533.695.01

Paper No. 195-4

Unsteady Force on a Cambered Blade Passing Through an Oblique Wake*

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Theoretical and experimental investigations are reported in this paper, concerning the wake interaction in a turbomachine. Theoretical calculations are made on the unsteady force induced on a cambered blade with angle of attack due to the passing wake of an upstream moving cylinder. The results are compared with the experiments. The reason why discrepancies arise is considered by the experimental data of the three-dimensional unsteady velocity distribution around the blade. Discussions are also made about the wake interaction.

1. Introduction

Each blade of a turbomachine has a viscous wake trailing behind it. Owing to the relative motion of adjacent blade rows, the wakes shed from an upstream blade row are encountered by the following row. This phenomenon gives rise to an unsteady force on each blade and is one of the more common sources of high cycle fatigue failure of a blade in a turbomachine. The estimation of the unsteady force will be of further importance to the design procedures of the high speed and large scale machine.

Many theoretical studies^{(1)~(7)} have been published about the unsteady force on the blade travelling through gusts. Most of these analyses described previously treated the blade as a flat-plate airfoil under a transverse gust. Horlock⁽⁴⁾ determined the unsteady lift on a flat-plate airfoil with angle of attack due to a longitudinal gust. His analysis showed that the amplitude of the unsteady lift due to a longitudinal gust cannot be neglected. Naumann-Yeh⁽⁵⁾ developed the unsteady lift for a blade considering both camber and angle of attack, under longitudinal as well as transverse gusts. Knowledge gained by their analysis indicated design trends to obtain minimum lift fluctuations.

However, only a few studies, which compared the theoretical results with the experimental ones, have been published. Lefcort⁽³⁾ extended Meyer's analysis⁽²⁾ to the case when an arbitrary wake passed

perpendicular to the blade, and compared with the experimental results for the first time. His results obtained support qualitatively Meyer's predictions. In previous report⁽⁶⁾⁽⁷⁾, the authors treated the unsteady force acting on the blade due to the wake shed by the upstream cylinder moving perpendicular to the free-stream. Comparison between the theoretical and the experimental results was made for various values of the velocity of cylinder and the axial distance between the blade and the cylinder. It was shown that the agreement of both results was fairly good when the velocity of cylinder was very high.

In this paper, the analysis developed by Naumann-Yeh⁽⁵⁾ is extended to cover a wake of arbitrary shape. The unsteady force on the blade due to the wake shed by the upstream cylinder moving perpendicular to the free-stream is calculated and compared with the experiments for the case when the velocity of cylinder is much lower than the free-stream velocity. And also the three-dimensional unsteady velocity distributions around the blade are measured in order to consider the reason for the discrepancy between the calculated and experimental unsteady forces. Discussions are made about the wake interaction.

Nomenclature

ΔC_L	: unsteady lift coefficient
c	: chord length
d	: diameter of cylinder
F_1, F_2, F_3	: functions [Eq. (13)]
f	: ratio of maximum camber over half-chord length
$H_m^{(2)}$	: Hankel function of the second kind
J_m	: Bessel function of the first kind
ΔL	: unsteady lift
l	: axial distance between center of cylinder and leading edge of blade
t	: time
U	: main airstream velocity

* Received 28th April, 1979.

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- u_b : velocity of cylinder
 u_g, v_g : x,y-components of the velocity perturbation
 w : velocity defect of wake
 X : distance along wake axis, measured from center of cylinder [Eq.(16)]
 X_n : distance between center of cylinder and measuring positions of wake
 x_t : coordinate
 x, y : coordinates
 Y : distance from center line of wake [Eq.(17)]
 Z : number of blades
 α : angle of attack
 β : angle between center line of wake and x-axis
 ρ : density
 ω : nondimensional frequency
 Superscript
 ' : nondimensional distance and time divided by half-chord length
 Subscripts
 1 : effect of camber
 2 : effect of angle of attack
 u : effect of x-component of velocity perturbation
 v : effect of y-component of velocity perturbation
 sum : value summed up

