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Research on Bending Strength Properties of Spur Gears with a Thin Rim*

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In the present paper, the stresses and deformation at tooth and rim of gears with a thin rim caused by loading on tooth were examined by various experiments and finite element method.

Moreover, the bending fatigue strength of gears with a thin rim was compared with that of solid gears.

The following conclusions were obtained.

(1) The change of stresses at the root fillet of tooth of gears with a thin rim is partial alternating. For this reason the bending fatigue strength is lower than that of solid gears.

(2) A practical formula of stresses at loaded side of the root fillet of tooth corresponding to the center of sector hole can be introduced, and consequently the bending fatigue limit of gears with a thin rim can be estimated from the endurance limit diagram.

1. Introduction

Recently, gears with a thin rim are drawing attention in the field of high speed gears for the purpose of material reduction (smaller size and weight). However, there are few researches which dealt with gears with a thin rim, hitherto. For the design of gears with a thin rim, deformation and stress in the parts of gear rim as well as those at the root fillet of gear tooth are important problems, and it is necessary that the deformations give no influence on normal running properties of gears with a thin rim.

Considering such a situation, gears with a thin rim were chosen as a subject of authors study, and researches on those gears from various points of view have been made for several years. The stresses at the root fillet of tooth and at the part of rim corresponding to the center of sector hole of gears with a thin rim having a specific shape were examined in previous report^{a)} by means of strain gauge and finite element method (F.E.M.). In this paper, the deformation of each tooth and the stresses at the root fillet of teeth corresponding to the spoke and no spoke parts of gears with a thin rim having a general shape were examined and practical formulas for those stresses were discussed. Moreover, the relations between the radius of curvature at the joint part of spoke with rim and the

stresses at the joint part and the root fillet of tooth were also examined. In this case, the experimental results of the stresses at the root fillet of tooth in the static bending test and the running test were compared with the calculated results by F.E.M., and consequently it was confirmed that those results show similar tendencies. From these facts, subsequent discussion are made only for the calculated results by F.E.M..

From the experimental results mentioned above, it is found that, in the case of gears with a thin rim, the stress induced at loaded side of the root fillet of tooth partially alternate, and so a general type pulsater used for pulsating load was reconstructed into one that can be used for partial alternating loading condition. By using this reconstructed pulsater, a bending fatigue test of gears with a thin rim is done and the results are compared with those of solid gears. Bending fatigue limits of gears with a thin rim are also discussed from the endurance limit diagram and an estimation method is shown.

2. Experimental method

2.1 Test gears

The dimensions and details of the size of gears with a thin rim used in this experiment are shown in Table 1 and Fig. 1. Those test gears are made of S45C, and each tooth surface is induction hardened (60kW, 30kHz, heating during 10 seconds) after gear-hobbing. Solid spur gears of the same dimensions, material and heat treatment as those of the spur gears with a thin rim are prepared for a comparative test.

2.2 Experimental device and method

2.2.1 Static bending test

Load P was imposed on position A as shown in Fig. 1 by a universal tester of Amslar type (Akashi Factory Co. Ltd.), and

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every time the load was increased by 98.07 N (10 kgf) step, the stresses in each part (positions A, B, C, and D in Fig.1) were measured. Fig.2 shows the set condition of a gear in the static bending test.

Table 1 Dimensions of test gears

Tooth profile (clearance)	B (0.25 m)
Module	$m = 4$
Pressure angle	$\alpha = 20^\circ$
Number of teeth	$Z = 30$
Pitch diameter	120.0 mm
Face width	$b = 10$ mm
Number of spoke	6
Diameter of boss	$d_b = 60$ mm
Rim thickness	$T = 4.0$ mm $T = 6.0$ mm $T = 7.9$ mm $T = 10.0$ mm
Radius of curvature	$\rho = 4$ mm

Fig.1 Gear with a thin rim

Fig.2 Static bending test of tooth

2.2.2 Gear driving test

The power circulating type gear test rig [maximum circulating power 73.55kW (100ps)] used in this test is one made by Osaka Precision Machinery Co. Ltd., and revolution of test gear is held at 600rpm constant and loaded torque is in the range of 9.81~78.45kN-mm (1000~8000kgf-mm). In this experiment, a solid gear meshing with test gears had entirely the same dimensions and sizes as those of test gears.

