533.6.011

Paper No. 195-15

Thermally Induced Acoustic Oscillations in a Pipe*

(1st Report : Oscillations Induced by

Plane Heat Source in Air Current)

By Haruki MADARAME**

Thermally induced acoustic oscillations in a pipe have been studied analytically and experimentally. Temperature distributions have been obtained near a heat source in a uniform flow with an acoustic field by solving heat-conduction equations. The temperature change of gas causes a volumetric change, and it may convert thermal energy to acoustic energy under certain conditions which have already been clarified only qualitatively. Using the calculated temperature distributions, the amount of converted energy in a cycle has been computed and proved to be a function of the heat source location, the current velocity, the frequency and the thermal diffusivity of the fluid. Experimental results agreed well with the analysis where the modified current velocity is used considering the boundary-layer around the heater. Some other characteristics of the oscillation have been discovered; for example, the growth rate of oscillation changes when turbulent transition occurs over a certain amplitude.

Subsequently, experimental results are shown and compared with the theory for discussion.

1. Introduction

An acoustic oscillation may be induced in a pipe in which a heat source is installed and an air current exists. It is a well-known phenomenon called "a singing flame" or "a gauze tone". Saito⁽¹⁾ calculated the acoustic energy converted from a thermal energy using a linearized temperature variation near the heat source. He succeeded in explaining the interesting fact that the position of heat source is an important factor in the growth of oscillation. This has clarified the mechanism of the phenomenon but only qualitatively. Recently, Katto et al.⁽²⁾⁽³⁾ found that the oscillation can be induced without the air current on condition that a cooler is placed near the heater and the temperature gradient between them is beyond a certain value which is a function of the pipe length. Besides they showed experimentally that the oscillation growth in the pipe with a heater and an air current depends on the flow rate, the pipe length and so forth. The author⁽⁴⁾ previously found from calculation that the energy supply to the oscillation in the pipe without air current as shown in Fig.1 is in proportion to

$$\theta \sin \frac{\pi l_1}{l} \cos \frac{\pi l_1}{l} \operatorname{Im} \left\{ \frac{(e^{\sqrt{i\omega/a\delta}} - 1)(e^{-\sqrt{i\omega/a\delta}} - 1)}{\sqrt{i\omega/a\delta} \sinh(\sqrt{i\omega/a\delta})} \right\}$$

which explains the experimental result fairly well. In this paper, the energy supply in the case with air current is calculated.

Fig.1 Tube without air current

2. Analysis

2.1 Temperature change in a pipe

Fig.2 Tube with air current

Fig.2 shows an air current with velocity u_0 through a pipe of length l in which a heater is installed at the distance l_1 from the inlet. It is assumed that the heater can be regarded as an ideal plane heat source, that is, it is so small that the perturbation effect on the flow can be neglected and its temperature is constant. The acoustic oscillation in the pipe is described by Eqs.(1) and (2):

$$\tilde{u} e^{i\omega t} = K \theta_0 \cos(n\pi \frac{x}{l}) e^{i\omega t} \quad (1)$$

$$\tilde{p} e^{i\omega t} = -i K k p_0 \sin(n\pi \frac{x}{l}) e^{i\omega t} \quad (2)$$

where $\tilde{u}$ and $\tilde{p}$ are fluctuating components of velocity and pressure, K indicates a dimen-

* Received 1st October, 1979.

** Associate Professor,
Faculty of Engineering,
University of Tokyo
7-3-1 Hongo Bunkyo-ku Tokyo

sionless constant representing the amplitude of oscillation, a_0 is the sound velocity, p_0 is the external pressure, κ is the ratio of specific heats and n denotes a modal number. The temperature distribution in the pipe is derived from the following equation:

$$\frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial x^2} - (u_0 + \tilde{u} e^{i\omega t}) \frac{\partial \theta}{\partial x} \quad (3)$$

where α is the thermal diffusivity of air. The boundary conditions are $\theta = \theta_0$ at $x = l_1$ and $\theta = 0$ at $x \rightarrow -\infty$, where θ_0 is a constant. Assuming a solution of the form:

