boundary, i.e.

$$\left. \begin{aligned} \phi(X, Y, t) &= 0, & X \geq 0, & Y = 0^+ \\ & & X \geq c, & Y = 0^- \end{aligned} \right\} \quad \text{.....(11)}$$

- (2) The perturbations u , v , and P are zero at $X = -\infty$; therefore F is zero at $X = -\infty$.
- (3) The Kutta condition is satisfied; i.e., $F(Z, t)$ is continuous at the trailing edge $Z = c$ for all time t .
- (4) The normal component of the relative velocity on the wetted part of the foil surface is equal to zero, i.e.

$$v = \frac{\partial Y}{\partial t} + V \frac{\partial Y}{\partial X} \quad \text{.....(12)}$$

which can be reduced as follows, by using ψ ;

Fig.2 Z-plane

Fig.3 ζ-plane

$$\left. \begin{aligned} \frac{\partial \phi}{\partial X} &= -\frac{\partial v}{\partial t} - V \frac{\partial v}{\partial X} \\ &= -\frac{\partial^2 Y}{\partial t^2} - 2V \frac{\partial^2 Y}{\partial X \partial t} - V^2 \frac{\partial^2 Y}{\partial X^2} \end{aligned} \right\} \dots\dots\dots (13)$$

Integration of Eq.(13) yields

$$-\phi(X, t) = \int \frac{\partial^2 Y}{\partial t^2} dX + 2V \frac{\partial Y}{\partial t} + V^2 \frac{\partial Y}{\partial X} + \Psi(t) \dots\dots\dots (14)$$

where $\Psi(t)$ is an unknown function of time.

For seeking a general solution, it is convenient to transform the Z-plane (Figure 2) into the upper half of the ζ -plane (Figure 3) by conformal transformation

$$\zeta = (Z/c)^{1/2} \dots\dots\dots (15)$$

It is seen that the upper cavity boundary becomes the positive real axis, and the foil and the lower cavity boundary extend the negative real axis. The solution of a mixed boundary value problem of Riemann-Hilbert type is

$$\phi(\xi, \eta, t) = \frac{1}{\pi} \int_{-1}^0 \frac{\phi(\xi', 0, t)}{(\xi' - \xi)^2 + \eta^2} d\xi' \dots\dots\dots (16)$$

$$\phi(\xi, \eta, t) = -\frac{1}{\pi} \int_{-1}^0 \frac{\phi(\xi', 0, t) (\xi' - \xi)}{(\xi' - \xi)^2 + \eta^2} d\xi' \dots\dots\dots (17)$$

From Eq.(17)

$$\phi(\xi, 0, t) = -\frac{1}{\pi} \int_{-1}^0 \frac{\phi(\xi', 0, t)}{\xi' - \xi} d\xi' \dots\dots\dots (18)$$

The inversion of Eq.(18), satisfying the Kutta condition at the trailing edge, is reduced to be

$$\phi(\xi, 0, t) = \frac{1}{\pi} \sqrt{\frac{\xi+1}{-\xi}} \int_{-1}^0 \sqrt{\frac{-\xi'}{\xi'+1}} \frac{\phi(\xi', 0, t)}{\xi' - \xi} d\xi' \dots\dots\dots (19)$$

Galerkin's method is introduced to solve Eqs.(5) and (6), and the solutions of $\bar{w}(X)$ and $\tilde{w}(X, t)$ are assumed to be

$$\bar{w}(X) = \sum_{n=0}^N a_n w_n(X) = \sum_{n=0}^N a_n c \left(\frac{X}{c}\right)^n \left(1 - \frac{X}{c}\right)^2 \dots\dots\dots (20a)$$

$$\tilde{w}(X, t) = w(X) e^{j\omega t} = e^{j\omega t} \sum_{n=0}^N a_n w_n(X) = e^{j\omega t} \sum_{n=0}^N a_n c \left(\frac{X}{c}\right)^n \left(1 - \frac{X}{c}\right)^2 \dots\dots\dots (20b)$$