2.2.3 Miscellaneous experimental apparatus

For the measurement of stress, strain gauge (FIP-5-30 type made by Shinko Communication Industry Co. Ltd. 0.5mm x 1.5mm), strain meter (DS6/MTC type made by Shinko Communication Industry Co. Ltd.), synchroscope (DS-6020 type made by Iwasaki Correspondence Co. Ltd.) and universal photograph apparatus, etc, and for the measurement of hardness, Vickers hardness tester were respectively used.

2.2.4 Bending fatigue test

(a) Reconstruction of pulsater

Fig.3 and Fig.4 show respectively an outline photograph and a block diagram of the pulsater reconstructed from a pulsating loading into an alternating loading structure. In Fig.4, two cam shafts (5) (joined by universal joint (6)) moving two injection pumps (plunger type) are driven by one motor through one step V-belt drive, and repeated loads are imposed on each tooth by loading bars (1, 3).

In this case, position of one cam is so set as to keep phase difference 180° of load wave appearing in synchroscope for the position of the other cam, that is, loading bar (1) imposes a load on one tooth and then adjacent loading bar (3), moved by a cam having a phase difference 180° imposes a load on the neighbouring tooth of test gears. Load imposed on loading bars

Fig.3 Reconstructed pulsater

①, ③ is adjustable independently by pressure control of injection pump.

Bending fatigue test was done by adjusting the load imposed on loading bars to the condition of stresses at the root fillet of tooth (stress wave shown in synchroscope), obtained by the experimental results in driving test (by the power circulating type gear test rig mentioned in above 2.2.2 section and theoretical results by F.E.M.). Fig.5 shows a synchroscope photograph of change of oil pressure applied to the loading bars ①, ③ of the reconstructed pulsater. This pulsater gives 900 cycles of loading per minute.

(b) Bending fatigue test and measurement of stress

Loading positions on test gear and measuring positions of stresses at the root fillet of tooth by strain gauge are shown in Fig.6. Considering the results of driving test (section 2.2.2), load P imposed on tooth is adjusted in the range of 3.92~9.81kN(400~1000kgf). After here,

Fig.4 Block diagram of bending fatigue test

Fig.5 Synchroscope photograph of load wave of reconstructed pulsater

a tooth corresponding to the spoke part in Fig.6 is called No.0 tooth and adjacent teeth are called clockwise Nos.1, 2, 3 and counterclockwise Nos.-1, -2, -3 respectively. For loading position, two combinations of teeth are chosen, that is, No.1-No.2 teeth and No.2-No.3 teeth and the stresses at the root fillet of tooth are measured respectively at ①, ② positions in case of No.1-No.2 teeth test and at ③, ④ positions in case of No.2-No.3 teeth test (①, ②, ③ positions mean Hofer's weakest section).

3. Calculated results by F.E.M. and discussions

3.1 Gears chosen as the object of the calculation by F.E.M.

Dimensions of gears chosen as the object of this calculation are the same as the test gear shown in Fig.1. The radius of curvature at joint part of spoke with rim and thickness of gear rim are respectively $\rho = 6.0, 4.0, 2.0, 1.0$ mm, $T = 7.9, 6.0, 4.0$ mm.

3.2 Mesh division

Fig.7 shows representatively a model of mesh division diagram (number of elements 445) in case of $\rho = 4.0$ mm, $T = 6.0$ mm. Method of mesh division of the other gears is similar to the case of Fig.7. As for condition of constraint, it is assumed that rim and spoke are fixed rigidly at both ends of rim and joint part of spoke with boss respectively.

Fig.6 Measuring positions of stress in bending fatigue test

Fig.7 Mesh division diagram (number of nodal points 307, number of elements 445)

3.3 Comparison of the experimental results of the stresses at the root fillet of tooth with the calculated results by F.E.M.

In Fig.8, the experimental results of static bending test in case of $\rho = 4.0\text{mm}$ constant, $T = 7.9, 6.0, 4.0\text{mm}$ are shown. Broken line in this figure shows the calculated results by F.E.M.. Both results show a similar tendency, but the calculated results are a little smaller usually than the experimental ones. The difference may be due to the fact that the calculated results by F.E.M. are not actual stresses at the root fillet of tooth but average values of divided triangular elements and loading position shifts by deformation of tooth.