$\theta(x, t) = \theta_0(x) + \theta_1(x) e^{i\omega t} + \theta_2(x) e^{i2\omega t} + \dots$ (4)
the static temperature θ_0 can be easily obtained in the form:

$$\theta_0 = \theta_0 \exp\left\{\frac{u_0}{\alpha}(x - l_1)\right\} \quad (x \leq l_1) \\ \theta_0 = \theta_0 \quad (x > l_1) \quad \left. \vphantom{\theta_0 = \theta_0 \exp\left\{\frac{u_0}{\alpha}(x - l_1)\right\}} \right\} \quad (5)$$

The fluctuating component of temperature with the fundamental frequency θ_1 is derived from Eq. (6).

$$i\omega\theta_1 = \alpha \frac{d^2\theta_1}{dx^2} - u_0 \frac{d\theta_1}{dx} - \tilde{u} \frac{d\theta_0}{dx} \quad (6)$$

Substituting Eqs. (1) and (5) into Eq. (6), Eq. (7) is obtained in the range $x \leq l_1$, the solution of which is given by Eq. (8).

$$\frac{d^2\theta_1}{dx^2} - \frac{u_0}{\alpha} \frac{d\theta_1}{dx} - i\omega\theta_1 = \frac{\kappa c_0 u_0 \theta_0}{\alpha^2} \cos \frac{n\pi x}{l} \exp\left\{\frac{u_0}{\alpha}(x - l_1)\right\} \quad \dots (7)$$

$$\theta_1 = K \frac{c_0}{\sqrt{a\omega}} \frac{-UL^2}{U^2 L^2 + (1 + iL^2)^2} \left[\left\{ UL \sin \xi_1 - (1 + iL^2) \cos \xi_1 \right\} \exp\left\{ \frac{1 + \sqrt{1 + i4/U^2}}{2} UL(\xi - \xi_1) \right\} - \left\{ UL \sin \xi - (1 + iL^2) \cos \xi \right\} \exp\{UL(\xi - \xi_1)\} \right] \quad \dots (8)$$

where $U = u_0/\sqrt{a\omega}$, $L = \sqrt{\omega/a}/n\pi$, $\xi = n\pi x/l$, $\xi_1 = n\pi l_1/l$. In the range of $x > l_1$, θ_1 is invariably 0. Fig. 3 shows the temperature fluctuations in a dimensionless form. When the air current velocity is low and the acoustic frequency is high, $u_0/\sqrt{a\omega} \leq 0.01$, and there arises a phase lag of the temperature from the velocity change, that is, the temperature on the upstream side of the heater is the lowest 90° behind the moment of maximum velocity. In this case the amplitude decreases with an increasing frequency and a decreasing current velocity. In the case of $u_0/\sqrt{a\omega} \geq 10$, the temperature is the lowest simultaneously with the maximum velocity. The amplitude and the area of temperature change are in inverse proportion to the current velocity u_0 .

2.2 Energy supply to oscillation

The temperature change of gas causes a volumetric change, and it converts thermal energy to acoustic energy the amount of which in a cycle is obtained as follows.

$$\Delta E = A \beta \int_0^l \int_0^{2\pi/\omega} \text{Re}(\tilde{p} e^{i\omega t}) \frac{\partial}{\partial t} \text{Re}(\theta_1 e^{i\omega t}) dt dx \\ = \pi K^2 \kappa p_0 (c_0/\sqrt{a\omega}) \beta \theta_0 A L \text{Im}(G) \quad (9)$$

where A is the flow area and β is the coefficient of expansion. The function G is written in the form:

$$G = \frac{i}{n\pi} \frac{UL^2}{U^2 L^2 + (1 + iL^2)^2} \left\{ UL \sin \xi_1 - (1 + iL^2) \cos \xi_1 \right\} \\ \times \left[(1 + \sqrt{1 + i4/U^2}) \frac{UL}{2} \sin \xi_1 - \cos \xi_1 \right. \\ \left. + \exp\{-(1 + \sqrt{1 + i4/U^2}) UL \xi_1 / 2\} \right]$$

