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Analyses of Cutting Conditions of Spiral Bevel Gears*

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This paper is intended to find out the best cutting condition for the purpose of improving the cutting efficiency of a spiral bevel gear and the roughness of its finished surface. But the cutting experiment by the gear generator requires too much trouble. Therefore the authors analysed conditions in which a single blade of the cutter cuts the spiral bevel gear, and clarified the cutting conditions of the single blade, with a view to simplifying the experiment by using a single blade instead of the cutter.

1. Introduction

This paper is intended to find out the best cutting conditions for the purpose of improving the cutting efficiency of a spiral bevel gear. The form and thickness of the undeformed chip are calculated. Few reports are published about the load of the cutter blade and the relation between cutting conditions and cutter wear in cutting of a spiral bevel gear.⁽¹⁾

Three methods of cutting a spiral bevel gear are known: (1) a method based on generating for both roughing and finishing, (2) a method based on forming for roughing and based on generating for finishing, and (3) a method based on forming for both. The cutting states in the case of method (1) which is usually used to cut a pinion is analysed.

It requires much gear blanks and trouble to find out the wear and the life of a blade by using the spiral bevel gear cutting machine. In order to simulate the spiral bevel gear cutting states by using a lathe or a milling machine, the form and thickness of the chip and the cutting states of the cutter blade in cutting a spiral bevel gear are analysed. The engage angles which are closely associated with the tool wear are known from the result of this analysis.

2. Position of the cutter blade

As shown in Fig.1, the cutter turns on its own axis O_c at the rate of n_c rpm.⁽²⁾

As the cutter moves round the rotor (cradle in Gleason gear generator, virtual crown gear) axis O at the rate of n_R rpm, the gear blank rotates at n_g rpm, and the cutter generates a spiral bevel gear. In this case, the cutting speed n_c is suitably chosen independently of n_g and n_R .

The relation between n_R and n_g is given by

$$n_R/n_g = Z_g/Z_c \quad (1)$$

where Z_c and Z_g are the numbers of teeth in virtual crown gear and in work gear, respectively

This study takes up the case of cutting by the inside blade of a cutter, which has $2Z_b$ blades inside and outside on the cutter body.

In Fig.1 the following equations are obtained.

$$S^2 = R_m^2 + r_c^2 - 2 R_m r_c \cos(90^\circ - \beta_0) \quad (2)$$

$$r_c^2 = S^2 + R_m^2 - 2 S R_m \cos \lambda_0$$

Fig.1 Generation of spiral bevel gear

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$$\therefore \lambda_0 = \cos^{-1} \left(\frac{S^2 + R_m^2 - r_c^2}{2 S R_m} \right) \quad (3)$$

where β_0 denotes the mean spiral angle, r_c the cutter radius, R_m the mean cone distance, S the distance between O and O_c , and λ denotes the angle $\angle POO_c$. As R_m , β_0 , r_c are already known, S and λ can be calculated from these equations.

The spiral angle at Q , (the point at any cone distance R) can be calculated by the following equations (in Fig.1).

$$S^2 = R^2 + r_c^2 - 2 R r_c \cos(90^\circ - \beta)$$

Substituting Eq.(2) into the above equation gives

$$\beta = \sin^{-1} \left(\frac{R^2 - R_m^2 + 2 R_m r_c \sin \beta_0}{2 R r_c} \right) \quad (4)$$

, the angle $\lambda = \angle QOO_c$ is given by

$$\lambda = \cos^{-1} \left(\frac{S^2 + R^2 - r_c^2}{2 S R} \right) \quad (5)$$

In Fig.2, while the blade at point P_1 turns round the cutter axis O_{c1} by the angle $\Delta\epsilon = \angle P_1 O_{c1} Q_1$, the rotor rotates by the angle $\Delta\theta = \angle P_1 O P_2$. Then, it reaches the point Q_2 . As the angle $\angle Q_1 O Q_2$ is equal to the angle $\Delta\theta$, the relation between $\Delta\theta$ and $\Delta\epsilon$ is

$$\frac{\Delta\epsilon}{\Delta\theta} = \frac{n_c}{n_R} \quad (6)$$

In this study, by giving an adequate value to $\theta_1 (= \angle P O P_1)$, the cutting state in the neighborhood of P_1 is analysed. The cutting state at any position of generating the gear can be determined by values of θ_1 . The values of θ_1 and $\Delta\epsilon$ are positive, provided that they rotate counterclockwise from $OP, O_{c1}P_1$. Then θ_1 in Fig.2 is negative and $\Delta\epsilon$ in it is positive.

$$\theta_{a1} = \angle Q_1 O Q = \lambda_0 + \theta_1 - \lambda \quad (7)$$

In Fig.2 ϵ denotes the angle $\angle P_1 O_{c1} O$.