On the other hand, the experimental results of static bending test and the calculated results by F.E.M. and by Aida and Terauchi's equation¹⁰⁾ for the case of solid gear are shown in Fig.9. Considering from these results, after here, only the calculated results by F.E.M. are discussed. Fig.10 shows measured results of the stresses at the root fillet of tooth in case when a load shifts clockwise from -No.4 tooth to No.4 tooth, and meas-

ured results of stresses at loaded side of the root fillet of No.0 tooth are shown in Fig.11. Both figures show a similar tendency (partial alternating loading condition) to the calculated results by F.E.M. mentioned later.

3.4 A load imposed on tooth and the stresses at the root fillet of tooth

In case of increasing load by 98.07N (10kgf) step, the stresses at loaded and opposite sides of the root fillet of tooth corresponding to the parts of spoke become smaller in order of $T = 4.0, 6.0, 7.9\text{mm}$ and solid gear, but the stresses of tooth corresponding to the parts with no spoke show a reverse order.

3.5 The stresses at the root fillet in case when a load shifts to the neighbouring teeth

Fig.12(a)~(c), Fig.13(a)~(c) show respectively the stresses at loaded and opposite sides of root fillet of No.0, No.1, No.2 teeth. The stresses at loaded side of root fillet of No.0 tooth are pulsating in case of solid gear and partial alternating in case of the other, and the stresses of No.2 tooth are almost

Fig.8 Results of static bending test of tooth (No.0 tooth, $\rho = 4.0\text{mm}$)

Fig.9 Results of static bending test (solid gear)

Fig.10 Measured stresses at loaded side of the root fillet of No.0 tooth in case when a load shifts from -No.4 tooth to No.4 tooth [$P = 980.7\text{ N}$ (100kgf)]

Fig.11 Measured stresses at loaded side of the root fillet of No.0 tooth in driving test [torque 58.8 kN (6000 kgf-mm)]

complete alternating. Tensile stresses in case of No.0, No.1 teeth are large, and tensile stresses of No.2 tooth have no effect of thickness of rim and are constant, while compression stresses becomes as large as the rim becomes thin.

On the other hand, the stresses at opposite side of the root fillet of No.0, No.1, No.2 teeth are partial alternating and tensile stresses are small. Both stresses of tension and compression are larger as thickness of rim is smaller. These tendencies are similar in case when a load is imposed at geometrically worst loading position.

(a) Case of No.0 tooth

(b) Case of No.1 tooth

(c) Case of No.2 tooth

Fig.12 Calculated stresses by F.E.M. at loaded side of the root fillet of each tooth in case when a load shifts from -No.4 tooth to No.4 tooth [$P = 980.7 \text{ N}$ (100 kgf)]

3.6 The calculated results of deformation

Deformations of tooth and rim in case when a load is imposed on No.0 and No.2 teeth are shown in Fig.14(a),(b). In cases when a load is imposed on tooth tip and the worst loading position of No.0 tooth, a deformation (of tooth and rim) occurs in the whole range of teeth from -No.4 tooth to No.4 tooth, but the deformation in case of loading on No.2 tooth is almost zero in range from -No.4 tooth to No.0 tooth and is larger partially in range of No.2 tooth to No.3 tooth.

3.7 The radius of curvature and the

(a) Case of No.0 tooth

(b) Case of No.1 tooth

(c) Case of No.2 tooth

Fig.13 Calculated stresses by F.E.M. at opposite side of the root fillet of each tooth [$P = 980.7 \text{ N}$ (100 kgf)]

stresses at joint part of spoke with rim

Comparison of the radius of curvature at joint part of spoke with rim with the stresses at the same part and at the root fillet of No.0 tooth in case of $T=4.0\text{mm}$ is shown in Fig.15. Broken line in this figure shows the maximum stresses at joint part of spoke with rim, that is, respectively the stresses at positions of C and D in case when a load is imposed on No.1 and -No.2 teeth. The stresses at loaded and opposite sides of joint part of spoke with rim become larger as radius of curvature ρ becomes smaller, but the stresses at the root fillet of tooth show almost no effect of ρ .