Fig. 3 Temperature fluctuations on upstream side of heater

$$-\frac{1}{2} \left\{ 1 - \frac{UL(3 + iL^2)}{U^2 L^2 + 4} \sin 2\xi_1 - \frac{U^2 L^2 - 2(1 + iL^2)}{U^2 L^2 + 4} \cos 2\xi_1 - \frac{6 + i2L^2}{U^2 L^2 + 4} \exp(-UL\xi_1) \right\} \quad (10)$$

In the case of $UL > 10$, the temperature change is limited to the neighborhood of the heater. Thus

$$\left. \begin{aligned} G &= \frac{1}{2n\pi L} \sin 2\xi_1 \cdot F \\ F &= -\frac{iU^2}{4} (1 - \sqrt{1 + i4/U^2})^2 \end{aligned} \right\} \quad (11)$$

Fig. 4 $\text{Im}(F)$ in Eq. (11)

The function $\text{Im}(F)$ shown in Fig. 4 is always positive. It is known from the calculation mentioned above that the oscillation may most possibly be induced when $U = 1$ due to a large value of $\text{Im}(F)$ and is never induced when $\sin(2n\pi l_1/l)$ is negative. The energy

of the standing wave in the pipe described by Eqs.(1) and (2) is written as follows.

$$E = \frac{1}{4} K^2 \kappa p_0 A l \quad (12)$$

Neglecting the dispersed energy, the logarithmic increment ζ can be written in the form:

$$\left. \begin{aligned} \zeta &= \frac{1}{2} \ln(1 + \Delta E/E) \\ \Delta E/E &= \frac{28\theta}{n} \sin 2\xi_1 \cdot \text{Im}(F) \end{aligned} \right\} \quad (13)$$

2.3 Effects of other factors

When the heat transmission to the tube wall and the heat capacity of the heater are taken into consideration, it is necessary to use Eq.(14) in place of Eq.(11) on the assumptions that the temperature distribution of wall along the axis is the same as that of the air and the heat generation rate in the heater is constant.

$$F = \frac{UL - iW + 4(H + i)L/U + (UL + iW)\sqrt{1 + 4(H + i)/U^2}}{(1 + \sqrt{1 + 4(H + i)/U^2})(UL\sqrt{1 + 4(H + i)/U^2} + iW)} \quad (14)$$

Herein $H = 4h/\rho c_p \omega D$ and $W = \rho' c' \omega l d / 4\pi \lambda$ are dimensionless values where h is the heat transfer coefficient on the wall, D is the tube diameter; d , ρ' and c' are the diameter, density and specific heat of the heater; and ρ , c_p and λ are the density, specific heat and thermal conductivity of the air. The imaginary part of Eq.(14) is shown in Fig.5. Their effects cannot be neglected when $W/L > U^3$ or $H > 1$.

Fig.5 $\text{Im}(F)$ in Eq.(14)

3. Experiment

3.1 Experimental method

Fig.6 illustrates the constitution of experimental devices. The air current is made by the blower and measured by the rotameters. The surge tank is installed at the inlet of the test pipe. Three types of

Fig.6 Experimental apparatus

Fig.7 Heater

brass tubes of inner diameter 30 mm are used as the test pipe, the lengths of which are 640 mm, 1280 mm and 2560 mm. The heater is a kanthal wire of diameter 1 mm formed into a vortex shape as illustrated in Fig.7, which is inserted into the pipe from the outlet. The pressure pickup of strain-gauge type, capacity 0.1 atm and frequency range DC ~ about 300 Hz is used to measure the oscillation. The outer surface of the pipe is cooled only by the natural convection of air except the neighborhood of the pickup. After the volumetric air flow rate $\dot{V}$ and the heater input $\dot{Q}$ are set, the heater temperature reaches a constant in a few minutes. After the plug is closed which suppresses the oscillation, the oscillation growth rate and the final amplitude are measured as the oscillation grows.

