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Study on a Turbulent Boundary Layer with Injection*

— Turbulent Scales —

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Turbulent integral and differential scales have been measured in a turbulent boundary layer with injection and the effects of injection on these scales are investigated. In the inner layer, the longitudinal integral scale decreases with increasing the injection rate, but the transverse integral scale remains unchanged. On the other hand, both of these scales decrease in the outer layer. The ratio of transverse to longitudinal integral scales increases with increasing the injection rate and approaches a value of 0.5. This result suggests that turbulence tends to be isotropic with injection. The ratio of differential to integral scales becomes greater near the wall with increasing the injection rate, mainly due to the reduction in the turbulent Reynolds number.

1. Introduction

A turbulent boundary layer with injection is encountered in many engineering applications. One of the applications is to protect the surface from a hot freestream, as seen in the transpiration cooling of gas turbine blades or in the ablation cooling of high-speed reentry space-vehicles. There have been many investigations⁽¹⁾⁻⁽¹⁶⁾ on heat transfer coefficient or skin friction coefficient, since the reduction of these coefficients due to injection is of primary importance in the engineering applications.

In order to clarify the mechanism of heat transfer, it is very important to investigate the turbulence structure experimentally. However, there have been only a few studies on turbulence structure in the transpired turbulent boundary layer: Peterson and Isaacson⁽¹⁷⁾ and Andersen et al.⁽¹⁸⁾ in an isothermal turbulent boundary layer, and Pimenta et al.⁽¹⁹⁾ in a nonisothermal one on a rough wall.

The present authors⁽²⁰⁾ have carried out simultaneous measurements of velocity and temperature fluctuations in a nonisothermal turbulent boundary layer with uniform injection of heated air, and have investigated the effects of injection on the turbulence structure related to the heat transfer by obtaining several statistical turbulent quantities. It has also been suggested that the integral turbulent length scales of velocity and temperature fluctuations appear to decrease with increasing the injection rate. The integral length scale of turbulence represents the scale of large eddies, which mainly bear the turbulent exchange of

momentum or heat. It may be useful, therefore, to examine this characteristic scale of turbulence in a more systematic way.

Essentially, turbulence has a three-dimensional character by nature and in thin shear flows, eddies are stretched in one direction by the mean shear stress acting in the fluid. It is not sufficient to investigate the characteristic length scale of one direction only. Thus, attention is paid to both the longitudinal and transverse integral scales of turbulence at the same time, and the effect of injection on them is studied. Particularly it is interesting to confirm whether the turbulence tends to become more isotropic with injection, as suggested by Wooldridge and Muzzy⁽¹³⁾ in the case of a turbulent boundary layer with injection and combustion.

Baker and Launder⁽⁹⁾ measured the mixing length in a turbulent boundary layer with injection. It may be meaningful to examine the relationship between their results and the statistical length scales of large eddies to be studied here.

On the other hand, turbulence consists of eddies with different scales. Only with the knowledge about a characteristic length scale, turbulence can not be understood correctly. Among the scales a differential length scale of turbulence, which relates to the dissipation rate of turbulent kinetic energy, is another characteristic length scale indispensable for the understanding of the turbulence structure. It may be worthwhile to discuss about the ratio of this differential and the integral scales, which can be considered to be an important parameter in turbulent flows.

In this paper, several length scales of turbulence are measured in an isothermal turbulent boundary layer with injection, and the effects of injection on these scales are discussed. They are the longitudinal (streamwise direction) and the transverse (normal direction to the wall) integral scales, and the differential scale of the streamwise velocity fluctuation.

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2. Experimental apparatus and procedure

2.1 Experimental apparatus

A schematic diagram of the experimental apparatus used in this study is shown in Fig. 1. The air stream is sucked into the low speed wind tunnel by the blower 1 through the honeycomb 2, the three screens 3 consisting of different mesh sizes, and the contraction 4 having a contracting ratio 1/9. The test section is 300 mm × 300 mm in cross section and is 2100 mm in length. A two-dimensional turbulent boundary layer developing on the bottom flat plate 8 is studied. At the leading edge of the plate, the boundary layer on the bottom wall of the wind tunnel developed upstream is sucked by the blower 5. The flow rate of this blower is adjusted so that the velocity distributes uniformly over the cross section at the leading edge. The transition to turbulent flow is hastened by designing the leading edge in a semicircular shape and roughening the following part of the wall of 500 mm in length with a glued sandpaper. The porous part of the plate starts at 1200 mm downstream from the leading edge, and covers a length of 900 mm downstream. The porous plate 8 consists of a part of grinding wheel and its porosity is about 40 percent. The surface of the porous plate is polished and it can be considered aerodynamically smooth. The injection gas (air) supplied by the compressor 7 is blown into a developed turbulent boundary layer uniformly through the permeable part of the plate. The flow rate of the injection gas is measured with the orifice flow meter 9. The test section has a movable upper wall 6 being used to minimize the static pressure variation along the test section. For this purpose, seventeen static pressure taps are provided on the side wall of the wind tunnel.

