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Transient Flow Caused by a Rotationally Decelerated Disk Enclosed in a Housing*

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The transient flow caused by a rotationally decelerated disk enclosed in a housing is studied theoretically and experimentally, in order to determine the influence of non-steadiness on the flow at the stopping period of the disk. The main results obtained are as follows:

During the whole deceleration period there exists, despite the deceleration-rate, a core region similar to that of the steady-state with uniformly decreasing tangential velocity, and its flow pattern is well approximated by a forced vortex. This uniform deceleration of the core is mainly due to the secondary flow in the boundary layer while the disk speed is larger than the core velocity, and due to the radial outward flow induced in the core when the disk speed becomes smaller than the core velocity.

The theoretical analysis based on the assumption of a forced vortex gives satisfactory results, though the steady-state characteristics of the boundary layer are used.

1. Introduction

One of the serious problems associated with the transient operation of radial and mixed flow pumps is transient axial thrust, which acts in the opposite direction to that of the steady operation, and sometimes causes great damage to the machine.

Fang⁽¹⁾ and Miyashiro⁽²⁾ have given a reasonable description of this phenomena, but its cause is not yet clear. In order to determine the cause of the phenomena, it is necessary to examine the transient flow in the back spaces of an impeller at the starting and the stopping periods of the machine.

Two of the present authors have investigated the transient flow⁽³⁾ caused by the rotationally accelerated disk enclosed in a casing, and have determined the mechanism of the transient flow at the disk start-up. The main results were as follows:

(1) Soon after the initiation of disk rotation, not only the fluid in the disk boundary layer but the fluid in the endwall boundary layer begin to rotate, and between them a core region similar to that of the steady-state is formed due to the effect of the angular-momentum transfer by the secondary flow in the boundary layers. The rotation of the core fluid is dominant over the whole phenomena.

(2) The theoretical analysis gives satisfactory results, though the steady-state characteristics of the boundary layer are

assumed.

In the present report, the rotationally decelerated flow at the stopping period of the disk is examined theoretically and experimentally.

2. Nomenclature

- a, a', b, c : experimental constants, Eq. (23)
- c_p : pressure coefficient, Eq. (25)
- c_T : axial force coefficient, Eq. (26)
- $K' = \beta/\omega_0 = U/r\omega_0$: core rotation parameter
- L : angular-momentum of fluid, Eq. (2)
- M : wall friction moment, Eqs. (3)~(5)
- p : pressure
- r : radius
- s : axial space between disk and end-wall
- t : time
- u, v, w : tangential, radial and axial velocity components in boundary layers, respectively, Eqs. (7)~(12)
- U, V : tangential and radial velocity components of core fluid, Eq. (6)
- X : radial distance from outer cylindrical casing, Eq. (13)
- z, z' : axial distances from endwall and disk, respectively
- β : angular velocity of core fluid
- ϑ, δ, ξ : boundary layer thicknesses on end-wall, disk and cylindrical casing wall, Eqs. (15), (16) and (19)
- ϵ : annular gap at the disk tip
- τ : wall shearing stress, Eq. (14)
- ν : kinetic viscosity of fluid
- ω : disk angular velocity
- ρ : fluid density

Subscripts

- R, S, C : representing the rotating disk, the stationary endwall and the cylindrical casing wall, respectively.
- θ : tangential component
- 0 : steady-state
- 1 : disk outer periphery

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3. Theoretical analysis

Now we consider the axisymmetric transient flow along a rotationally decelerated disk enclosed in a cylindrical housing, as shown in Fig. 1, during the period from the steady rotation until the still-state of the fluid. The balance of angular-momentum of the fluid in the casing at each moment is given as

$$\frac{dL}{dt} = M_R - M_S - M_C \quad (1)$$

where the angular-momentum L of the fluid and the friction moments M_R , M_S and M_C of the disk, the stationary endwall and the outer cylindrical casing wall, respectively, are given as follows (see Fig. 1).

$$L = \int_0^{r_1+\epsilon} \int_0^S 2\pi r^2 u \, dz \, dr \quad (2)$$

$$M_R = \int_0^{r_1} 2\pi r^2 \tau_{\theta R} \, dr \quad (3)$$

$$M_S = \int_0^{r_1+\epsilon} 2\pi r^2 \tau_{\theta S} \, dr \quad (4)$$

$$M_C = \int_0^S 2\pi (r_1+\epsilon)^2 \tau_{\theta C} \, dz \quad (5)$$

The boundary layer flow might undergo a transition from the turbulent to the laminar just before the stop of rotation. But when the steady-state Reynolds number Re_0 ($= r_1^2 \omega_0 / \nu$) is larger than 10^6 , which is usually encountered in the operation of turbomachine, the period of laminar flow is much shorter than that of turbulent flow. Therefore, turbulent flow is dominant over the phenomena, and we assume that the flow is turbulent throughout the whole period.