The foil thickness distribution is assumed to be

$$T = T_0 \left(\frac{X}{c}\right)^\beta \dots\dots\dots (21)$$

and the foil stiffness is given by

$$D = D_0 \left(\frac{X}{c}\right)^{3\beta} \dots\dots\dots (22)$$

Equations (20) and (22) satisfy the boundary conditions, i.e., Eqs.(7) and (8). By substituting Eqs.(3) and (20), Eq.(14) is rewritten in the following form:

$$\begin{aligned} -\tilde{\psi} = \sum_{n=0}^N a_n \left[-\omega^2 c^2 \left(\frac{\xi^{2n+6}}{n+3} - \frac{2}{n+2} \xi^{2n+4} + \frac{\xi^{2n+2}}{n+1} \right) + 2jV\omega c (\xi^{2n+4} - 2\xi^{2n+2} + \xi^{2n}) \right. \\ \left. + V^2 \{ (n+2)\xi^{2n+2} - 2(n+1)\xi^{2n} + n\xi^{2n-2} \} \right] + \psi_0 \dots\dots\dots (23) \end{aligned}$$

where

$$\Psi(t) = \psi_0 e^{j\omega t} \dots\dots\dots (24)$$

and

$$\phi(\xi, t) = \bar{\psi}(\xi) + \tilde{\psi}(\xi) e^{j\omega t} \dots\dots\dots (25)$$

Martin⁽⁵⁾ used a power series for describing the amplitude of oscillation. Integrating Eq.(19) after substitution of Eq.(23), the following equation can be obtained.

$$\bar{\phi}(\xi) = \sqrt{\frac{\xi+1}{-\xi}} \left\{ \sum_{n=0}^N a_n \left(-\omega^2 c^2 \sum_{k=0}^{2n+6} A_{nk} \xi^k + 2jV\omega c \sum_{k=0}^{2n+4} B_{nk} \xi^k + V^2 \sum_{k=0}^{2n+2} C_{nk} \xi^k \right) + \phi_0 \right\} \quad (26)$$

where

$$\sum_{k=0}^{2n+6} A_{nk} \xi^k = \frac{1}{n+3} G(2n+6) - \frac{2}{n+2} G(2n+4) + \frac{1}{n+1} G(2n+2) \quad (27)$$

$$\sum_{k=0}^{2n+4} B_{nk} \xi^k = G(2n+4) - 2G(2n+2) + G(2n) \quad (28)$$

$$\sum_{k=0}^{2n+2} C_{nk} \xi^k = (n+2)G(2n+2) - 2(n+1)G(2n) + nG(2n-2) \quad (29)$$

$$G(m) = \xi^m - \frac{1}{2} \xi^{m-1} + \frac{1 \cdot 3}{2^2 \cdot 2!} \xi^{m-2} - \frac{1 \cdot 3 \cdot 5}{2^3 \cdot 3!} \xi^{m-3} + \dots + \frac{1 \cdot 3 \cdot 5 \dots (2m-1)}{2^m m!} \quad (30)$$

and

$$\phi(\zeta, t) = \bar{\phi}(\zeta) + \bar{\phi}(\zeta) e^{j\omega t} \quad (31)$$

Eq.(31) represents the hydrodynamic force acting on the foil through equation (9), provided the unknown constant ψ_0 is determined. By introducing Schwartz's principle of symmetry, it can be shown that on the lower half plane

$$F(\zeta) = -\bar{F}(\bar{\zeta}) \quad (32)$$

where $\bar{}$ means complex conjugate. $F(\zeta)$ can be obtained as follows, after some calculations using Eqs.(16), (26), (32) and (2);