Fig.2 Position of the cutter blade

For the triangles $\Delta P_1 O_{c1} O$ and $\Delta Q_1 O_{c1} O$, the following equations are obtained.

$$R_m^2 = S^2 + r_c^2 - 2 S r_c \cos \epsilon$$

$$\therefore \epsilon = \cos^{-1} \left(\frac{S^2 + r_c^2 - R_m^2}{2 S r_c} \right)$$

$$R^2 = S^2 + r_c^2 - 2 S r_c \cos(\epsilon + \Delta\epsilon)$$

$$\therefore \epsilon + \Delta\epsilon = \cos^{-1} \left(\frac{S^2 + r_c^2 - R^2}{2 S r_c} \right)$$

$$\therefore \Delta\epsilon = \cos^{-1} \left(\frac{S^2 + r_c^2 - R^2}{2 S r_c} \right) - \cos^{-1} \left(\frac{S^2 + r_c^2 - R_m^2}{2 S r_c} \right) \quad (8)$$

$$\theta_{a2} = \angle Q_2 O Q = \theta_{a1} + \Delta\theta \quad (9)$$

2.1 Cutting by one cutter blade

The cutter cuts a gear from the toe to the heel. The cutter blade at the position $P_1(R_m, \theta_1)$ moves to the position $Q_2(R, \theta_{a2})$ cone distance of which is R . Calculating Eqs. (5), (7), (8) and (6) in turn, λ , θ_{a1} , $\Delta\epsilon$ and $\Delta\theta$ are successively obtained, and θ_{a2} is given by Eq.(9).

2.2 Cutting by the following cutter blade

The inside blade cuts the tooth profile at the position P_1 (the mean cone distance R_m), and the next inside blade cuts at the position P_2 when the cutter rotates by the angle $\Delta\epsilon = 2\pi/Z_b$. The angle $\theta_{P2} = \angle P_2 O P$ is obtained from $\Delta\theta$ given by Eq.(6) as

$$\theta_{P2} = \theta_1 + \Delta\theta \quad (10)$$

Here, before the following inside blade cuts the tooth profile at P_2 , the outside blade which lags by the angle $\Delta\epsilon = 2\pi/2Z_b$ from the foregoing inside blade cuts the opposite tooth surface. In the same way, θ_{P2} can be calculated at any cone distance R .

3. The sectional form of an undeformed chip

The relation between work gear and the cutter (crown gear) is approximately assumed to be the gearing of the virtual spur gear and the rack as shown in Fig.3. In this case, pitch radius r_o of the virtual spur gear pitch circle at the mean cone distance R_m can be calculated by the following equation.

Fig.3 Virtual spur gear

$$r_0 = R_m \tan \delta \quad (11)$$

As for a cone distance R , R_m is replaced by R in above equation.

Figure 4 shows an enlarged side view in Fig.3. Referring to Fig.2 and Fig.4,

$$l_1 = r_0 \gamma_1 \doteq -R_m \theta_1 \quad (12)$$

$$\gamma_1 = l_1 / r_0 \quad (13)$$

where γ_1 is the angle $\angle PO_P P_1$ (γ_1 is positive, that is clockwise with respect to $O_P P$) and $PP_1 = PP'_1 = l_1$.