2.2 Experimental procedure

Prior to the experiments the upper wall is carefully adjusted so as to keep uniformity along the test section of the ratio between the static pressure and the dynamic pressure in the freestream to within ± 0.5 percent. This uniformity is the same as that of the experiments by Klebanoff (21).

The injection rate F is defined as the mass flow rate ratio of the injection gas to the main flow: $F = (\rho V)_w / (\rho U)_e$, where ρ is the density, U and V the streamwise and normal velocity components. The suffixes w and e denote the values at the wall and in

Figure 1. Schematic diagram of experimental apparatus

the freestream, respectively. The value of F is varied in four steps; 0.0, 0.00214, 0.00351 and 0.00457. The freestream velocity is kept constant and equal to 8.0 m/s.

The measurements are carried out at a station 680 mm downstream from the starting point of injection. According to the preliminary experiments, already at a station 400 mm downstream from the leading edge of the porous plate, the effect of injection is confirmed to reach the boundary layer edge.

The measurements of turbulent quantities are carried out with a constant-temperature hot wire anemometer. The hot wire consists of a 5 μ m diameter tungsten wire with copper plated ends, and its effective length is 1.0 mm. The overheat ratio of the hot wire is 0.4. The longitudinal integral scale is evaluated from the autocorrelation coefficient, obtained with a single hot wire by making use of Taylor's hypothesis. The transverse integral scale is calculated from the cross-correlation coefficient measured with two parallel hot wires separated in the direction normal to the wall. The differential length scale is obtained by passing the output signal of a hot wire anemometer through a differentiating circuit and measuring its root mean square value.

3. Experimental results and discussion

3.1 Mean velocity profiles

In Fig. 2 are shown the profiles of mean velocity U , where x_p denotes the distance from the upstream edge of the porous plate. The solid line in the figure represents the following equation of the Coles⁽²²⁾ wake law.

$$\Phi = -\frac{1}{\kappa} \ln \frac{y}{\delta} + \frac{\Pi}{\kappa} \{2 - \omega(\frac{y}{\delta})\} \quad (1)$$

Here, κ is the von Kármán constant, Π the Coles wake parameter, $\omega(y/\delta)$ the Coles wake function, y the distance from the wall and δ the boundary layer thickness. In the transpired boundary layer, the ordinate Φ is defined as follows after Stevenson⁽²³⁾.

Figure 2. Mean velocity profiles

$$\Phi = \frac{\sqrt{2cf}}{F} \left\{ (1+B)^{0.5} - \left(1+B \frac{U}{U_e}\right)^{0.5} \right\} \quad (2)$$

where cf is the skin friction coefficient and B the similarity parameter in the transpired boundary layer defined as $2F/cf$. The mean velocity profiles reduced in the form of Eq. (2) agree well with the extended Coles law of the wake, and Fig. 2 confirms that the velocity boundary layer with injection is in a fully developed condition. Incidentally, Φ defined above tends to the form $(U_e - U)/u^*$ when F approaches zero, where, u^* is the friction velocity.

In this study the skin friction coefficient cf is evaluated from the measured momentum thickness θ with the Kays⁽²⁾ empirical formula.

$$\frac{cf}{cf_0} \bigg|_{Re_\theta} = \left\{ \frac{\ln(1+B)}{B} \right\}^{1.25} (1+B)^{0.25} \quad (3)$$

where $cf_0 = 0.026 Re_\theta^{-0.25}$, and Re_θ is the Reynolds number based on the momentum thickness and cf_0 the skin friction coefficient in the case of no injection.

3.2 Turbulent intensity of streamwise velocity fluctuation

In Fig. 3 are shown the intensities of streamwise velocity fluctuation. With increasing the injection rate, the intensity increases in the boundary layer except in the region close to the wall where the intensity decreases. The peak of the intensity profile becomes less pronounced with increasing the value of F . The present results agree with those of Pimenta et al.⁽¹⁹⁾ obtained in the nonisothermal transpired turbulent boundary layer and the present authors' previous results⁽²⁰⁾.