$$\begin{aligned} F(\zeta) = & i \left\{ \phi_0 \left(\sqrt{\frac{\zeta+1}{\zeta}} - 1 \right) + \sum_{n=0}^N a_n \left[-\omega^2 c^2 \sum_{k=0}^{2n+6} A_{nk} \left\{ \left(\sqrt{\frac{\zeta+1}{\zeta}} - 1 \right) \zeta^k - \sum_{l=0}^{k-1} F_{kl} \zeta^l \right\} \right. \right. \\ & + 2jV\omega c \sum_{k=0}^{2n+4} B_{nk} \left\{ \left(\sqrt{\frac{\zeta+1}{\zeta}} - 1 \right) \zeta^k - \sum_{l=0}^{k-1} F_{kl} \zeta^l \right\} \\ & \left. \left. + V^2 \sum_{k=0}^{2n+2} C_{nk} \left\{ \left(\sqrt{\frac{\zeta+1}{\zeta}} - 1 \right) \zeta^k - \sum_{l=0}^{k-1} F_{kl} \zeta^l \right\} \right] \right\} \quad (33) \end{aligned}$$

where

$$\left. \begin{aligned} F_{0l} = 0, \quad F_{10} = \frac{1}{2}, \quad F_{11} = 1, \quad F_{20} = -\frac{1}{2^2 2!}, \quad F_{21} = \frac{1}{2}, \quad F_{22} = 1 \\ \dots \dots \dots \\ F_{kl} = -\frac{(2k-2l-3)(2k-2l-5) \dots 3 \cdot 1}{2^{k-l}(k-l)!} (-1)^{k-l} \end{aligned} \right\} \quad (34)$$

By assuming the motion of the foil to be simple harmonic, the oscillating component of v can be expressed as follows;

$$\bar{v}(X) = -\frac{1}{V} \exp\left(-j\frac{\omega}{V}X\right) \int_{-\infty}^X \frac{\partial \bar{\phi}}{\partial X} \exp\left(j\frac{\omega}{V}X\right) dX \quad (35)$$

Integrating equation (35) by parts and setting the end at the origin yields

$$\bar{v}(0)V = -\bar{\phi}(0) + j\kappa \int_0^\infty \bar{\phi}(x) e^{-j\kappa x} dx \quad (36)$$

where

$$\kappa = \frac{\omega c}{V} \quad (37)$$

$$x = \frac{|X|}{c} \quad (38)$$

By substituting the imaginary part of Eq.(33) into Eq.(36), ψ_0 is obtained as follows;

$$\phi_0 = -\frac{V^2 j\kappa}{1+j\kappa I_0} \left\{ a_0 + \sum_{n=0}^N a_n (-\kappa^2 J_{2n} + 2j\kappa J_{1n} + J_{0n}) \right\} \quad (39)$$

$$I_0 = \int_0^\infty \left\{ \left(\frac{\sqrt{1+(1/x)}+1}{2} \right)^{1/2} - 1 \right\} e^{-jks} dx \dots\dots\dots(40)$$

$$J_{2n} = \sum_{k=0}^{n+3} A_{n,2k} (-1)^k \int_0^\infty \left\{ \left(\frac{\sqrt{1+(1/x)}+1}{2} \right)^{1/2} - 1 \right\} x^k e^{-jks} dx \\ + \sum_{k=1}^{n+3} A_{n,2k-1} (-1)^{k+1} \int_0^\infty \left(\frac{\sqrt{1+(1/x)}-1}{2} \right)^{1/2} x^{(2k-1)/2} e^{-jks} dx \\ - \sum_{k=0}^{2n+6} A_{n,k} \left\{ \sum_{l=0}^{[(k-1)/2]} F_{k,2l} \int_0^\infty (-x)^l e^{-jks} dx \right\} \dots\dots\dots(41)$$

$$J_{1n} = \sum_{k=0}^{n+2} B_{n,2k} (-1)^k \int_0^\infty \left\{ \left(\frac{\sqrt{1+(1/x)}+1}{2} \right)^{1/2} - 1 \right\} x^k e^{-jks} dx \\ + \sum_{k=1}^{n+2} B_{n,2k-1} (-1)^{k+1} \int_0^\infty \left(\frac{\sqrt{1+(1/x)}-1}{2} \right)^{1/2} x^{(2k-1)/2} e^{-jks} dx \\ - \sum_{k=0}^{2n+4} B_{n,k} \left\{ \sum_{l=0}^{[(k-1)/2]} F_{k,2l} \int_0^\infty (-x)^l e^{-jks} dx \right\} \dots\dots\dots(42)$$