Two coordinate systems $O_v - X_s Y_s$ and $O_v - x_{s1} y_{s1}$ are taken as shown in Fig.4. The dimensions of the cutter blade are shown in Fig.5, and the coordinate system $P - x_c y_c$ is set up on this cutter blade as shown in this figure. The coordinates (x_c, y_c) of a point A on the cutter blade are (X_{s1}, Y_{s1}) and (x_{s1}, y_{s1}) in $O_v - X_s Y_s$ and $O_v - x_{s1} y_{s1}$ coordinate systems respectively. These values are obtained from the following equations.

$$\begin{aligned} X_{s1} &= l_1 + x_c / \cos \beta \\ Y_{s1} &= r_0 + y_c \end{aligned} \quad (14)$$

$$\tan \alpha_s = \tan \alpha_n / \cos \beta \quad (15)$$

$$x_{s1} = X_{s1} \cos \gamma_1 - Y_{s1} \sin \gamma_1 \quad (16)$$

$$y_{s1} = X_{s1} \sin \gamma_1 + Y_{s1} \cos \gamma_1$$

where α_s is the transverse pressure angle and α_n is the normal pressure angle.

The coordinates (x_c, y_c) of a point

Table 1 Equations of x_c, y_c

The position of A_1 (in Fig.5)	Equations of x_c, y_c
Inside blade	
1-2 (straight line)	$x_c = -h \tan \alpha_n$ $y_c = h$ ($h_f + r_b - r_b \sin \alpha_n \leq h \leq h_k$)
2-3 (arc line)	$x_c = -(h_f + r_b) \tan \alpha_n + r_b / \cos \alpha_n - r_b \cos \xi$ $y_c = h_f + r_b - r_b \sin \xi$ ($20^\circ \leq \xi \leq 90^\circ$)
3-4 (straight line)	$x_c = x_{c3}$ ($x_{c3} \leq x_c \leq x_{c4}$) $x_{c3} = -(h_f + r_b) \tan \alpha_n + r_b / \cos \alpha_n$ $x_{c4} = -h_f \tan \alpha_n + 0.95$ $y_c = h_f$
4-5 (straight line)	$x_c = -2h_f \tan \alpha_n + 0.95 - h_f (1 + \tan \alpha_n)$ $y_c = h$ ($h_f \leq h \leq h_f + 0.35$)
5-6 (straight line)	$x_c = -2h_f \tan \alpha_n + 1.3 + h \tan \alpha_n$ $y_c = h$ ($h_f + 0.35 \leq h \leq h_k$)
Outside blade	
1'-2' (straight line)	$x_c = -2h_f \tan \alpha_n + 1.5 + h \tan \alpha_n$ $y_c = h$ ($h_f + r_b - r_b \sin \alpha_n \leq h \leq h_k$)
2'-3' (arc line)	$x_c = -h_f \tan \alpha_n + 1.5 - r_b (1 - \sin \alpha_n) / \cos \alpha_n + r_b \cos \xi$ $y_c = h_f + r_b - r_b \sin \xi$ ($20^\circ \leq \xi \leq 90^\circ$)
3'-4' (straight line)	$x_c = x_{c3'}$ ($x_{c3'} \leq x_c \leq x_{c4'}$) $x_{c3'} = -h_f \tan \alpha_n + 1.5 - r_b (1 - \sin \alpha_n) / \cos \alpha_n$ $x_{c4'} = 0.55 - h_f \tan \alpha_n$ $y_c = h_f$

r_b : tip radius h_f : dedendum
 h_k : addendum

A_1 on the cutting blade are shown in Table 1. As mentioned in §2.2, after the inside blade at the position of P_1 moves round by $\Delta \epsilon = 2\pi / Z_b$, the next inside blade comes at the position P_2 . The coordinates (X_{s2}, Y_{s2}) of a point A_2 , that is the same point A_1 on the cutter blade, are obtained by calculating Eqs.(10),(12),(13) and (14) in turn. If the coordinate system $O_v - x_{s2} y_{s2}$ is chosen as shown in Fig.4, the coordinates (x_{s2}, y_{s2}) of A_2 are derived by replacing γ_2 for γ_1 in Eq.(16).