Figure 3. Intensity profiles of streamwise velocity fluctuation

3.3 Longitudinal correlation coefficient and integral length scale

The longitudinal correlation coefficient $R_{11}(\Delta x)$ defined in Eq. 4 is obtained from the auto-correlation $u(t)u(t+\Delta t)$ by transforming the time delay Δt into the space Δx following Taylor's hypothesis.

$$R_{11}(\Delta x) = \frac{u(x)u(x+\Delta x)}{\sqrt{u(x)^2} \sqrt{u(x+\Delta x)^2}} \quad (4)$$

The longitudinal integral scale Λ_F is defined as follows:

$$\Lambda_F = \int_0^\infty R_{11}(\Delta x) d(\Delta x) \quad (5)$$

In the above equation, integral calculus is performed up to the position Δx where the correlation coefficient crosses zero for the first time.

(a) $F = 0.0$

(b) $F = 0.00351$

Figure 4. Longitudinal correlation coefficient

Figure 5. Effect of injection on longitudinal correlation coefficient

The results of the longitudinal correlation coefficient are shown in Figs. 4(a) and 4(b). As seen easily in Fig. 4(a) ($F=0.0$), in the inner layer, the correlation coefficient tends to distribute more widely at a position of increased distance from the wall, and therefore, the integral scale increases as the distance is increased from the wall. In the outer layer, however, the correlation coefficient is found not to vary with the position and to be independent of the distance from the wall. The characteristics of the correlation coefficient discussed above are also applicable to the transpired boundary layer qualitatively, as seen in Fig. 4(b). But, quantitatively, the correlation coefficient in the case of injection is seen to distribute less widely in the inner layer than that in the case of no injection, if compared at the same distance from the wall with each other.

In order to investigate the effect of injection on the correlation coefficient, the results at the same distance from the wall are compared in Fig. 5. Two typical

comparisons, one in the inner layer and the other in the outer layer respectively, are shown in the figure. In the inner layer, the extent of the distribution of the correlation coefficients becomes narrower with increasing the injection rate, and thus, the longitudinal integral scale decreases. This is reasonable because the reduction in the mean velocity gradient near the wall due to injection would result in a decrease of the stretching rate of the eddies. In the outer layer, the distributions of the correlation coefficients at different injection rates are almost the same. Therefore, the injection does hardly affect the integral scale of the streamwise direction in this region.

In Figs. 6(a), 6(b), 6(c) and 6(d) are shown the one-dimensional energy spectra of the streamwise velocity fluctuation. The abscissa k_1 is the wave number evaluated by introducing Taylor's hypothesis. There exists the relationship shown below between the spectrum $E(k_1)$ at vanishing k_1 value and the integral scale Λ_f defined by Eq. (5).

(a) $F = 0.0$ (c) $F = 0.00351$ (b) $F = 0.00214$ (d) $F = 0.00457$

Figure 6. One-dimensional energy spectrum of streamwise velocity fluctuation

$$\Lambda_f = \frac{\pi}{2} \lim_{k_1 \rightarrow 0} \frac{E(k_1)}{u'^2} \quad (6)$$

In the low wave number range, the spectrum obtained in the inner layer is seen to reduce in its height with increasing the injection rate. This confirms the above mentioned result about the longitudinal integral scale decreasing in the inner layer at the increased injection rate.

The integral scales non-dimensionalized with the boundary layer thickness Λ_f/δ are shown in Fig. 7. In the inner layer, the longitudinal integral scale is found to be the same order of magnitude as the boundary layer thickness, but Λ_f/δ decreases with increasing the injection rate. The effect of injection becomes more noticeable as the wall is approached. In the outer layer, Λ_f/δ seems to decrease by an increment of the boundary layer thickness, since the absolute value of the integral scale Λ_f remains unchanged.