3.2 Experimental results

Figs.8 ~ 15 summarize the experimental results. In these figures the flow rate readings on the rotameters are directly used, which should be corrected using a multiplier 1.022 because of the difference between the room temperature 33°C in the experiment and the rotameter calibration temperature 20°C. Figs.8 ~ 13 are the results in case of $l_1 = l/4$, the condition in which the fundamental mode of oscillation may most possibly be induced. In this case the pressure fluctuation measured by the pickup installed at the point of $l/2$ was observed to change in a sinusoidal manner.

The lower limits of the heater input for the oscillation growth are plotted against the air flow rate in Fig.8. In case of lower flow rate an unstable oscillation may occasionally occur, namely the growth and reduction of oscillation may appear alternately. In case of extremely high flow rate, an oscillation of large amplitude may be able to be sustained under the condition that no oscillation can grow without initial disturbance. The author defined the unstable region of the system only where the oscillation grows from standstill condition

Fig. 8 Unstable region boundary

Fig. 9 Growth of oscillation

Fig. 10 Logarithmic increment versus heater input

to a certain final amplitude.

Fig. 9 shows examples of processes of the oscillation growth. Usually the logarithmic increment of amplitude is constant until it is saturated, but near the upper flow rate boundary of unstable region the increment may become large when the amplitude exceeds a certain value. In such case, logarithmic increments in small amplitude and large amplitude were read separately, as shown in Figs. 10 and 12. The turning point exists around the amplitude of $0.003 \sim 0.005$ atm, while the amount of scatter is large. The frequencies of oscillation were about 285 Hz, 142 Hz and 71 Hz in 640 mm, 1280 mm and 2560 mm lengths of pipes respectively, but they vary a little with the

Fig. 11 Final amplitude versus heater input

Fig. 12 Logarithmic increment versus flow rate

Fig. 13 Final amplitude versus flow rate

flow rate and the heater input. The frequency of final oscillation is slightly low compared with the growing oscillation frequency.

Figs. 10 and 11 illustrate the relation between the heater input and the logarithmic increment or the final amplitude when the pipe length is 1280 mm. The mean values of the unsteady final amplitudes are adopted when the flow rates are very low. Both the increment and the amplitude increase with the heater input, while the increasing rates decrease. By comparison between Fig. 10 and 11, it is seen that the heater input of zero increment does not give zero final amplitude, the tendency being more evident in the

high flow rate region.

Figs.12 and 13 illustrate the relation between the air flow rate and the logarithmic increment or the final amplitude when the heater input is 200 W. The increment has a maximum with a certain flow rate which is a little higher than that with which the minimum heater input causes the oscillation as shown in Fig.8. The final amplitude increases with the flow rate while it decreases a little near the upper flow rate limit. The values over the limit in Fig.13 mean the amplitudes which can be obtained by changing the flow rate with the oscillation generated within the unstable region.

Figs.14 and 15 are the results when the heater and the pressure pickup are installed at the point of $l/8$ and $3/4l$ from the inlet and the heater input keeps 200 W. In this case, the oscillation of 2nd mode alone can grow at a flow rate over 30 l/min, while at a lower rate the oscillations of fundamental and 2nd modes can exist together, hence the pressure does not change in a sinusoidal manner. Fig.14 shows the amplitudes of each frequency component in the final constant oscillation obtained by the Fourier analyzer, where amplitudes of 3rd or much higher modes were so small that they were neglected. In the region of $15 \sim 27.5$ l/min, the 2nd mode grows at first, then the fundamental mode grows, and finally they coexist in the ratio shown in Fig.14. Fig.15 shows the growing process, the data in which the values are averaged over 30 msec by the analyzer at each time. The amplitude of 2nd mode decreases with the growth of the fundamental mode before it takes the final value. The interval between the growths of 2nd and fundamental modes is long when the flow rate is high.