3.1 Consideration of flow characteristics

When a disk is steadily rotating in a cylindrical casing, in which the axial spacing s between the disk and the endwall is not too small, there exists a core region, in which the tangential velocity U is nearly constant in the axial direction and the flow pattern is well approximated by a forced vortex ($\beta_0 = \text{const.}$). In the case of the transient flow along a rotationally accelerated disk at the start-up, a core region similar to that of the steady-state is formed soon after the initiation of rotation, as mentioned above, and is maintained until the steady rotation. The flow in the core gives dominant effect on the characteristics of the transient flow⁽³⁾. Therefore, in the case of the transient flow along the rota-

tionally decelerated disk at the stopping period, it is also expected that a core region similar to that of the steady-state is maintained during the whole deceleration period till the still-state.

When the deceleration-rate of the disk is small enough to be approximated to the quasi-steady-state, the core fluid will maintain the flow pattern of a forced vortex during the deceleration period. But when the rate of the disk deceleration is relatively large, the flow condition in the core will differ largely from that of the quasi-steady-state.

During the period from the initiation of deceleration till the time of $\omega = \beta$, the radial outward flow in the disk boundary layer and the inward flow in the endwall boundary layer will decrease with a decrease of the disk rotation. But as the inertia of the boundary layer fluid is much smaller compared with that of the core fluid, the outward flow in the disk boundary layer will decrease much faster than the inward flow in the endwall boundary layer, which results in that the radial outward flow is induced in the core to satisfy the continuity equation.

When the disk speed further decreases and becomes lower than the core velocity ($\omega < \beta$), the radial velocity in the disk boundary layer changes its direction from the outward to the inward, and the fluid in both the disk and the endwall boundary layers flows inward. In this case the radial outward flow-rate in the core becomes equal to the sum of the inward flow-rates in both boundary layers, and the equilibrium of the forces acting on the fluid particle in the disk boundary layer might be similar to that in the endwall boundary layer.

After all, during the whole period from the initiation of deceleration until the still-state of the fluid, the radial outward flow will be induced in the core. This outward flow takes the inner fluid of lesser angular-momentum to the outer region and mixes it with the outer fluid of larger angular-momentum, which results in a decrease of the tangential velocity of outer fluid. Thus, mainly due to the outward secondary flow induced in the core, the rotational speed of the core fluid will decrease uniformly in the radial direction with a decrease of the disk speed.

According to the above-described consideration, and also for the sake of simplicity of the analysis, we assume that the core fluid rotates as a forced vortex during the whole deceleration period, and that the outward flow induced in the core is uniform in the axial direction.

3.2 Distributions of velocities, shearing stresses and boundary layer thicknesses

In the analysis of the transient flow caused by the rotationally accelerated disk, satisfactory results were obtained even at a rapid start-up, with the assumptions that the distributions of velocities and shearing stresses in the transient state are expressed as in the steady-state, shown in the following equations (6)~(12) and (14)⁽³⁾. It is then also expected that the expressions

Fig. 1 Flow along an enclosed rotating disk at the stopping period

given below are applicable to the rotationally decelerated flow.

Core region:

$$U = r\beta, \quad V = V(r) \quad (6)$$

Disk boundary layer:

$$u = r\omega\{1-(z'/\delta)^{1/7}\} + r\beta(z'/\delta)^{1/7} \quad (7)$$

$$v = a'r(\omega-\beta)(z'/\delta)^{1/7}(1-z'/\delta) \quad (8)$$

Stationary endwall boundary layer:

$$u = r\beta(z/\delta)^{1/7} \quad (9)$$

$$v = -ar\beta(z/\delta)^{1/7}(1-z/\delta) \quad (10)$$

Cylindrical casing wall boundary layer:

$$u = [r\beta]_{X=\xi}(X/\xi)^{1/7} \quad (11)$$

$$w = \hat{w}(X/\xi)^{1/7}(1-X/\xi) \quad (12)$$

where X is the distance from the cylindrical wall and is given as

$$X = r_1 + \varepsilon - r \quad (13)$$

The shearing stresses are given by the Blasius formula:

$$\tau = c_p[U_{rel}^2(\nu/U_{rel}z)^{1/4}]_{z \rightarrow 0} \quad (14)$$

where U_{rel} is the relative velocity of the fluid to the wall, and z is the distance from the wall.