$$J_{n0} = \sum_{k=0}^{n+1} C_{n,2k} (-1)^k \int_0^\infty \left\{ \left(\frac{\sqrt{1+(1/x)}+1}{2} \right)^{1/2} - 1 \right\} x^k e^{-jks} dx \\ + \sum_{k=1}^{n+1} C_{n,2k-1} (-1)^{k+1} \int_0^\infty \left(\frac{\sqrt{1+(1/x)}-1}{2} \right)^{1/2} x^{(2k-1)/2} e^{-jks} dx \\ - \sum_{k=0}^{2n+2} C_{n,k} \left\{ \sum_{l=0}^{[(k-1)/2]} F_{k,2l} \int_0^\infty (-x)^l e^{-jks} dx \right\} \dots\dots\dots(43)$$

where $\left[\frac{k-1}{2} \right]$ is the greatest integer included in $(k-1)/2$.

4. Application of Galerkin's method

4.1 Static chordwise deformation of foil

In order to solve Eq.(5), Galerkin's method is introduced. By using Eqs.(9) and (26) (in this case $\omega=0$), the static hydrodynamic force $P_{\bar{w}}$ acting on the foil can be obtained, though there still remain unknown coefficients a_n . By multiplying Eq.(5) by $w_m(X)$ and integrating over the length of the foil after substitution of Eq.(20) and $P_{\bar{w}}$, the following equation can be obtained;

$$\sum_{n=0}^N a_n (\Lambda E F_{nn} + P_{0nn}) = \frac{1}{\rho V^2 c} \int_0^1 P_1(X) w_m(X) dX \dots\dots\dots(44)$$

where

$$\Lambda = \frac{D_0}{\rho V^2 c^2} \dots\dots\dots(45)$$

$$E F_{nn} = \int_0^1 x^{3/2} \frac{d^2}{dx^2} \{ x^n (1-x)^2 \} \frac{d^2}{dx^2} \{ x^n (1-x)^2 \} dx \dots\dots\dots(46)$$

$$P_{0nn} = 2 \int_{-1}^0 \sqrt{-\xi(\xi+1)} \left(\sum_{k=0}^{2n+2} C_{nk} \xi^k \right) \{ \xi^{2n} (1-\xi^2)^2 \} d\xi \dots\dots\dots(47)$$

The unknown coefficients a_n can be obtained by solving Eq.(44) for a chosen value of Λ and an initial foil shape. And by substituting a_n 's into Eqs.(20) and (26), the static foil deformation and the static hydrodynamic force due to the chordwise deformation are obtained.

4.2 Cavitation inception on the pressure side

Excepting the neighborhood of the leading edge, the hydrodynamic force acting on the foil decreases due to the chordwise bending deformation. In this report, the separation is assumed to occur at the leading and trailing edges, and so the pressure acting on the wetted surface of the foil must be larger than the cavity pressure. When the local minimum pressure acting on the wetted surface decreases to zero between the leading and trailing edges on the pressure side, a cavity initiates there. The critical condition for cavitation inception on the pressure side can be determined by the trial and error method for a chosen value of Λ and an initial foil shape.

4.3 Stress due to the chordwise bending

For the stress analysis, it is assumed that the neutral plane of the foil is flat and the deflection of the foil due to the chordwise bending is small. Then the bending stress

σ_x can be given as follows;

$$\sigma_x = -\frac{E}{1-\nu^2} Y \frac{d^2 w}{dX^2} \dots\dots\dots(48)$$

By using Eqs.(20), (21) and (48), the maximum stress due to the chordwise bending is given by

$$\sigma_{x\max} = -\frac{E}{2(1-\nu^2)} \delta \sum_{n=0}^N a_n \left\{ (n+2)(n+1) \left(\frac{X}{c}\right)^{n+\beta} - 2(n+1)n \left(\frac{X}{c}\right)^{n+\beta-1} + n(n-1) \left(\frac{X}{c}\right)^{n+\beta-2} \right\} \dots\dots\dots(49)$$

The critical condition due to the chordwise bending stress can be obtained for a chosen value of ultimate stress by trial and error method using Eq.(49).