2. Theoretical calculation

2.1 Equation of unsteady lift

The x,y-components of the flow perturbation u_g, v_g can be represented by a sinusoidal wave travelling with the main stream velocity U . Thus,

$$u_g(x, t) = \bar{u}_g(x) e^{i\omega t} = \bar{u}_g \exp[i\omega(t' - x')] \quad \dots(1)$$

$$v_g(x, t) = \bar{v}_g(x) e^{i\omega t} = \bar{v}_g \exp[i\omega(t' - x')] \quad \dots(2)$$

where ω is the nondimensional frequency. Also the nondimensional time t' and the nondimensional distance x' are

$$t' = 2Ut/c \quad \dots(3)$$

$$x' = 2x/c \quad \dots(4)$$

where c denotes the chord length. For the case of a blade that has both camber line

$$y = f\left(-\frac{2}{c}x^2 + \frac{c}{2}\right) \quad \dots(5)$$

and angle of attack α , under u_g and v_g , as shown in Fig.1, Naumann-Yeh⁽⁵⁾ have shown that the unsteady lift is given by

$$\Delta \bar{L} = \Delta \bar{L}_{u1} + \Delta \bar{L}_{u2} + \Delta \bar{L}_v \quad \dots(6)$$

$$\Delta \bar{L}_{u1} = \pi c \rho \bar{u}_g f F_f(\omega); \quad F_f(\omega) = F(\omega) + \frac{4}{\omega} J_1(\omega) \quad \dots(6.a)$$

$$\Delta \bar{L}_{u2} = \pi c \rho \bar{u}_g \alpha F_a(\omega); \quad F_a(\omega) = J_0(\omega) + i J_1(\omega) \quad \dots(6.b)$$

Fig.1 Cambered blade with angle of attack

$$\Delta \bar{L}_v = \pi c \rho U \bar{v}_g S(\omega) \quad \dots(6.c)$$

In the above equations, f is the ratio of maximum camber over the half-chord length, J_0 and J_1 are Bessel functions of the first kind, $S(\omega)$ is the Sears function⁽¹⁾ and the functions $F(\omega)$ is defined as

$$F(\omega) = T(\omega) \left[J_0(\omega) - \frac{J_1(\omega)}{\omega} - i J_1(\omega) \right] - \left[J_0(\omega) - \frac{J_1(\omega)}{\omega} + i J_1(\omega) \right] \quad \dots(7)$$

$$T(\omega) = \frac{H_0^{(2)}(\omega) + i H_1^{(2)}(\omega)}{-H_0^{(2)}(\omega) + i H_1^{(2)}(\omega)} \quad \dots(8)$$

where $H_0^{(2)}$ and $H_1^{(2)}$ are Hankel functions of the second kind.

2.2 Extension to arbitrary wake

Consider a blade passing through the wake at an angle β to the x-axis, in Fig.2. It is assumed that the wake axis passes through the origin of the x,y-coordinate system at the time $t=0$. Similarly to previous studies⁽³⁾⁽⁷⁾, it is considered that the wake is divided by h sheets of thin wake with small width Δy_h (see Fig.3). Representing the x,y-components of the velocity defect in the wake as a complex Fourier integral, the results are

$$u_g = -\frac{\cot \beta}{\pi c} \sum_{h=-\infty}^{\infty} w(y_h) \sum_{k=-\infty}^{\infty} \exp[-i\omega_k(x' - t_h')] \Delta \omega_k \Delta y_h \quad \dots(9)$$

$$v_g = \frac{1}{\pi c} \sum_{h=-\infty}^{\infty} w(y_h) \sum_{k=-\infty}^{\infty} \exp[-i\omega_k(x' - t_h')] \Delta \omega_k \Delta y_h \quad \dots(10)$$

Fig.2 Wake passing through blade

Fig.3 Divided wake of small width Δy_h

where

$$t_h = \frac{2U}{c} \left(t - \frac{y_h}{U \sin \beta} \right) \dots \dots \dots (11)$$

Substituting Eqs.(9),(10) for u_y, v_y into Eq.(6), the unsteady lift ΔL is defined as

$$\Delta L = \Delta L_{u1} + \Delta L_{u2} + \Delta L_v \dots \dots \dots (12)$$

$$\Delta L_{u1} = -\rho U f \cot \beta \int_{h=-\infty}^{\infty} w(y_h) F_1(t_h) \Delta y_h \dots \dots \dots (12.a)$$

$$\Delta L_{u2} = -\rho U a \cot \beta \int_{h=-\infty}^{\infty} w(y_h) F_2(t_h) \Delta y_h \dots \dots \dots (12.b)$$

$$\Delta L_v = \rho U \int_{h=-\infty}^{\infty} w(y_h) F_3(t_h) \Delta y_h \dots \dots \dots (12.c)$$

where

$$\left. \begin{aligned} F_1(t_h) &= \int_{\kappa=-\infty}^{\infty} F_f(\omega_\kappa) \exp(i\omega_\kappa t_h) \Delta \omega_\kappa \\ F_2(t_h) &= \int_{\kappa=-\infty}^{\infty} F_a(\omega_\kappa) \exp(i\omega_\kappa t_h) \Delta \omega_\kappa \\ F_3(t_h) &= \int_{\kappa=-\infty}^{\infty} S(\omega_\kappa) \exp(i\omega_\kappa t_h) \Delta \omega_\kappa \end{aligned} \right\} \dots \dots \dots (13)$$