The thickness of the disk boundary layer is assumed as follows so that it is equal to the expression by Schultz-Grunow⁽⁴⁾ for the steady-state ($\omega = \omega_0$, $\beta = \beta_0$):

$$\delta = b(\beta_0/\omega_0)^{11/5}(\omega/\beta_0 - 1)^2 r / (\beta r^2/\nu)^{1/5} \quad (15)$$

The thickness of the endwall boundary layer is also similarly expressed as

$$\delta = fr / (\beta r^2/\nu)^{1/5} \quad (16)$$

where the function f is to be determined from the equation of continuity:

$$\int_0^\delta v dz = 0 \quad (17)$$

Considering that the radial velocity of the core is zero in the steady-state, we assume the following expression for the function f :

$$f = a'b(\beta_0/\omega_0)^{11/5}(\omega_0/\beta_0 - 1)^3/a \quad (18)$$

The boundary layer thickness ξ on the cylindrical casing wall might vary little in the axial direction, and then we assume that ξ is constant in the axial direction and is equal to that of the endwall boundary layer thickness at $r = r_1$. Thus,

$$\xi = fr_1/(\beta r_1^2/\nu)^{1/5} \quad (19)$$

The parameter $\hat{w}$ in Eq.(12) is also to be determined by the equation of continuity (17) at the outer edge of the boundary layer on the cylindrical casing wall:

$$\frac{\partial}{\partial z} \left[\int_0^\xi w dX \right] + [V]_{X=\xi} = 0 \quad (20)$$

together with the continuity condition at the intersecting plane of the outer edges of the boundary layers on the endwall and the cylindrical wall:

$$\left[\int_0^\xi w dX \right]_{z=\delta} + \left[\int_0^\delta v dz \right]_{X=\xi} = 0 \quad (21)$$

In Eqs.(20) and (21), the variation of radius r is small enough to be approximated constant in the range of $0 < X < \xi$, and the parameter $\hat{w}$ is obtained as a linear function of z .

The radial velocity in the core region is also determined from the equation of continuity as

$$V(s-\delta) = \int_0^\delta v dz + \int_0^\delta v' dz' \quad (22)$$

In the above described equations, a , a' , b and c are the experimental constants, which are given as follows in the steady-state⁽⁵⁾:

$$a = 0.374, \quad a' = 0.220, \quad b = 0.400, \quad c = 0.0256 \quad (23)$$

These values have given satisfactory results in the analysis of the transient flow caused by a rotationally accelerated disk⁽³⁾, and then they might be applicable to the present analysis.

3.3 Numerical calculations

Substituting Eqs.(6)~(23) into Eq.(1) yields the ordinary differential equations as for the angular velocity β as a function of time t . In the first step of the calculation, the steady-state solution β_0 is obtained by putting $d/dt = 0$, and then using the steady-state solution as the initial value, the time dependence of the angular velocity ratio β/ω_0 is calculated with the aid of the Runge-Kutta method.

Pressure distribution in the radial direction at each moment might be determined mainly by the balance of the forces in the core region. The radial velocity in the core is small enough to be neglected in comparison with the tangential velocity, and therefore the pressure distribution is determined by

$$dp/dr = \rho r \beta^2 \quad (24)$$

Here, we define the pressure coefficient c_p and the axial force coefficient c_T as follows:

$$c_p = (p-p_1)/\frac{1}{2}\rho r_1^2 \omega_0^2 \quad (25)$$

$$c_T = \frac{\int_{r_1}^{r_2} 2\pi r(p-p_1) dr}{\pi \rho r_1^4 \omega_0^2} \quad (26)$$

where the lower limit of the integration r_1 in the definition of c_T is chosen to be $0.23r_1$ so as to compare with the experimental results.