As the coordinate system $O_v - x_{s2} y_{s2}$ rotates round O_v by $(\gamma_1 - \gamma_2)$, the coordinate system $O_v - x_{s2} y_{s2}$ coincides with the system $O_v - x_{s1} y_{s1}$, as shown in Fig.6, and P_1' coincides with P_2' . In this new coordinate system $O_v - x_{s1} y_{s1}$, the values of (x_{s1}, y_{s1}) and (x_{s2}, y_{s2}) obtained from Eq.(16) are directly used as they are. Calculating the coordinates of the cutter blade in the new

Fig.4 Gearing of virtual spur gear and rack

Fig.5 Dimensions and coordinates of the cutter blade

system for various values (x_c, y_c) , the cutting edge profiles can be drawn as indicated by E_1 and E_2 in Fig.6. The blade E_1 cuts the tooth profile at the position P_1 and E_2 at the position P_2 . The outside blade which lags by the angle $\Delta\epsilon=2\pi/2Z_b$ from the foregoing inside blade, is drawn in the same way. According to this drawing, the parts to be cut by the E_2 blade, the shaded area in Fig.6, that is the sectional forms of undeformed chip are made clear.

The above discussion has been done on the transverse plane. Now, the normal plane is considered. In this coordinate system $O_v - x_n y_n$, y_n axis coincides with y_s axis, and the angle between x_n axis and x_s axis is equal to the spiral angle β . Therefore, (x_n, y_n) is obtained

$$x_n = x_s \cos \beta, \quad y_n = y_s \quad (17)$$

$$\begin{aligned} x_n &= ((l+x_c/\cos\beta)\cos\gamma - (r_0+y_c)\sin\gamma)\cos\beta \\ y_n &= ((l+x_c/\cos\beta)\sin\gamma + (r_0+y_c)\cos\gamma) \end{aligned} \quad (18)$$

As an example, the calculation for the pinion shown in Table 2 and Fig.7 is performed. The dimensions of the cutter blade and the values of n_c and n_R , are given in Table 2 and Fig.5.

Figure 8 shows the normal section obtained from Eq.(18) and parts of tip. In Fig.2, the inside blade starts from the position $P_1(R=96.8, \theta_1=-8^\circ)$ and arrives at the position $Q_2(R=99.8)$. The cutting edge at the position Q_2 is E_1 in Fig.8 and E_2 cuts after E_1 rotates by the angle $\Delta\epsilon=60^\circ/2$ ($360^\circ/6$). The outside blade F_1 cuts after

E_1 rotates by the angle $\Delta\epsilon=30^\circ$ ($360^\circ/(2*6)$). It is apparent from Fig.8 that the undeformed chip is the thickest at the rounded corner and the top cutting edge, and the load on the cutting edge is the highest on this portion.

As the coner 4-5 causes an unusual wear, it is necessary to have the roundness at the portion 4-5.

4. Cutting area of one blade

In Fig.6, the coordinates of the point B that is the non-rounded corner of the cutting edge (in Fig.5) are obtained from Eq.(18). Also those of the intersecting point C of E_1 and E_2 (in Fig.5) are obtained from Eqs.(18) and (19). In Fig.4 those of the intersecting point G of E_2 and the tip circle of the gear are obtained from the following Eq.(20). The tooth profile of the gear is cut by the portion BC of the cutting edge E_2 . However, when G is inside C, that is, C is outside of the gear work, it is cut by the portion BG.

$$x_{n_1} = x_{n_2}, \quad y_{n_1} = y_{n_2} \quad (19)$$

$$x_{s_2}^2 + y_{s_2}^2 = (r_0 + h_k)^2 \quad (20)$$

The face width is divided into n equal intervals, and the cutting portion BC or BG of the cutting edge on E_2 blade is calculated at the $(n+1)$ positions of inner cone distance R_i , cone distance $R_1=R_i+b/n$, $R_2=R_i+2b/n$, ---- mean cone distance $R_m=R_i+(n/2)(b/n)$, ---- outer cone distance $R_a=R_i+b$. When these are expressed from inner cone distance as $B_iC_i(B_iG_i), B_1C_1(B_1G_1), \dots, B_mC_m(B_mG_m)$ (at the position of mean cone distance), ----, $B_aC_a(B_aG_a)$ (at the position of outer cone distance) and are drawn in turn, the cutting state of the blade turns out as illustrated in Fig.9. In this illustration, the moving state of the cutting edge from the toe to the heel and cutting state of it can be seen. The illustration in Fig.9 is drawn as a three dimensional model by obtaining the cutting portion of the blade at 11 positions of the cone distance divided $b=30$ by $n=10$. The broken lines show the normal teeth pro-