The functions F_1, F_2 and F_3 are graphically represented in Fig.4. The function $F_3(t')$ is the same as Meyer's $T(t')$ multiplied by π . $\pi T(t')$ and $F_3(t')$ were compared as in-

Table 1 Functions $F_1(t'), F_2(t'), F_3(t')$ and $T(t')$

t'	$F_1(t')$	$F_2(t')$	$F_3(t')$	$\pi T(t')$
-1.0	$-\infty$	∞	∞	∞
-0.8	-0.2554	6.1812	3.1018	3.0103
-0.6	3.9060	4.0295	2.0450	2.0288
-0.4	6.0191	3.0657	1.5881	1.5821
-0.2	7.2504	2.4547	1.3129	1.3091
0.	7.9026	2.0039	1.1236	1.1206
0.2	8.0852	1.6367	0.9829	0.9795
0.4	7.8193	1.3127	0.8728	0.8699
0.6	7.0427	1.0027	0.7838	0.7813
0.8	5.5040	0.6666	0.7103	0.7075
1.0	1.7886	0.1368	0.6485	0.6450
1.5	0.6176	0.0057	0.5280	0.5240
2.0	0.5123	0.0052	0.4404	0.4367
2.5	0.4333	0.0022	0.3727	0.3707
3.0	0.3690	-0.0021	0.3187	0.3186
3.5	0.3144	-0.0071	0.2746	0.2771
4.0	0.2665	-0.0111	0.2384	0.2428
4.5	0.2251	-0.0126	0.2090	0.2149
5.0	0.1914	-0.0108	0.1856	0.1910
7.5	0.1482	0.0252	0.1267	0.1150

Fig.4 Functions $F_1(t'), F_2(t'), F_3(t')$

indicated in Table 1 in order to confirm the precision of the calculation for Eq.(13). $F_1(t')$ and $F_2(t')$ are tabulated in Table 1, too.

The unsteady lift acting on the blade due to the passing wake can be obtained by Eq.(12), but these equations involve a very large amount of calculation. In order to reduce the amount of calculation, t_h in these equations are replaced with x_t , which are the fixed points on the x -axis downstream from the leading edge of the blade (see Fig.5). Then, Eq.(12) is rewritten as

$$\Delta L_{u1} = -\rho U f \cos \beta \int_{x_t=-l}^{\infty} w(x_t, t) F_1(x_t) \Delta x_t \dots \dots \dots (14.a)$$

$$\Delta L_{u2} = -\rho U a \cos \beta \int_{x_t=-l}^{\infty} w(x_t, t) F_2(x_t) \Delta x_t \dots \dots \dots (14.b)$$

$$\Delta L_v = \rho U \sin \beta \int_{x_t=-l}^{\infty} w(x_t, t) F_3(x_t) \Delta x_t \dots \dots \dots (14.c)$$

Using the above equations, ΔL can be obtained by changing $w(x_t, t)$ at the positions x_t . Thus, $F_1(x_t) \sim F_3(x_t)$ can be used repeatedly. In the representation of the calculated results, the unsteady lift coefficient ΔC_L is used.

$$\Delta C_L = \Delta L / \left(\frac{1}{2} \rho U^2 c \right) \dots \dots \dots (15)$$

Fig.5 Velocity defect $w(x_t, t)$ at fixed point x_t

Fig.6 Wake position at time t'

2.3 Unsteady velocity due to wake

In the first step to take account of reduction in the wake velocity defect varying with distance from the cylinder, $w(x_t, t)$ at x_t is expressed as follows.