4.4 Chordwise bending divergence

Critical condition for chordwise bending divergence can be given by the following equation;

$$|\Delta EF_{mn} + P0_{mn}| = 0 \dots\dots\dots(50)$$

4.5 Chordwise bending flutter

Unsteady hydrodynamic force due to the chordwise bending can be obtained by using Eqs.(9) and (26). By multiplying Eq.(6) by $w_m(X)$ and integrating over the chordwise length of the foil after substituting Eq.(26) and $P_{\tilde{w}}$, the following equation is obtained,

$$\sum_{n=0}^N a_n W_{mn} = \sum_{n=0}^N a_n (\Delta EF_{mn} - \lambda \kappa^2 IF_{mn} - \kappa^2 P2_{mn} + 2j\kappa P1_{mn} + P0_{mn}) - \frac{j\kappa T_m}{1+j\kappa I_0} \left\{ a_0 + \sum_{n=0}^N a_n (-\kappa^2 J_{2n} + 2j\kappa J_{1n} + J_{0n}) \right\} = 0 \dots\dots\dots(51)$$

where

$$IF_{mn} = \int_0^1 x^{\beta+m+n} (1-x)^4 dx \dots\dots\dots(52)$$

$$P2_{mn} = \sum_{k=0}^{2n+6} A_{nk} 2 \int_{-1}^0 \sqrt{-\xi(\xi+1)} \xi^k \xi^{2m} (1-\xi^2)^2 d\xi \dots\dots\dots(53)$$

$$P1_{mn} = \sum_{k=0}^{2n+4} B_{nk} 2 \int_{-1}^0 \sqrt{-\xi(\xi+1)} \xi^k \xi^{2m} (1-\xi^2)^2 d\xi \dots\dots\dots(54)$$

$$T_m = 2 \int_{-1}^0 \sqrt{-\xi(\xi+1)} \xi^{2m} (1-\xi^2)^2 d\xi \dots\dots\dots(55)$$

$$\lambda = \frac{\gamma T_0}{\rho c} \dots\dots\dots(56)$$

From the criterion for existence of solution, the critical condition for chordwise bending flutter can be given by the following equation;

$$|W_{mn}| = 0 \dots\dots\dots(57)$$

5. Numerical results

The parameter β for the foil thickness distribution used in the numerical calculation is 6/12 to 12/12 (Figure 4). In order to investigate the effect of number of terms in Eq.(20), the static chordwise deformation and the pressure distribution have been calculated for $\Lambda=1$. The difference between five and six terms has been 1%, 2%, 4%, and 7% for $\beta=6/12$ to 9/12, 10/12, 11/12, and 12/12 respectively. For $\beta=6/12$ to 11/12, the solution using six terms has apparently converged.

Fig.4 Foil Thickness Distribution ($\delta=0.1$)

5.1 Cavitation inception on the pressure side

The critical values of Λ for cavitation inception on the pressure side, represented by Λ_{cavi} , are shown in Figure 5 versus the angle of attack α_i . The results for a flat plate show the insensitivity of Λ_{cavi} to the angle of attack. But for a circular arc Λ_{cavi} increases with an increase of angle of attack. An increase in camber-chord ratio decreases Λ_{cavi} . The effect of decreasing β , i.e., increasing the thickness near the leading edge, is seen in Figure 6 to decrease Λ_{cavi} . Figure 6 also shows the sensitivity of Λ_{cavi} on β when β is close to unity, which means that a slight increase of foil thickness near the leading edge increases the critical speed of cavitation inception on the pressure side considerably. Figure 7 shows the critical speed of cavitation inception on the pressure side versus thickness-chord ratio for a flat plate. Poisson's ratio and Young's modulus of the foil material are assumed to be 0.3 and 2.0594×10^{11} Pa respectively. The fluid is water at 20°C.