Fig.6 The section of undeformed chip

Fig.7 Pinion used for example of calculation

Fig.8 The normal section of undeformed chip

files at each cone distance, and the assembly of those lines represents the tooth profile which is to be finished. The cutter blade moves from B_1G_1 (at inner cone distance) to B_1G_1 , ----, B_mG_m (at mean cone distance) in the direction of the arrow, and the intersecting point C comes into the work gear between B_6G_6 and B_7G_7 , finishing the cutting at B_8C_8 (at outer cone distance). As the distance between the point C and the contact point D_2 is very small in Fig.6, the line of contact is almost the same as the line of CC_8 in Fig.9, and is slightly to the side of the root of the gear. The range enclosed by $B_1B_8C_8G_1$ is the cutting area of one blade, and H denotes the tooth surface which is already finished. Figure 9 shows a case in which the blade E_2 cuts with the delayed angle $\Delta\epsilon=60^\circ$ after the E_1 blade has cut from the toe to the heel through the point P_1 ($R_m=96.8, \theta_1=-8^\circ$).

The tip portion of B_6G_6 at the position with the cone distance $R_6(=R_i+6*30/10$

Fig.9 Cutting area of one blade: I
 B_1, B_2 ----: Corner of a cutting blade tip
 C_7, C_8 ----: An intersecting point of cutting blade
 G_1, G_2 ----: A point of intersecting of cutting blade and top circle of the gear

$=99.8$) is shown in Fig.8. By changing the value of θ_1 as the generating proceeds, the cutting state at any position of generating can be seen. Figure 10 shows a case of $\theta_1=-12^\circ$ and 0° . In the figure (a), the cutter blade goes into the gear blank from the position a little to the inner side of G_2 . As the generating goes on, the line CC_8 and the contact line in Fig.9 change to those in Fig.10(b).

5. Undeformed chip thickness

The form of undeformed chip is shown in Fig.8. Taking the intersecting point between E_1 and the normal line of the point $J(x_J, y_J)$ on E_2 as $K(x_K, y_K)$, JK is the undeformed chip thickness t , which is obtained from the following equation.

Table 2 Gear, cutter and cutting conditions used for example

	Pinion	Gear
Number of teeth	20	40
Outer module	5	
Pressure angle	20	
Shaft angle	90	
Outside diameter	100	200
Whole depth	Parallel depth tooth 7.54	
Addendum h_k	4.59	2.20
Deddendum h_f	2.95	5.34
Cone distance R	inner	$R_i = 81.80$
	mean	$R_m = 96.80$
	outer	$R_o = 111.80$
Spiral angle β_0	35(right)	35(left)
Cutter		
Mean diameter : 7.5 inch = 190.5 mm		
Point width : PW = 1.5 mm		
$r_c = 190.5/2 - 1.5/2 - 2.95 \tan 20^\circ = 93.43$ mm		
Inside blade $\therefore Z_b = 6$, Outside blade $\therefore Z_b = 6$		
n_c : Mean cutting speed 20m/min		
$n_c = 20 * 1000 / (\pi * 190.5) = 33.42$ rpm		
n_R : Mean generatig speed 50mm/min		
$n_R = 50 / (2 * \pi * R_m) = 0.0822$ rpm

Fig.10 Cutting area of one blade II

$$t = \sqrt{(x_J - x_K)^2 + (y_J - y_K)^2}$$

Next, how to calculate t at any position on E_2 is described. Equation of straight line 1-2 on the cutting edge is obtained by substituting the equations in Table 1 and Eq.(15) into Eq.(18).

$$\begin{aligned} x_n &= ((l - h \tan \alpha_s) \cos \gamma - (r_0 + h) \sin \gamma) \cos \beta \\ y_n &= (l - h \tan \alpha_s) \sin \gamma + (r_0 + h) \cos \gamma \end{aligned} \quad (21)$$

Eliminating h in Eq.(21) gives the following equation.

$$y_n = -\frac{1}{\tan(\alpha_s + \gamma) \cos \beta} x_n + \frac{(l + r_0 \tan \alpha_s)}{\sin \gamma + \cos \gamma \tan \alpha_s}$$

Therefore, the normal line at the point $J_1(x_{J1}, y_{J1})$ on the cutting edge is given by

$$y_n = \tan(\alpha_s + \gamma) \cos \beta x_n + y_{J1} - \tan(\alpha_s + \gamma) \cos \beta x_{J1} \quad (22)$$