As shown in Fig.6, the parallel distance X from the center of cylinder to the fixed point x_t is

$$X = \left[\left(1 + (t^* + 1) \frac{c}{2} \right) \sec \beta + (x_t^* + t^*) \frac{c}{2} \cos \beta \right] \dots \dots \dots (16)$$

Fig.7 Measuring positions X_n of wake velocity defect w_n

Fig.8 Cylinder wake velocity profile

- ① Pitot-static tube
- ② Yawmeter
- ③ Cylinder $d=\phi 15$
- ④ Blade NACA65(75)10
- ⑤ Pressure transducer
- ⑥ Amplifier
- ⑦ Data processing device
- ⑧ Photoelectric pickup
- ⑨ Tachometer

Fig.9 Experimental apparatus

Also the perpendicular distance Y from the center line of the wake to x_t can be written as

$$Y = (x_t^* - t^*) c \sin \beta / 2 \dots \dots \dots (17)$$

Here, X_n is the parallel distance from the center of cylinder and w_n is the velocity defect at the same perpendicular distance from the center line of the wake, as shown in Fig.7. Thus $w_x(y)$ at X is represented in three ways

$$\left. \begin{aligned} w_x(y) &= w_1 & \text{for } X < X_1 \\ &\text{linear reduction} & \text{for } X_1 \leq X < X_n \\ w_x(y) &= w_n & \text{for } X \geq X_n \end{aligned} \right\} \dots \dots (18)$$

Then, for the range $X_{n-1} \leq X < X_n$, $w_x(y)$ can be written as

$$w_x(y_h) = \left\{ (w_n - w_{n-1}) \frac{X - X_{n-1}}{X_n - X_{n-1}} w_n + X_{n-1} w_{n-1} \right\} / (X_n - X_{n-1}) \dots \dots (19)$$

Similarly, $w(x_t, t)$ at the point x_t with the distance Y from the center line of the wake can be written by using $w_x(y_h)$ as follows:

$$w(x_t, t) = \left[\{ w_x(y_h) - w_x(y_{h-1}) \} Y - y_{h-1} w_x(y_h) + y_h w_x(y_{h-1}) \right] / (y_h - y_{h-1}) \dots \dots (20)$$

In our calculation, $w_1, w_2, \dots, w_n$ are evaluated as experimental values on the wake velocity profile shed by a stationary cylinder in the airstream. The Reynolds number $Re (= Ud/\nu)$ is 2×10^4 . The measuring locations X_n were taken such that the difference between the measured values and $w_x(y_h)$ calculated from Eq.(19) was within 1% of the main stream velocity U .

3. Calculated and experimental results

A schematic of the apparatus is shown in Fig.9. In the experiments, the blade stagger angle is adjusted to -6.64° to make the outflow axial. The blade profile used is NACA65(7.5)10 (chord length $c=150$ mm). There are 12 blades in the stator blade row, which gives a space/chord ratio of 0.83. Two cylinders of diameter $d=\phi 15$ mm are mounted on a wheel. The unsteady pressure acting on the blade is measured by a semi-conductor pressure transducer. The output signal of the pressure transducer is sampled and averaged 27 times periodically and then, by integrating these pressure values, the unsteady lift is obtained⁽⁶⁾. In the theoretical calculations, f in Eq.(5) would be 0.08272. Figure 10 shows a comparison between the camber line of NACA65(7.5)10 and the one described by Eq.(5).

Figure 11 shows the experimental results and the calculated ones for the case when the circumferential velocity of cylinder u_b is 3 m/s and the axial distance between the center of cylinder and the leading edge of blade l/d is 2.5. The velocity defect perpendicular to the blade chord has the same influence as an increase in angle of attack. Therefore, the unsteady

lift coefficient ΔC_{L_v} is positive when the wake reaches the blade, as plotted in Fig. 11. As the velocity defect parallel to the blade chord has the same influence as a decrease in the stream velocity, $\Delta C_{L_{u_1}}$ due to the camber is negative. But, because the angle of attack is negative in our study, $\Delta C_{L_{u_2}}$ is positive. $\Delta C_{L_{u_2}}$ is almost in

phase with ΔC_{L_v} and reaches a peak at $t' = -0.5$. While, $\Delta C_{L_{u_1}}$ lags and its amplitude takes a maximum at $t' = 0$. Therefore, $\Delta C_{L_{sum}}$ as the sum total of three values is maximum at $t' = -1$. As t' is increased, it decreases rapidly and then becomes negative. In the experimental results, ΔC_L becomes negative at $t' = 8 \sim 12$. This is the reason why ΔC_L on the neighboring blade is positive.