Fig.14 Frequency components

Fig.15 Growth of oscillation

4. Discussion

4.1 Unstable region

The experimental results in Fig.8 are arranged in Fig.16 using $U = u_m / \sqrt{a\omega}$ and $\theta = \dot{Q} / \rho c_p \dot{V}$, where $u_m = \dot{V} / A$ denotes the mean current velocity, which is accelerated by heating near the heater. The thermal diffusivity a is also a function of the temperature, therefore the arrangement is affected by the temperature used in the calculation. The properties at 500°C were used in calculating the value U and the value of ρc_p at the room temperature was used in calculating θ , while the value $\rho \sqrt{a}$ at 1000°C is 0.776 times the value at the room temperature.

Fig.16 Dimensionless unstable region boundary

The dimensionless heat transfer coefficient H of the pipe 1280 mm long is 0.013 when the heat transfer coefficient on the tube wall is as large as $100 \text{ W/m}^2\cdot\text{K}$. The dimensionless heat capacity of the heater W is about 8×10^6 and L is about 10^3 . Hence the effects of them can be neglected in the neighborhood of $U = 1$. It can be comprehended from Fig.4 that the oscillation is supplied with the maximum energy when $U \approx 1$, whereas the oscillation grows with the least value of θ when $U = 2 \sim 3$ in the experiment as shown in Fig.16. This is a significant difference even considering the error due to the choice of the air properties. Thus the effect of heater size must be taken into consideration. Fig.3 indicates that the area where the temperature changes is bounded in the range of $\Delta x = 1.0 l / nUL$ from the heater. In the case of the fundamental oscillation in the pipe 1280 mm long where $n = 1$ and $L = 1130$, Δx is computed as 1.13 mm when $U = 1$. The heater of diameter $d = 1$ mm was used in the experiment, thus the Reynolds number based on d is 1.28 when $U = u_m / \sqrt{a\omega} = 1$. If the heater is regarded as a single straight cylinder perpendicular to the air current, the boundary-layer thickness around the cylinder is computed as about 1 mm⁽⁵⁾. Therefore the current velocity u_0 which has been used in the temperature calculation must be affected by the heater wall and be less than the free-stream velocity u_m . Thus the results in Fig.16 are rearranged in Fig.17 using $U = u_0 / \sqrt{a\omega}$ where u_0 is the product of u_m and the constant $C = 0.4$ which has been chosen for the agreement of experiment and analysis. The broken lines in the figure

are the least temperature rise $\Theta_{\min}$ for the oscillation growth from the following equation:

$$\Theta_{\min} = \frac{n}{2\beta \sin 2\xi_1} \frac{1}{\text{Im}(F)} \left(\frac{\Delta E}{E}\right)_{\min} \quad (15)$$

where it is taken into consideration that the coefficient of expansion β is in inverse proportion to the absolute temperature. The main cause of the difference between the experiment and analysis in small- U region of Fig.17 is radiative heat loss to the tube wall as was already pointed out by Katto et al.⁽²⁾ That is, the heat transmitted to the air is less than the heater input especially when U is small because of the radiation from the heater at an elevated temperature. The value $(\Delta E/E)_{\min}$ means the amount of dispersed energy in the pipe, which is affected by the heater shape, the pipe length and so on, and can be measured with a suitable apparatus. But the measurement is beyond the scope of this study. In the author's experiment, $(\Delta E/E)_{\min}$ increases with the pipe length, while it is estimated to be large when the pipe is very short from the experiment made by Katto et al.⁽²⁾

Fig.17 Modified dimensionless unstable region boundary

4.2 Increment and final amplitude

It is strange that the logarithmic increment abruptly increases at a certain amplitude when the flow rate is near the upper limit of the unstable region. Besides the oscillation can be maintained out of the limit if the amplitude is sufficiently large. It seems to be maintained in the region where the extrapolated value of the logarithmic increment in the large amplitude may be positive. When the oscillation grows, the heat transfer coefficient becomes large, and the radiative heat loss to the wall decreases because of the falling heater temperature, which means a larger energy supply. Therefore the growth of oscillation is accelerated with an increasing amplitude. This may be the reason for the oscillation growth at a low flow rate and a small heater input where the oscillation cannot grow without a certain disturbance. However, this effect does not excessively multiply the energy supply. Moreover it cannot explain the fact that the energy supply changes at a certain amplitude. The