As for the curve line 2-3 on the cutting edge, substituting equations of Table 1 into Eq.(18) gives

$$\begin{aligned} x_n &= (A \cos \gamma - B \sin \gamma) \cos \beta - (r_b \cos \gamma) \cos \xi + (r_b \sin \gamma \cos \beta) \sin \xi \\ y_n &= (A \sin \gamma + B \cos \gamma) - (r_b \sin \gamma / \cos \beta) \cos \xi - (r_b \cos \gamma) \sin \xi \end{aligned} \quad (23)$$

where

$$A = l - h_f \tan \alpha_s - r_b \tan \alpha_s + r_b / (\cos \beta \cos \alpha_n), \quad B = r_0 + h_f + r_b$$

The coordinates $J_2(x_{J2}, y_{J2})$ at the angle ξ on the cutting edge are obtained from the above equations. And the slope of the tangent line at this point is expressed as

$$\frac{dy_n}{dx_n} = \frac{dy_n/d\xi}{dx_n/d\xi} = \frac{r_b \sin \xi \sin \gamma / \cos \beta - r_b \cos \xi \cos \gamma}{r_b \sin \xi \cos \gamma + r_b \cos \xi \sin \gamma \cos \beta} = -\frac{1 - \tan \xi \tan \gamma / \cos \beta}{\tan \xi + \tan \gamma \cos \beta}$$

Therefore, the equation of normal line at $J_2(x_{J2}, y_{J2})$ is

$$y_n = \frac{\tan \xi + \tan \gamma \cos \beta}{1 - \tan \xi \tan \gamma / \cos \beta} x_n + (y_{J2} - \frac{\tan \xi + \tan \gamma \cos \beta}{1 - \tan \xi \tan \gamma / \cos \beta} x_{J2}) \quad (24)$$

Table 3 and 4 show the equations for the inside blade and outside blade.

Equations of 1₁-2₁ and 2₁-3₁ in Fig.8 are obtained by taking l_1 and γ_1 for l and γ in Eqs.(21) and (23), and resultant equations are (21)-1 and (23)-1. The point $K(x_{K1}, y_{K1})$ is the intersecting point of Eq.(21)-1 and Eq.(22) which is an expression of normal line on the point $J_1(x_{J1}, y_{J1})$ of

the 1₂-2₂ line. Now, it is investigated whether the point K is on the 1₁-2₁ line or on its prolonged line. If K is on the prolonged line, the intersecting point of Eq.(23)-1 (2₁-3₁) and Eq.(22), K , is adopted. Also whether the point K is inside the gear or not, and whether the point K is inside of the point C or not is considered. When the point K is outside of them the undeformed chip thickness is zero. Similarly,

Table 3 Equations of inside blade and its normal line
(1) Inside blade (2) Its normal line

1-2 (straight line)	(1) In the text (2)
2-3 (curve line)	(1) In the text (2)
3-4 (straight line)	(1) $y_n = (\tan \gamma / \cos \beta) x_n + (r_0 + h_f) / \cos \gamma$ (2) $y_n = -(\cos \beta / \tan \gamma) x_n + y_{J1} + (\cos \beta / \tan \gamma) x_{J1}$
4-5 (straight line)	(1) $y_n = ((1 + D \tan \gamma) / ((D - \tan \gamma) \cos \beta)) x_n - (C - r_0 D) / (D \cos \gamma - \sin \gamma)$ (2) $y_n = -((D - \tan \gamma) \cos \beta) / (D \tan \gamma + 1) x_n + y_{J1} + (((D - \tan \gamma) \cos \beta) / (D \tan \gamma + 1)) x_{J1}$ $C = l - 2h_f \tan \alpha_s + (0.95 - h_f) / \cos \beta$ $D = (1 + \tan \alpha_n) / \cos \beta$