5.2 Chordwise bending stress

The critical speed due to the chordwise bending stress versus thickness-chord ratio is shown and compared with Grumman's results in Figure 7. Material properties used in the calculation are as follows;

Young's modulus; 2.0594×10^{11} Pa
Poisson's ratio; 0.3
ultimate strength; 1.372×10^9 Pa

Fig.5 Critical Condition of Cavitation Inception on the Pressure Side

Fig.6 Critical Condition of Cavitation Inception on the Pressure Side

initial angle of attack; 0.2094 rad. The critical speed V_{str} increases linearly with an increase of thickness-chord ratio δ . Present theory predicts smaller values of V_{str} than Grumman's theory. The effect of thickness distribution V_{str} is shown in Figure 8. A slight increase of foil thickness near the leading edge increases V_{str} considerably.

5.3 Chordwise bending divergence

Values of Λ which satisfy Eq.(50) are not shown, because they are smaller than Λ_{cavi} . Accordingly the cavitation inception speed V_{cavi} is smaller than the chordwise bending divergence speed which is not shown either.

5.4 Chordwise bending flutter

Among the values of Λ which satisfy Eq.(51) calculated for chosen values of λ

Fig.7 Critical Speed Due to the Chordwise Bending Stress

Fig.8 Critical Speed Due to the Chordwise Bending Stress

and κ , positive and real ones are meaningful physically, and shown in Fig.8. The range of reduced frequencies used in the numerical calculation is 0 to 20. The range of thickness chord ratios of typical supercavitating foil is 2 to 10 per cent. The specific gravity is between 3.0 (for aluminum alloys) and 8.0 (for steel or high tensile-strength alloys). With these informations the range of λ used in this numerical calculation is 0.05 to 0.8. An increase of λ increases Λ_{flutter} linearly. Figure 9 also shows the effect of thickness distribution on Λ_{flutter} . It is shown that a slight increase of thickness near the leading edge increases the chordwise bending flutter speed considerably. An example of chordwise bending flutter mode is shown in Figure 10. By the way the reduced frequencies of chordwise bending flutter are in high range, e.g. 19 to 20 for $\beta=12/12$, so the assumptions used in this theory are doubtful. Further investigation is necessary about chordwise bending flutter.

5.5 Static performance of chordwise flexible foil

Drag-lift ratio versus lift coefficient of chordwise flexible foil is shown in Figure 11. The effect of thickness distribution on drag-lift ratio is shown in Figure 12. A slight increase of foil thickness near the leading edge prevents an

increase of drag-lift ratio due to the chordwise bending deformation.

6. Conclusions

A linearized theory for the prediction of steady and unsteady flows around, and hydrodynamic forces on a chordwise flexible

Fig.9 Critical Condition of Chordwise Bending Flutter

Fig.10 Mode of Chordwise Bending Flutter

Fig.11 Drag-Lift Ratio of Chordwise Flexible Hydrofoil

Fig.12 Drag-Lift Ratio of Chordwise Flexible Hydrofoil

supercavitating foil with clamped trailing edge has been developed. By using this theory cavitation inception on the pressure side, chordwise bending stress, chordwise bending divergence, chordwise bending flutter, and steady lift and drag coefficients of chordwise flexible foil have been investigated numerically. Some specific points deduced from the investigations are as follows;

- (1) For a flat plate foil the critical value of Λ for cavitation inception on the pressure side is insensitive to the angle of attack. But for a circular arc foil Λ_{cavi} increases with an increase of angle of attack, and decreases with an increase of camber-chord ratio.
- (2) The critical speed V_{str} for the chordwise bending stress increases linearly with an increase of thickness-chord ratio.
- (3) The critical speed V_{cavi} for cavitation inception on the pressure side is smaller than the critical speed of chordwise bending divergence.

(4) The critical value of chordwise bending flutter $\Lambda_{flutter}$ increases with an increase of density ratio λ . But the reduced frequency of chordwise bending flutter is so large that the assumptions used in the present theory become doubtful. Further investigation is necessary for the chordwise bending flutter.

(5) To give a slight thickness near the leading edge is very effective for increasing the critical speed of chordwise bending flutter and cavitation inception on the pressure side or decreasing the stress due to chordwise bending.

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