Reynolds number of the static current is less than 100, hence the flow in the pipe is laminar. The pressure amplitude of 0.005 atm where the increment changes is translated into the velocity amplitude of 1.30 m/s using $|\tilde{u}| = |\tilde{p}|c_0/\kappa p_0$. The Reynolds number is found to be 2300 based on this velocity amplitude, the pipe diameter and the kinetic viscosity at the inlet temperature. It is deduced that the transition to turbulent flow participates in the change of the logarithmic increment at the moment the Reynolds number reaches the critical value. If the flow is turbulent, the eddy diffusivity for heat ϵ_H must be taken into consideration, thus the dimensionless current velocity U must be rewritten in the form $U = u_0/\sqrt{(\alpha + \epsilon_H)\omega}$. Therefore, in the turbulent flow U takes a smaller value than in the laminar flow, which means that the unstable region shifts to the higher flow rate and that the energy supply may increase by the transition when the flow rate is high. The experimental results can be explained well by the above consideration.

For the same reason the maximum-amplitude flow rates in Fig.13 are shifted compared with the minimum-heater-input flow rates in Fig.8. The maximum-increment flow rates in Fig.12 are shifted for other reasons, one of which is the radiative heat loss mentioned previously. Another cause is the changes of properties. The energy supply is in proportion to $\beta/\rho c_p \sqrt{\alpha c_0}$ which decreases with the temperature, whereas the friction of a flow passing the heater increases due to the viscosity change.

The logarithmic increment ζ is computed as 0.2 in the case $U = 1$ from Eq. (13), which agrees fairly well with the results in Fig.12 considering the dissipated energy.

4.3 Higher mode oscillations

It is explained by Saito's analysis⁽¹⁾ that the higher mode of oscillation may also be induced if the heater is installed at the point of $l/8$ from the inlet. The energy supply to the fundamental and the 2nd modes of oscillations is illustrated in Fig.18 which is obtained from Eq.(11). The logarithmic increment of the fundamental mode oscillation is 2 times as large as the 2nd mode at low flow rate whereas the latter is $\sqrt{2}$ times at high flow rate neglecting the dissipation energy. Hence the final amplitude of the fundamental mode is large at low flow rate while the 2nd mode is large at high flow rate as shown in Fig. 14. The flow rate of 30 l/min or more is in the unstable region of the 2nd mode alone, thus the fundamental mode cannot grow. The cause of the delayed growth of fundamental mode in Fig.15 may be related with the turbulence transition, that is, the growth of 2nd mode induces the transition which enables the growth of fundamental mode by the extension of the unstable region. The cause of the interference of two modes, that is, the decrease of the 2nd mode amplitude by the growth of fundamental mode cannot be made clear in this report.

Fig.18 Energy supply to two modes

5. Conclusions

Thermally induced acoustic oscillations in a pipe with a plane heat source and an air current have been studied analytically and experimentally. Temperature distributions and their changes have been obtained near a heat source in a uniform flow with an acoustic field by solving heat-conduction equations. Then the amount of energy supply to the oscillation has been computed and the following results have been obtained from the analysis. An oscillation may most possibly be induced when the dimensionless current velocity $U = u_0/\sqrt{a\omega}$ equals 1, while it may hardly be induced when U is far from 1. The amplitude ratio of some oscillation modes which may coexist is a function of the current velocity, namely the lower mode amplitude is large at low velocity and the higher mode at high velocity.

The experimental results have supported the analysis, provided that the current velocity is modified considering the boundary-layer developed around the heater of finite size used in the experiment. In this report, the fine structure of velocity distribution in the boundary-layer has not been discussed and a constant has been introduced to explain the difference between the analysis and the experiment which represents the ratio of the free-stream velocity and the velocity in the layer. The transition to turbulent flow occurs above the critical velocity amplitude, which is expected to shift the unstable region toward higher flow rate from the analysis. In the experiment, the transition caused the change of logarithmic increment at a certain amplitude.