Figure 7. Distribution of longitudinal integral length scales

3.4 Transverse correlation coefficient and integral length scale

The transverse correlations have been measured by using two hot wire probes located at the same position in the streamwise and spanwise directions and held perpendicular to the flow and parallel to the wall. One of the probes which lies nearer to the wall is fixed in position and the other probe is traversed outwards in the boundary layer. The transverse correlation coefficient $R_{11}(\Delta y)$ and the transverse integral scale Λ_g are, respectively, defined as follows:

$$R_{11}(\Delta y) = \frac{u(y)u(y+\Delta y)}{\sqrt{u(y)^2} \sqrt{u(y+\Delta y)^2}} \quad (7)$$

$$\Lambda_g = \int_0^\infty R_{11}(\Delta y) d(\Delta y) \quad (8)$$

where Δy denotes the space between two hot wires. In Eq. (8), integral calculus is performed up to the position of Δy where the correlation coefficient crosses zero for the first time.

The results of the transverse correlation coefficient are shown in Figs. 8(a) and 8(b). Like the longitudinal correlation coefficient, in the inner layer, the transverse correlation coefficient tends to distribute more widely as the normal distance

from the wall increases, and therefore, the transverse integral scale becomes larger. In order to investigate the effect of injection on the correlation coefficient, two typical comparisons are made at the same distance from the wall, one in the inner layer and the other in the outer layer. The comparison is shown in Fig. 9. The result indicates that the transverse integral scale in the inner layer increases with increasing the injection rate, which is the opposite to

(a) $F = 0.0$

(b) $F = 0.00351$

Figure 8. Transverse correlation coefficient

Figure 9. Effect of injection on transverse correlation coefficient

the tendency found for the longitudinal integral scale. The reduction in the mean shear stress due to injection, resulting in a smaller stretching rate of eddies, brings about turbulent eddies less reduced in size in the direction normal to the streamwise one. On the basis of this consideration, the results in Fig. 9 are considered to be consistent with those in Fig. 5. On the other hand, the injection does hardly affect the distribution of the correlation coefficients in the outer layer.

The transverse integral scales non-dimensionalized with the boundary layer thickness Λ_g/δ are shown in Fig. 10. In the region close to the wall ($y/\delta < 0.1$), the data of Λ_g/δ obtained at different injection rates tend to fall on the profile obtained for the case without injection. Λ_g/δ is, thus, hardly affected by the injection. In the outer layer, however, Λ_g/δ decreases by an increment of the boundary layer thickness, since the absolute value of the transverse integral scale remains unchanged with injection.

Baker and Launder⁽⁹⁾ investigated the mixing length in a transpired turbulent boundary layer by measuring the mean velocity profiles. Their results indicate that in the inner layer the mixing length is proportional to the distance from the wall and the proportional constant (von Kármán constant) does not change due to injection, and that in the outer layer the mixing length distributes almost uniformly and it decreases with increasing the injection rate, when non-dimensionalized with the boundary layer thickness. The present statistical results of the transverse integral scale correspond qualitatively well to their phenomenological results for the mixing length.

The effects of injection on the longitudinal and transverse integral scales are summarized in Table 1, where *Absolute value* denotes the value of the integral scale itself and *Relative value* denotes the scale non-dimensionalized with the boundary layer thickness.

Figure 10. Distribution of transverse integral length scales

3.5 Ratio of longitudinal and transverse integral scales

In Fig. 11 is shown the ratio of the transverse to the longitudinal integral scales Λ_g/Λ_f . In general, the eddies are strained and stretched by the mean shear stress in shear flows. There exists a strong mean shear stress in the boundary layer near the wall and eddies are supposed to be stretched there considerably in the streamwise direction. Therefore, the scale ratio Λ_g/Λ_f would be expected to take a very small value. This is clearly shown by the experimental results of space-time correlation of Favre⁽²⁴⁾. A reduction of the velocity gradient due to injection near the wall is expected to result in a reduction of the stretching rate of eddies. As seen in the figure, the integral scale ratio Λ_g/Λ_f increases with increasing the injection rate, approaching a value of 0.5. In isotropic turbulence, the ratio Λ_g/Λ_f should be 0.5: Comte-Bellot and Corrsin⁽²⁵⁾. The present result supports the suggestion by Wooldridge and Muzzy⁽¹³⁾ that, as the injection rate is increased, a progressively larger portion of the boundary layer tends to become isotropic in character.