3.8 Relation between thickness of rim and the stresses at the root fillet of tooth

Fig.16 shows the relation T/m vs. the stresses at opposite side of the root fillet of tooth. From this figure, it is found that the increasing rate of the stresses at opposite side of the root fillet of tooth becomes larger in the range $T/m \approx 1.6 \sim 1.8$, and this result almost agrees with $T/m=1.92$ obtained by V.V. Dolgosolov⁽⁴⁾ with planet gears. In the range of experiment, it seems that $T/m \approx 1.6 \sim 1.8$ is the practical lower limit of T for the stresses. In previous study⁽⁴⁾ on gears with a thin rim having specific shape, limit value of T/m for the stresses at loaded and opposite sides of the root fillet of tooth is $1.6 \sim 1.7$ and this result agrees well with the present experimental results. Thickness of rim calcu-

lated from this result is $6.4 \sim 7.2\text{mm}$, and judging from the maximum stresses at the root fillet of tooth, rim thickness beyond this value may be taken as zone of safety.

4. Experimental results and discussions

4.1 The results of bending fatigue test

4.1.1 No.1-No.2 teeth repeated loading test

The results of bending fatigue test in case when the load is imposed on No.1 and No.2 teeth are shown in Fig.17. As seen from Fig.17, the bending fatigue limit in case of thickness of rim $T=6.0, 7.9\text{mm}$ is respectively 32.4%, 18.9% lower than that of solid gear, but a drop of the bending fatigue limit in case of $T=10\text{mm}$ hardly occurs. The reason for this drop is considered to be due to partial alternating condition of the stresses at

Fig.14 Calculated deformation of each tooth and rim in case when a load is imposed on a tooth [$P=980.7\text{N}$ (100 kgf)]

Fig.15 Comparison of radius of curvature with stresses at joint part of spoke with rim and at the root fillet of tooth

Fig.16 Relation between T/m and stresses at opposite side of the root fillet of tooth

the root fillet of tooth caused by the deformation of rim in case of gears with a thin rim of $T=6.0, 7.9\text{ mm}$.

4.1.2 No.2-No.3 teeth repeated loading test

Fig.18 shows the results of bending fatigue test in case when the load is imposed on No.2 and No.3 teeth. The bending fatigue limit in this case is lower than that of solid gear in the same manner as the case of section 4.1.1 mentioned above, but the rate of drop is smaller than that in No.1-No.2 teeth test (27% in case of $T=6.0\text{ mm}$, 10.8% in case of $T=7.9\text{ mm}$). The reason is that deformation of rim in case of No.1-No.2 teeth test becomes larger and consequently alternation of the stresses at the root fillet of tooth becomes very large.

Next, Fig.19(a),(b), Fig.20(a),(b) show respectively synchroscope photographs of the stresses at the root fillet of tooth in cases of test by loading bar ① only and test by loading bars ①, ③. The stresses in test by loading bar ① only of Fig.19(a),(b) show complete tension, and on the other hand the stresses in case of Fig.20(a) ($T=7.9\text{ mm}$, No.1-No.2 teeth test) show partial alternation of tension and compression. The stresses at the root fillet of Fig.20(b) ($T=10\text{ mm}$) show tension in the same manner as case of solid gear. From these facts, it is found that the difference between bending fatigue limit of gears with a thin rim ($T=10\text{ mm}$) and

Fig.17 P-N curve in No.1-No.2 teeth repeated loading test (No.2 tooth ruptured)

Fig.18 P-N curve in No.2-No.3 teeth repeated loading test (No.3 tooth ruptured)

(a) Case of $T = 6.0\text{ mm}$

(b) Case of $T = 7.9\text{ mm}$

Fig.19 Synchroscope photographs of stress at loaded side of the root fillet of tooth in case when a load is imposed by loading bar ① only (⑥ position in Fig.6)

(a) Case of $T = 7.9\text{ mm}$

(b) Case of $T = 10\text{ mm}$

Fig.20 Synchroscope photographs of stress at loaded side of the root fillet of tooth in case when a load is imposed by loading bars ①, ③ (No.1-No.2 teeth repeated loading test, ⑥ position in Fig.6)

that of solid gears is very small.