The mechanism of thermally induced oscillations in a pipe has been made clear so long as a simple shaped heater is used. It is known from the experiment of Katto et al. (2) that a heater of complicated shape may induce a phenomenon which cannot be explained by the theory used here. Besides the phenomenon using a flame as a heat source should be studied in order to make clear the mechanism of combustion oscillation which causes several industrial troubles. They are subjects for a future study.

Appendix

The acoustic energy is produced without air current in the pipe where the heater and the adjacent cooler are installed as shown in Fig.1, the amount of which can be obtained in the following manner. The acoustic

oscillation is described by Eqs.(1) and (2). The temperature distribution is derived from Eq.(3) with the provision that u_0 is 0 in this case. The solution can be written in the form as shown in Eq.(4), thus θ_0 and θ_1 are expressed by Eqs.(16) and (17) when $l_1 \leq x \leq l_1 + \delta$:

$$\theta_0 = \theta(x - l_1)/\delta \quad (16)$$

$$\begin{aligned} \theta_1 = & -iK \frac{a}{\omega \delta} \theta \cos(n\pi \frac{x-l_1}{l}) \\ & \times \{ (e^{\sqrt{i\omega/a}\delta} - 1)(e^{-\sqrt{i\omega/a}(x-l_1)} - 1) \\ & - (e^{-\sqrt{i\omega/a}\delta} - 1)(e^{\sqrt{i\omega/a}(x-l_1)} - 1) \} \\ & / (e^{\sqrt{i\omega/a}\delta} - e^{-\sqrt{i\omega/a}\delta}) \end{aligned} \quad (17)$$

where the boundary conditions of $\theta = 0$ at $x = l_1$ and $\theta = \theta$ at $x = l_1 + \delta$ are used. The distance between the heater and the cooler δ is assumed to be sufficiently short compared with the acoustic wave length $2l/n$. Fig.19 shows the temperature fluctuations in a dimensionless form.

Fig.19 Temperature fluctuations between heater and cooler

The amount of energy conversion between the heater and the cooler is obtained as follows.

$$\begin{aligned} \Delta E = & A\beta \int_0^l \int_0^{2\pi/\omega} \text{Re}(\tilde{p}e^{i\omega t}) \frac{\partial}{\partial t} \text{Re}(\theta_1 e^{i\omega t}) dt dx \\ = & \pi K^2 \kappa p_0 (a_0/\omega) \beta \theta A \cdot 2 \sin(2n\pi l_1/l) \\ & \times \text{Im} \left\{ \frac{(e^{\sqrt{i\omega/a}\delta} - 1)(e^{-\sqrt{i\omega/a}\delta} - 1)}{\sqrt{i\omega/a}\delta \sinh(\sqrt{i\omega/a}\delta)} \right\} \end{aligned} \quad (18)$$

The value of $\text{Im}\{\}$ is shown in Fig.20 graphically, which indicates that the thermally induced acoustic oscillation in the pipe without air current may most

Fig.20 $\text{Im}\{ \}$ in Eq.(18)

possibly be induced when the dimensionless distance $\sqrt{\omega a \delta}$ equals $\sqrt{10}$.

References

- (1) Saito, T., Bull. JSME, Vol.8, No.32 (1965), p.651.
- (2) Katto, Y. and Sajiki, A., Bull. JSME, Vol.20, No.147 (1977-9), p.1161.
- (3) Katto, Y. and Takano, K., Bull. JSME, Vol.20, No.147 (1977-9), p.1169.
- (4) Madarame, H., Text of Course, Jpn. Soc. Mech. Eng. (in Japanese), Vol.449 (1977-5), p.67.
- (5) Schlichting, H., Boundary-Layer Theory, (1968), p.160, McGraw-Hill.