3.6 Differential length scale

The differential length scale of the streamwise velocity fluctuation λ is defined by the following equation;

$$\frac{1}{\lambda^2} = - \left(\frac{\partial^2 R_{11}}{\partial x^2} \right)_{x=0} \quad (9)$$

By assuming the homogeneity of turbulence in the streamwise direction and introducing Taylor's hypothesis, Eq. (9) can be reduced to the following equation;

$$\lambda = \frac{u' U}{\sqrt{(\partial u / \partial t)^2}} \quad (10)$$

Figure 11. Ratio of transverse and longitudinal integral scales

	Inner layer		Outer layer	
	Absolute value	Relative value	Absolute value	Relative value
Λ_f	Decreasing	Decreasing	Unchanged	Decreasing
Λ_g	Increasing	Unchanged	Unchanged	Decreasing

Table 1. Effect of injection on the integral length scale

In this study the differential scale is obtained from Eq. (10) by passing a hot wire signal through a differentiating circuit. It was constructed with an amplifier LM301A commercially available. In Fig. 12 is shown the frequency response of the differentiating circuit used in this study. The cut-off frequency, at which the data deviate from the straight line in the figure, was set at about 10 kHz in this experiment, since there appeared no significant difference in the results when the cutoff frequency was varied from 5 kHz to 30 kHz. The circuit was constructed so as to avoid an undesirable electronic noise of higher frequency.

The present results of the differential length scale are shown in Fig. 13, compared with those of Narayanan et al.⁽²⁶⁾ in a non-transpired turbulent boundary layer. The Reynolds number of their experiment is almost the same as that of the present study ($Re_\theta \approx 3000$). The differential scale non-dimensionalized with the boundary layer thickness tends to decrease with increasing the injection rate in the region close to the wall. The absolute values of these differential scales also tend to decrease with injection. The present result might also be brought about by the reduction in the mean shear stress, since the turbulent Reynolds number is fairly small in this experiment and there might be a possibility that the mean flow field exerts influence on small eddies as well. On the other hand, they seem to distribute almost uniformly and are independent

Figure 12. Frequency response of differentiating circuit

Figure 13. Distribution of differential length scales

of the injection rate in the outer layer, although there is a little scattering of the data.

3.7 Ratio of differential and integral scales

The ratio of the streamwise differential to the integral scales is shown in Fig. 14. The ratio of these length scales of turbulence is almost constant in the outer layer and moreover does not depend on the injection rate. On the other hand, it increases with injection in the inner layer and the effect of injection on this scale ratio becomes noticeable as the wall is approached.

In general, the ratio of the small scale (differential scale) to the large scale (integral scale) depends on the turbulent Reynolds number⁽²⁷⁾; namely, it becomes smaller as the Reynolds number increases. In Fig. 15 are shown the present results of λ/Λ_f against the turbulent Reynolds number $u'\Lambda_f/\nu$. It can be seen in the figure that the ratio of the length scales λ/Λ_f decreases as a whole as the turbulent Reynolds number increases. A remarkable point is that the turbulent Reynolds number is reduced at increased injection rate even if

Figure 14. Ratio of differential and integral scales

Figure 15. Turbulent Reynolds number dependence of the ratio of differential and integral length scales

the measuring position is kept unchanged. Therefore, one of the reasons causing λ/Λ_f to increase near the wall due to injection (see Fig. 14) appears to be the reduction in the turbulent Reynolds number. However, when the results at different injection rates are compared at the same turbulent Reynolds number, the ratio of the length scales λ/Λ_f is found to be further dependent on the injection rate. Therefore, there might be another effect of injection on the ratio of these turbulent scales.

4. Concluding remarks

The integral and the differential length scales of turbulence have been measured with a hot wire anemometer in a turbulent boundary layer with injection, and the effects of injection on these scales have been discussed. The main results obtained are summarized as follows:

(1) In the near wall region, the longitudinal integral scale non-dimensionalized with the boundary layer thickness decreases with increasing the injection rate, and the effect of injection becomes more noticeable as the wall is approached. This appears to be caused by the reduction in the mean shear stress due to injection near the wall.

(2) In the inner layer, the transverse integral scale non-dimensionalized with the boundary layer thickness remains unchanged near the wall independent of the injection rate, but it decreases with injection in the outer layer. The qualitative tendency of the present results for the transverse scales corresponds well to those of the mixing length measured by Baker and Launder.

(3) The ratio of the transverse to the longitudinal integral scales increases and approaches a value of 0.5 with increasing the injection rate. This confirms that the turbulence in the boundary layer tends to be close to isotropy in character as the injection rate is increased.

(4) The differential scale of the streamwise velocity fluctuation decreases with injection near the wall. The ratio of this differential to the longitudinal integral scales becomes greater with increasing the injection rate, and it becomes more noticeable as the wall is approached. One of the reasons is the reduction in the turbulent Reynolds number due to injection.

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