4. Experimental equipment and procedures

General view of the test stand is shown in Fig.2. The inside surface of the cylindrical casing ① is smoothly finished and has nine static pressure holes and two holes for velocity measurements. The diameter of the smooth bronze disk ④ is 300 mm. The pressure distributions were measured by the electric pressure transducers attached on the casing endwall. The velocity and flow angle distributions were measured by a 3-hole-cobra probe in which three small pres-

sure transducers are mounted⁽⁵⁾.

The disk is driven by a variable speed DC-motor, in which an electro-magnetic brake system is assembled. Between the motor⁽³⁾ and the disk assemblies⁽¹⁾ and ⁽⁴⁾, the electro-magnetic clutch-brake⁽²⁾ is coupled, by which the rate of the disk deceleration is varied.

Three different methods of the disk deceleration were used:

(1) The motor rotates in the steady-state and the electro-magnetic clutch⁽²⁾ is suddenly released (called "slow deceleration").

(2) The clutch⁽²⁾ is suddenly released with the electro-magnetic brake⁽²⁾ put on at the same time (called "fast deceleration").

(3) The input power of the motor is suddenly cut off with the motor-brake put on at the same time (called "intermediate deceleration").

The time dependence of the disk angular velocity ω in each case is shown in Fig. 3. The time from the initiation of deceleration until the still-state was 10.5, 0.3 and 3.5 sec. in average, respectively.

The rotational speed of the disk in the steady-state is 1400 rpm, corresponding to the steady-state Reynolds number $Re_0 = 3.3 \times 10^6$. All measurements were performed with the axial spacing $s = 11.2$ mm between the disk and the endwall, and the annular gap $c = 0.5$ mm at the disk tip. The pressure in the center of the chamber was kept constant with the head tank⁽⁵⁾.

5. Experimental results and comparison with theory

5.1 Velocity distribution

Measured velocity distribution in each moment is illustrated in Figs. 4(a) and (b) in the case of the fast and the slow decelerations. It is clearly shown that the core region, in which U is nearly constant in the axial direction and V is very small, is maintained throughout the whole period from the steady rotation ($t=0$) until the still-state of the fluid. In the case of the slow deceleration, the disk speed is larger than the fluid rotational speed during the whole period, but when the fluid reaches the still-state ($t \approx 10$ sec.). However, in the case of the fast deceleration, the disk reaches the still-state much faster than the fluid. At $t = 0.2$ sec., the disk speed becomes nearly equal to the fluid rotational speed, at $t = 0.3$ sec., the disk stops and thereafter the disk works as a brake to the fluid. Accordingly, the centrifugal force acting on the fluid particle in the disk boundary layer decreases rapidly and becomes inferior to the radial pressure gradient, which results in that the radial outward flow in the disk boundary layer changes its direction to the inward. Therefore from the continuity condition, the radial outward flow is induced in the core region.

This outward flow has a role of transporting the inner fluid of lesser angular-momentum to the outer radii, which results in that the whole fluid in the core is decelerated, as described in Sec. 3.1. The theoretical results are also shown in Figs. 4(a) and (b) and are in satisfactory agreement with the measurements in both cases.

In order to have more detailed knowledge about the secondary flow induced in the

Fig. 2 Test stand

Fig. 3 Time dependence of disk speed

(a) Fast deceleration (b) Slow deceleration
Fig. 4 Velocity distribution in the space between the disk and the endwall at each moment

Fig. 5 Time dependence of radial velocity components in the boundary layers, and comparison with core velocity in the case of fast deceleration

boundary layers, its radial velocity component is compared with the core rotational velocity U in Fig. 5. $\bullet$ stands for the velocity in the endwall boundary layer and $\square$ in the disk boundary layer, and they are compared with the core velocity O . The core rotation ratio $U/r\omega$ changes little in the range of $t < 0.1$, which means that the core velocity decreases nearly in proportion to the disk speed just after the initiation of disk deceleration. This rapid decrease of the core speed is due to the effect of the secondary flow. The angular-momentum transferred by the secondary flow from the disk boundary layer to the endwall boundary layer decreases with a decrease of the disk speed, which results in the rapid deceleration of the core velocity from the surroundings⁽³⁾.

When the time becomes larger than 0.1 sec., the increase of $U/r\omega$ becomes rapid. The leading part in decelerating the core rotation shifts from the secondary flow in the boundary layer to that induced in the core, and the rate of the deceleration of the core rotation becomes lesser, which results in a rapid increase of the core rotation ratio $U/r\omega = \beta/\omega$. When the disk speed further decreases to be equal to that of the fluid ($U/r\omega=1$), the radial flow in the disk boundary layer becomes zero, and thereafter changes to the radial inward flow. In the endwall boundary layer the inward flow decreases gradually with a decrease of the disk speed.