Table 4 Equations of outside blade

2'-3' (curve line)	$x_n = (E \cos \gamma - F \sin \gamma) \cos \beta + (r_b \cos \gamma) \cos \xi + (r_b \sin \gamma \cos \beta) \sin \xi$ $y_n = (E \sin \gamma + F \cos \gamma) + (r_b \sin \gamma / \cos \beta) \cos \xi - (r_b \cos \gamma) \sin \xi$ $E = l - h_f \tan \alpha_s + 1.5 / \cos \beta - r_b (1 - \sin \alpha_n) / (\cos \alpha_n \cos \beta)$ $F = r_0 + h_f + r_b$
3'-4' (straight line)	$y_n = (\tan \gamma / \cos \beta) x_n + (r_0 + h_f) / \cos \gamma$
4'-5' (straight line)	$y_n = ((I \tan \gamma - 1) / ((\tan \gamma + I) \cos \beta)) x_n + (G + r_0 I) / (\sin \gamma + I \cos \gamma)$ $G = l + (0.55 + h_f) / \cos \beta$ $I = (1 + \tan \alpha_n) / \cos \beta$
5'-6' (straight line)	$y_n = -x_n / (\tan(\gamma + \alpha_s) \cos \beta) + (l + 0.2 / \cos \beta + r_0 \tan \alpha_s) / (\sin \gamma + \cos \gamma \tan \alpha_s)$

the lines of 1_1-2_1 , 2_1-3_1 , 3_1-4_1 , $3_1-4_1'$, 4_1-5_1 and 5_1-6_1 for the point $J_2(x_{J_2}, y_{J_2})$ on the line of 2_2-3_2 are investigated, and the distance between J_2 and $K_2(\bar{J}_2\bar{K}_2=t)$ which is the shortest, is adopted as the undeformed chip thickness. Here K_2 is not on the prolonged line but on the line. In the same way, the thickness at any point on 3_2-4_2 and 4_2-5_2 , and at any cone distance are calculated.

The results obtained in the above example are given in Table 5. The ordinate is the cone distance and abscissa is the point numbers on the cutting edge. As shown in Fig.11, 1-11, 11-21, 21-31 and 31-41 are equally divided into ten parts respectively, and the point numbers are 1,2,--- from the upper left hand. Table 5 is data for the point of odd numbers, and the values of θ_1 are the same as those in Figs. 9 and 10, and the values of thickness are for the blade E_2 (refer to Fig.6). The rows show the values of the undeformed chip thickness on a point of cutting edge at a cone distance. The columns show the changes of the undeformed chip thickness on the cutting edge when the blade cuts from the toe to the heel. From a column, an engage angle can be obtained.³⁾ More detailed calculations can be made by increasing the number of divisions in the tooth width. Figure 12 shows the case where θ_1 is -12° , and the undeformed chip thickness is drawn on the position of the cutting edge. B_2G_2 in Fig.10(a) corresponds to that in Fig.12 (a), and it is seen that the cutting edge of the rounded corner cuts into the tip of the gear work at first. As shown in Fig. 12, the undeformed chip is the thickest at the rounded corner, and it becomes thinner as cutting goes on. The cutting begins at the rounded corner and goes to the top next. As the cutting goes on, the cutting edge of the generating near rounded corner comes to take part in cutting first, i.e. the numbers 9,7,5 and 3 on the cutting edge gradually take part successively in cutting. The load is the highest at the rounded corner, and breakage and wear occur most in this portion. Figure 8 shows the aspect in the neighborhood of the tip ($\theta_1=-8^\circ$ and $R=99.8$, i.e. the values marked * in Table 5) and the values of Table 5 coincide well with form of the chip

Fig.11 The point numbers on the cutting edge (the numbers in () are ones in Fig.5)

in Fig. 8. In the case of $\theta_1=0^\circ$, E_1 cuts the pitch point ($R_p=96.8$, denoted by * in Table 5) and E_2 cuts after E_1 moves round by $\Delta\epsilon=60^\circ$. The chip thickness values of this case are shown in Table 5. At this position undeformed chip thickness on the point 7 of cutting edge (in Fig.11, above pitch point) is zero, and that on the point 9 is $4\mu\text{m}$. As the intersecting point of the cutting edges C (in Fig.6) is just below the pitch point, the thickness $4\mu\text{m}$ on the point number 9 of the cutting edge is considered reasonable. When the cutter blade generates the tooth profile at a place near the point C (in Fig.6), the undeformed chip is very thin. Therefore, the cutting edge should be sharp.

6. Conclusions

The cutting states, for example, the undeformed chip thickness, cutting area etc. have been analysed in this paper. The results are summarized as follows.

