536.242::532.525

Paper No. 196-16

A Study on the Laminar-flow Heat Transfer Between
a Two-dimensional Water Jet and a Flat Surface
with Constant Heat Flux*

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The authors obtained more accurate solutions of the boundary layer momentum and energy equations for a wedge-type flow by the electronic computer using the Runge-Kutta method and deduced an approximate equation of the temperature gradient at the wall with a high accuracy. When a two-dimensional water jet with finite nozzle width strikes normally on a flat surface, it forms a flow field which can be divided into two flow regions; the first one is an impingement zone and the other is a uniform parallel flow zone. It is the purpose of this paper to estimate local heat transfer coefficients in such a flow field, where the velocity gradient just outside the boundary layer changes along a flat surface and to study theoretically the effect of the nozzle height above the flat plate on the local heat transfer coefficient. Furthermore the calculated results were compared with experimental ones of local heat transfer coefficients along a flat surface with constant heat flux.

1. Introduction

Both impinging gas and liquid jets are used extensively in a wide variety of industrial processes for both cooling and heating purposes. Their attractive features are the simplicity of the equipment, the ease of control for a given mass flow rate and the attainment of high heat transfer coefficients with small temperature differences. Most recent work in this field has been stimulated by the application of gaseous impingement cooling to high temperature gas turbine engines; consequently, available informations are almost concentrated on gas jets and wall jets. A lot of measurements with water jet have been reported in the fields of the rapid quenching of steel and subcooled boiling with forced convection. When a two-dimensional water jet with finite nozzle width strikes normally on a flat surface, it forms a flow field which can be divided into two flow regions; the first one is an impingement zone and the other is a uniform parallel flow zone. Only a few studies of local heat transfer in such a flow field have been reported under the conditions that phase change or free convection does not occur. It is the purpose of this paper to estimate local heat transfer coefficients in such a flow

field, where the velocity gradient just outside the boundary layer changes along a flat surface and to study theoretically the effect of the nozzle height above the flat plate on the local heat transfer coefficient. In carrying out theoretical analysis, it is convenient to adopt Levy's solution⁽¹⁾ for heat transfer from a two-dimensional body, but the accuracy of Levy's approximate function which determines the heat transfer rate at the wall is not satisfactory. Then, the authors obtained more accurate solutions of the boundary layer momentum and energy equations for a wedge-type flow by the electronic computer using the Runge-Kutta method and deduced an approximate equation of the temperature gradient at the wall with a high accuracy. Furthermore they compared the calculated results with experimental results of local heat transfer coefficients along a flat surface with constant heat flux.

Nomenclature

- A : constant of proportionality
($T_w - T_\infty = Ax^r$)
 B : width of jet nozzle
 C : constant of proportionality
($u_{max} = Cx^n$)
 $F(k, \phi)$: incomplete elliptic integral of the first kind with modulus k and amplitude ϕ
 H : distance between nozzle and flat plate
 H_0 : thickness of jet liquid film at infinity
 K : complete elliptic integral of the first kind with modulus k

* Received 27th October, 1979.

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- K' : complete elliptic integral of the first kind with modulus k'
 m : Euler number ($u_{\max} = Cx^m$)
 n : index of Prandtl number
 P : static pressure along the flat plate
 $\dot{q}_w$: local heat transfer rate
 T : temperature in the boundary layer
 T_f : temperature of the fluid
 T_w : temperature of the plate surface
 T_∞ : temperature at the edge of the boundary layer $= T_f$
 $u_{\max}$: velocity just outside of the boundary layer
 u_0 : jet exit velocity
 u : velocity component of flow in x direction
 v : velocity component perpendicular to the plate surface
 x : distance along the plate surface from the stagnation point
 y : distance normal to the plate surface
 α : local heat transfer coefficient
 α_0 : heat transfer coefficient at the stagnation point
 β : velocity distribution parameter $= 2m/(m+1)$
 γ : the exponent controlling temperature distribution along the surface ($T_w - T_\infty = Ax^\gamma$)
 λ : thermal conductivity of fluid
 C_p : specific heat of fluid
 ρ : density of fluid
 μ : viscosity of fluid
 ν : kinematic viscosity of fluid
 ϕ : stream function
 f : nondimensional stream function
 η : nondimensional co-ordinates
 θ : nondimensional temperature $= (T - T_\infty)/(T_w - T_\infty)$
 N_{ux} : local Nusselt number $= \alpha x/\lambda$
 N_{uB} : Nusselt number $= \alpha B/\lambda$
 Pr : Prandtl number
 Re_s : local Reynolds number $= xu_{\max}/\nu$
 Re_B : nozzle Reynolds number $= Bu_0/\nu$