Photographs of teeth ruptured in the bending fatigue test are shown in Fig.21. Fig.21(a),(b) respectively show for the case of $T=7.9\text{mm}$ and $T=10\text{mm}$, and both teeth rupture at Hofer's weakest section. Besides, in our different experiment, that is, bending fatigue test of gears with a thin rim of $T=3\text{mm}$, gear tooth ruptured at the part of rim.

4.2 Measured results of hardness at the root fillet of tooth

Side surface hardness of tooth root

(a) Case of $T = 7.9\text{ mm}$

(b) Case of $T = 10\text{ mm}$

Fig.21 Photographs of gear tooth ruptured by the bending fatigue test

fillet area before and after the bending fatigue test was measured. The hardness before and after the bending fatigue test scarcely varied and the hardness after the test was 5~7% higher than that before the test.

5. Discussion of bending fatigue limit by endurance limit diagram

Fig.22 shows an endurance limit diagram in case of No.1-No.2 teeth loading test of gears with a thin rim ($T=7.9\text{mm}$). In this figure, residual stress value [$\sigma_r = 294.2\text{MPa}$ (30kgf/mm^2)] caused by induction hardening is adopted from the experimental result of separate test gear by boring method. Mean stress value [$\sigma_m = 156.9\text{MPa}$ (16kgf/mm^2)] is obtained from the synchroscope photograph of Fig.20(a). Difference [98.1MPa (10kgf/mm^2)] between bending fatigue limit [$2a_1 = 549.2\text{MPa}$ (56kgf/mm^2)] of solid gear and that of gear with a thin rim [$a_2 = 451.1\text{MPa}$ (46kgf/mm^2)] almost agrees with experimental result (18.9%) in the bending fatigue test of Fig.17.

On the other hand, judging from the calculated results by F.E.M. mentioned above, bending fatigue strength of gears with a thin rim having spokes is considered to be decided by the stresses at loaded side of the root fillet of No.2 tooth. Here, the stress σ_1 at loaded side of the root fillet of tooth of solid gear is obtained by Aida and Terauchi's equation⁽²⁾, and so compression stress σ_{th1} of No.2 in case when a load is imposed on No.1 tooth is expressed by the following Eq.(1) based on

$$\sigma_{th1} = -\eta\sigma_1 \dots\dots\dots(1)$$

Where, η is obtained from Fig.23 and the values in case of $T=7.9, 6.0, 4.0\text{mm}$ are respectively 0.66, 0.93, 1.52. This figure shows relation between T/m and η , and η is hyperbolical to T/m . Mean stress σ_m is expressed as follows.

$$\sigma_m = (\sigma_1 + \sigma_{th1})/2 \dots\dots\dots(2)$$

Thus the bending fatigue limit of gears with a thin rim can be estimated by

Fig.22 Endurance limit diagram

drawing an endurance limit diagram as shown in Fig.22.

Fig.23 Relation between T/m and r

6. Conclusions

(1) By this study, condition of stress at the root fillet of tooth of gears with a thin rim having spokes could be clarified, and it is found that the bending fatigue limit of gears with a thin rim should be considered for tooth corresponding to the part with no spoke.

(2) The change of stresses at the root fillet of tooth of gears with a thin rim is partial alternating, and con-

sequently the bending fatigue limit is lower than that of solid gear. The limit of thickness of rim judged from drop of the bending fatigue limit is under $T/m \approx 2.5$ in the range of this experiment.

(3) The bending fatigue limit of gears with a thin rim becomes lower in comparison with that of solid gear as the rim is smaller, and the drop rate of the bending fatigue limit in case of No.1-No.2 teeth test is larger than that in case of No.2-No.3 teeth test. The bending fatigue limit of gears with a thin rim having spokes is necessary to be considered for that of No.2 tooth in the range of gear dimensions in this experiment.

(4) A practical formula to decide the root fillet stress of tooth corresponding to the part with no spoke could be introduced and the bending fatigue limit of gears with a thin rim can be estimated from the root fillet stress calculated by this practical formula and the endurance limit diagram.

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