As shown in Fig. 11, taking account of the effects of angle of attack and camber due to the velocity defect parallel to the blade chord, the trend of the calculated results agrees fairly well with the experimental ones. But there is a discrepancy between both results as before. In order to make clear the cause of the discrepancy, Figs. 12 and 13 show the three-dimensional unsteady velocity distributions around the blade at A and B indicated by the arrow marks in Fig. 11. This measurement utilizing a hot-wire anemometer was carried out at the mean radius ($r = 237.5 \text{ mm}$)⁽⁶⁾. These figures indicate the deviation of velocity components from the steady state. The unsteady axial and tangential velocity components (ΔV_x and ΔV_θ) are shown by the vector that indicates the velocity magnitude ($\sqrt{\Delta V_x^2 + \Delta V_\theta^2}$) and the direction. The starting ends of the arrow mark correspond to the measuring stations. The unsteady radial velocity component ΔV_r is indicated by the symbol $\bullet$. In these figures, the observed blade is the one in

Fig. 10 Comparisons of camber lines

Fig. 11 Comparisons of unsteady lift coefficients ΔC_L

Fig. 12 Three-dimensional unsteady velocity distribution (A)

Fig.13 Three-dimensional unsteady velocity distribution (B)

the center.

It can be seen from Fig.11 that the peak value of the experimental unsteady lift coefficient ΔC_L is fairly larger and appears earlier than the calculated one ($\Delta C_{L, sum}$). As shown in Fig.12, there is little change in the flow on the upper side of the observed blade at the time indicated by the arrow mark A in Fig.11. While, on the lower side, the unsteady axial velocity component ΔV_x is negative because of the influence of the cylinder wake. Also, on the lower side in the neighborhood of the leading edge, the width of wake is very small and the velocity in it decreases as if the wake is pushing against the blade surface. Furthermore, it is apparent that the blade wake shifts in the direction of the cylinder wake, since ΔV_x becomes positive behind the observed blade. Consequently, in this instance, the experimental results show maximum values because the pressure on the lower side increases considerably.

At the time indicated by the arrow mark B in Fig.11, the experimental results do not become negative unlike the calculated ones. The cause of this discrepancy can be seen from Fig.13, which shows the unsteady velocity distributions in this instance. In Fig.13, the cylinder wake reaches both sides of the blade and it has a influence on the whole lower side. However, on the upper side, it has not a influence on the back portion of the blade. Therefore, the unsteady pressure on the

Fig.14 Comparisons of unsteady lift coefficients ΔC_L

lower side becomes positive continuously and on the upper side changes lately. Consequently, it can be considered that the experimental results are positive and then do not become negative.

Figure 14 shows a comparison between the experimental results and the calculated ones for $l/d=6.0, 14.0$. In this figure, the discrepancy between both results is reduced if l/d becomes large. It can be seen that the phenomenon, which the width of wake decreases as if the wake is pushing against the blade surface, appears clearly when l/d is small.

For reference, the comparison between the experimental results and the calculated ones for $u_b=24\text{m/s}$ is shown in Fig.15. This experiment was carried out for the case when the number of blades was 4 (space/chord ratio was 2.49 at the mean radius), the blade stagger angle was -7.11° and $l/d=6.0$. It can be seen that the trend of the calculated results agrees fairly

well with the experimental ones in the range of large values of u_b , even though the influence of the velocity defect parallel to the blade chord is neglected. If we take account of the effects of angle of attack and camber due to the velocity defect parallel to the blade chord, the calculated results agree well with the experimental ones all the more. Discussions about the induced mechanism of the unsteady force for large value of u_b have been made in the previous report⁽⁸⁾.

4. Conclusions

Theoretical calculations were made on the unsteady force induced on a cambered blade with angle of attack due to the wake shed by an upstream cylinder moving perpendicular to a free-stream. The calculated results were compared with the experimental ones for large values of the velocity defect parallel to the blade chord. The reason why discrepancies arise, was considered by the experimental data of the three-dimensional unsteady velocity distribution around the blade. Summarized in the following are the conclusions from the discussions.

(1) Taking account of the effects of angle of attack and camber due to the velocity defect parallel to the blade chord, the calculated results agree well with the experimental ones.

(2) The unsteady force due to the velocity defect perpendicular to the blade chord is almost in phase with it due to the angle of attack, but the one due to the camber lags. Therefore, the total unsteady force decreases rapidly after it reaches a peak.

(3) When the cylinder wake reaches the blade, the width of wake becomes smaller as if the wake is pushing against the lower surface of the blade.

(4) The blade wake shifts in the direction of the cylinder wake due to the interaction between the blade and the cylinder wake.

(5) Owing to the phenomena described in

Fig.15 Comparisons of unsteady lift coefficients ΔC_L

(3) and (4), the experimental unsteady force is fairly larger and appears earlier than the calculated one.

(6) In consequence of the phenomenon described in (3), the cylinder wake reaches the upper side lately.

(7) After taking a peak value, the experimental results do not become negative unlike the calculated ones because of the phenomenon described in (6).

(8) The phenomenon described in (3) appears intensely for small distance between the cylinder and the blade.

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