2. Fundamental equation

The solution of the laminar boundary layer equations for a two-dimensional incompressible flow over wedges is based on the following variation in free stream velocity which is proportional to some power of the distance from the stagnation point.

$$u_{\max} = Cx^m \quad \dots\dots\dots(1)$$

where m is the Euler number defined by,

$$m = \frac{x}{u_{\max}} \frac{du_{\max}}{dx} \quad \dots\dots\dots(2)$$

The local heat transfer coefficients on the plate surface for normal impingement of a water jet will be decided theoretically by employing Kezios's idea⁽⁵⁾ which is based on the assumption that heat transfer coefficient at any point along the meridian profile of a body with arbitrary cross section is the same as that on a wedge which has the same velocity and gradient (just outside the boundary layer) at the same distance from the stagnation point as the real profile.

According to this procedure the two-dimensional surface is replaced by a series of wedges, each having an angle uniquely determined by the value of Euler number.

2.1 Determination of the Euler number

The velocity distribution at the outer edge of the boundary layer may be computed by using Strand's analytical theory⁽⁶⁾ for two-dimensional solid jets impinging normally on a flat surface. Figure 1 shows the model for analysis.

Fig.1 Model for analysis

Strand took account of the distance between nozzle and flat plate as a parameter in his analytical theory. A nondimensional parameter H/B is transformed into a modulus k of elliptic integral, and is expressed by the following equation.

$$H/B = \frac{K' + \pi/2}{2[K + \ln\{k'/(1-k)\}]} \quad \dots\dots\dots(3)$$

where $0 \leq k \leq 1$ and $k' = \sqrt{1-k^2}$.

The nondimensional velocity $u_{\max}/u_0$ and the nondimensional distance x/B are expressed by the following equations:

$$u_{\max}/u_0 = \sqrt{(\epsilon-1)/(\epsilon+1)} \quad \dots\dots\dots(4)$$

$$\frac{x}{B} = \frac{\ln\{(\sqrt{\epsilon^2-k^2} + \sqrt{\epsilon^2-1})/k'\} + K - F(k, \phi)}{2[K + \ln\{k'/(1-k)\}]} \quad \dots\dots\dots(5)$$

meanwhile ϕ can be expressed in terms of ϵ as follows:

$$\sin \phi = 1/\epsilon \quad \dots\dots\dots(6)$$

where $\epsilon \geq 1$. The velocity distribution at the outer edge of the boundary layer is obtained by elimination of ϵ from Eqs.(4) and (5). From this velocity distribution by employing Eq.(2), the local value of Euler number at a given position along the flat plate is obtained as,

$$m = 2 \left\{ K + \ln \frac{k'}{1-k} \right\} \frac{\sqrt{\epsilon^2-k^2}}{(\epsilon+1)^2} \frac{x}{B} \frac{1}{\sqrt{\epsilon-1}} \quad \dots\dots\dots(7)$$

2.2 Boundary layer equations

According to Bernoulli's equation, the velocity $u_{\max}$ in the stream outside the boundary layer is related to the pressure gradient in the following way,

$$dP/dx = -\rho u_{\max} du_{\max}/dx \quad \dots\dots\dots(8)$$

As the boundary layer along the flat plate is very thin, the pressure gradient in this layer scarcely changes. Accordingly the pressure within the boundary layer is prescribed by the flow of the main stream outside the boundary layer. The boundary layer forms of the continuity, momentum,

and energy equations for an incompressible constant property two-dimensional flow are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \dots\dots\dots(9)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_{\max} \frac{du_{\max}}{dx} + \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2} \quad \dots\dots(10)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\lambda}{C_p \rho} \frac{\partial^2 T}{\partial y^2} \quad \dots\dots(11)$$

with the boundary conditions,

$$y=0; \quad u=v=0, \quad T=T_w(x),$$

$$y \rightarrow \infty; \quad u=u_{\max}, \quad T=T_{\infty}.$$

The solution is facilitated by introducing the Hartree variables,

$$\eta = \sqrt{[(1+m)/2]} (u_{\max}/\nu x) y, \quad$$

$$\psi = \sqrt{[2/(1+m)]} \nu x u_{\max} f(\eta)$$

where ψ is a stream function defined by

$$u = \partial \psi / \partial y, \quad v = -\partial \psi / \partial x.$$

Then Eqs.(10) and (11) become

$$f''' + f f'' + \beta(1-f'^2) = 0 \quad \dots\dots(12)$$

$$\frac{\partial^2 T}{\partial \eta^2} + P_r f \frac{\partial T}{\partial \eta} - P_r(2-\beta) f' x \frac{\partial T}{\partial x} = 0 \quad \dots\dots(13)$$

where $\beta = 2m/(m+1)$, and $P_r (= \mu C_p / \lambda)$ is the Prandtl number. The primes indicate the order of differentiation with respect to η . Equation (13) is changed to an ordinary differential equation by introducing the nondimensional temperature

$$\theta = (T - T_{\infty}) / (T_w - T_{\infty}), \quad \text{where } T_w - T_{\infty} = A x^r.$$