The radial distribution of the core rotation parameter $K' = U/r\omega_0 = \beta/\omega_0$ is illustrated in Fig. 6, in which the lines show the theoretical results. K' is somewhat larger in the outer radii than in the inner radii just after the initiation of deceleration, but at the latter part of deceleration it becomes somewhat smaller in the outer radii. Nevertheless, this figure shows that it is adequately assumed that the core fluid rotates as a forced vortex ($K' = \text{const. for } r$).

The time dependence of the core rotation parameter K' is shown in Fig. 7. The solid lines are the theoretical results and are in good agreement with the measurements. The dotted lines are the quasi-steady-state solutions, in which the non-steady acceleration

Fig. 6 Radial distribution of core rotation parameter ($K' = \beta/\omega_0$)

term dK'/dt in the basic equation is treated as zero, regardless of the deceleration-rate. The larger the deceleration is, the greater the difference becomes between the non-steady solutions and the quasi-steady-state ones. This is because the non-steady acceleration term (inertia term) is disregarded in the quasi-steady-state solutions, though the fluid still rotates due to its inertia after the stop of the disk rotation. In the case of the slow deceleration, the quasi-steady-state solutions seem to be in better agreement with the measurements, but this comes from the difference of the initial values between the theory and the measurements.

5.2 Pressure distributions

The time dependence of the pressure coefficient c_p measured at the radial position of $r/r_1 = 0.23$ is shown in Fig. 8. The non-steady-state solutions are seen to give fairly good results in every deceleration condition, although the quasi-steady-state ones give unsatisfactory results especially in the case of the fast deceleration.

Fig. 7 Time dependence of core rotation parameter K' and comparison with theory

Fig. 8 Time dependence of pressure coefficient c_p

The radial pressure distribution in each moment is shown in Figs. 9(a) and (b) in the case of the slow and the fast decelerations, respectively. In both cases the non-steady-state solutions are in good agreement with the measurements, which shows that the assumption of a forced vortex approximates adequately the behaviour of the transient flow in the core region during the whole deceleration period.

5.3 Axial force

Figure 10 shows the time dependence of the axial force acting on one side of the disk. The non-dimensional form of the axial force is defined by Eq. (26), and the non-steady-state solutions and the quasi-steady-state ones are also compared in the figure. The time dependence of c_T is seen to be very similar to that of c_p , and we recognize that the followability of pressure and axial force to the disk speed is very good.

From these figures we may think that the pressure in the back spaces of an impeller and the axial force acting on the back side of an impeller shroud decreases very soon with a decrease of the impeller speed.

(a) Slow deceleration (b) Fast deceleration

Fig. 9 Radial pressure distribution at each moment

Fig. 10 Time dependence of axial force coefficient c_T

6. Conclusions

The transient flow caused by a rotationally decelerated disk enclosed in a housing is examined theoretically and experimentally. The results obtained are summarized as follows:

- (1) During the whole deceleration period, a core region similar to that of the steady-state is maintained with uniformly decreasing tangential velocity, when the axial space is not too small and the steady-state Reynolds number Re_0 is larger than 10^6 . The flow pattern of the core is very well approximated by a forced vortex during the whole deceleration period.
- (2) The uniform deceleration of the core velocity is, despite the deceleration-rate, mainly due to the effects of the secondary flow induced in the boundary layers on the disk and the endwall during the period from the initiation of deceleration until the disk speed becomes equal to the core rotational velocity. But thereafter the uniform deceleration of the core is mainly due to the radial outward flow induced in the core region.

These secondary flows transfer lesser angular-momentum from the disk boundary layer or from the inner radii of the core into the region of larger angular-momentum of the endwall boundary layer or into that of the outer radii, respectively.

- (3) The non-steady-state solutions give satisfactory results even for the fast deceleration, though the steady-state characteristics of the boundary layer and a forced vortex flow pattern are assumed.

The larger the rate of deceleration is, the greater the difference between the non-steady-state solutions and the quasi-steady-state ones becomes.

- (4) In the case of the fast deceleration, the radial outward flow in the disk boundary layer changes its direction to the inward, when the disk speed becomes equal to the core rotational velocity.

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