(1) This study was done by an approximate analysis. The degree of approximation is not so bad, hence the result is sufficient to know the outline of cutting.

(2) The undeformed chip is the thickest at the rounded corner of the cutting edge, and the second thickest is at the top. The load on the cutting edge is the highest at the rounded corner, where the chipping, wear and breakage occur most.

Fig.12 Undeformed chip thickness (μm) and its position $\theta_1=-12^\circ$, (B_2G_2) , (B_2G_2) , --- are the generating position of the cutter blade in Fig.10(a). The values of thickness are those at the position 17.

Table 5 Undeformed chip thickness (μm)

R	Cutting edge used for generation					Cutting edge of tip radius					Top cutting edge						
	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33
81.80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
84.80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
87.80	0	0	0	0	0	70	97	115	125	129	120	0	0	0	0	0	0
90.80	0	0	0	0	0	67	92	110	120	124	118	60	61	61	61	61	0
93.80	0	0	0	0	46	63	87	105	115	118	115	57	58	58	58	59	0
96.80	0	0	0	36	43	59	82	99	108	112	109	54	55	55	55	55	0
99.80	0	0	0	33	40	54	76	92	101	104	102	51	51	51	52	52	0
102.80	0	0	23	30	37	50	70	84	93	96	94	47	47	47	48	48	0
105.80	0	0	19	26	33	45	63	76	84	87	85	42	43	43	43	44	0
108.80	0	8	15	22	29	39	55	67	74	77	76	38	38	38	39	39	0
111.80	0	4	11	18	25	34	47	57	64	66	65	32	33	33	33	34	0

$$\theta_1 = -12.0^\circ$$

R	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33
81.80	0	0	0	0	43	59	81	97	107	111	109	54	54	55	55	55	0
84.80	0	0	0	34	41	56	77	93	102	106	104	52	52	52	52	53	0
87.80	0	0	0	31	39	52	73	88	97	100	98	49	49	49	50	50	0
90.80	0	0	21	29	36	49	68	82	90	94	92	46	46	46	47	47	0
93.80	0	0	19	26	33	45	62	75	83	87	85	42	42	43	43	43	0
96.80	0	9	16	23	30	40	56	68	76	79	77	38	39	39	39	39	0
*99.80	0	5	12	19	26	36	50	61	67	70	69	34	34	35	35	35	0
102.80	0	2	9	16	22	31	43	52	58	60	59	29	30	30	30	31	0
105.80	0	0	5	12	18	26	36	43	48	50	49	24	25	25	25	25	0
108.80	0	0	1	8	14	22	29	34	38	39	38	19	19	19	20	20	0
111.80	0	0	0	3	10	17	21	24	26	27	26	13	13	13	14	14	0

$$\theta_1 = -8.0^\circ$$

R	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33
81.80	0	0	6	14	21	29	40	48	53	55	53	27	27	27	27	28	0
84.80	0	0	4	11	18	26	35	42	46	47	46	23	23	23	24	24	0
87.80	0	0	1	8	15	22	30	35	38	39	38	19	19	19	20	20	0
90.80	0	0	0	5	12	19	24	28	30	30	29	14	15	15	15	15	0
93.80	0	0	0	1	8	15	18	20	21	21	19	9	10	10	10	11	0
*96.80	0	0	0	0	4	11	12	12	11	10	9	4	5	5	5	5	0
99.80	0	0	0	0	0	7	5	4	2	0	0	0	0	0	0	0	0
102.80	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0
105.80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
108.80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
111.80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$$\theta_1 = 0.0^\circ$$

(3) When the cutter blade generates the tooth profile at a place near the point C (in Fig.6), the undeformed chip is very thin. Therefore, the cutting edge should be sharp to get the smooth tooth surface.

(4) When the cutter rotates clockwise as viewed from the rotor side, the chip is thick at first and becomes thin as the cutting goes on.

(5) In Fig.5, the portion of 4-5 needs to be rounded.

(6) The comparison of the data obtained here with the real chip of gear cutting, and the alternative method of using a lathe or a milling machine instead of a gear cutting machine will be reported on in the near future.

The calculation in this paper was performed by ACOS 600 of the Computer Center of Fukuoka University.

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