Thus,

$$\frac{d^2 \theta}{d\eta^2} + P_r f \frac{d\theta}{d\eta} - P_r(2-\beta) f' \theta = 0 \quad \dots\dots(14)$$

with the boundary conditions,

$$\eta=0; \quad \theta=1, \quad \eta \rightarrow \infty; \quad \theta=0.$$

When the heat transfer surface is heated by passing a current, generally the heat flux is kept at a constant state. In this case $r = (1-\beta)/(2-\beta)$, then equation (14) becomes,

$$\frac{d^2 \theta}{d\eta^2} + P_r f \frac{d\theta}{d\eta} - P_r(1-\beta) f' \theta = 0 \quad \dots\dots(15)$$

2.3 Relationship equation of heat transfer

The heat transfer rate at the wall can be expressed as,

$$q_w = -\lambda \left(\frac{\partial T}{\partial y} \right)_{y=0}$$

$$= -\lambda (T_w - T_{\infty}) \frac{1}{\sqrt{2-\beta}} \sqrt{\frac{u_{\max}}{\nu x}} \left(\frac{d\theta}{d\eta} \right)_{\eta=0} \quad \dots\dots(16)$$

or

$$N_{\text{us}} / \sqrt{R_{\text{ex}}} = \frac{\alpha x}{\lambda} \sqrt{\frac{u_{\max} x}{\nu}} = - \left(\frac{d\theta}{d\eta} \right)_{\eta=0} \frac{1}{\sqrt{2-\beta}} \quad \dots\dots(17)$$

where $-(d\theta/d\eta)_{\eta=0}$ is determined from Eq.(14) or Eq.(15). As is generally known, Eq.(14) has been solved by Pohlhausen⁽⁷⁾ and by Eckert⁽⁸⁾ for $r=0$ (an isothermal wall) and the solution can be obtained by quadratures. When r is not zero, a solution of Eq.(14) can be obtained numerically except for a particular case in which the factor $(2-\beta)r$ is equal to -1 . In this case, $(d\theta/d\eta)_{\eta=0} = 0$.

3. Numerical solution

3.1 Finite difference method

Levy⁽⁴⁾ solved Eqs.(12) and (14) making use of the finite difference method in the range of r from -0.455 to 4.0 and of Prandtl numbers from 0.7 to 10 for four values of β ($1.6, 1.0, 0, -0.199$). Values of $(d\theta/d\eta)_{\eta=0}$ reported by Levy are about $0.7 \sim 4.2$ percent lower than exact solutions derived by Eckert and the error in the approximate equation of $(d\theta/d\eta)_{\eta=0}$ obtained by Levy reaches $-8 \sim 4$ percent in comparison with exact solutions. In use of the finite difference method, the size of the interval $\Delta\eta$ and the total number of steps N must be chosen to make errors negligible in the final results. In Levy's analysis the interval $\Delta\eta$ was taken as 0.1 mainly because the Hartree functions⁽⁹⁾ are tabulated for each value of $\eta = 0.1$. In the present work the interval was changed from 0.1 to 0.01 , and it could be confirmed that the resulting error decreased as $\Delta\eta$ decreased.

3.2 The Runge-Kutta method

In the present work Eqs.(12) and (14) were solved mainly by the Runge-Kutta method. The calculation of the Runge-Kutta method was started with the initial values $f(0), f'(0), f''(0)$ and $(d\theta/d\eta)_{\eta=0}$, where $f''(0)$ and $(d\theta/d\eta)_{\eta=0}$ were chosen firstly as a close approximation to the required value. The values of $f''(0)$ and $(d\theta/d\eta)_{\eta=0}$ are corrected little by little by trial and error method until $f'(\infty)=1$ or $\theta(\infty)=0$ is satisfied. Calculations were made on two cases of $\Delta\eta=0.05$ and 0.01 . Reliable number of digits of $f''(0)$ and $(d\theta/d\eta)_{\eta=0}$ is up to a decimal place whose number agreed in two cases of $\Delta\eta$. Four decimal places have been employed in this study as a result of reliable number of digits. Values of $f''(0)$ are in agreement with those reported by Hartree up to the four decimal places. Values of $(d\theta/d\eta)_{\eta=0}$ also are in agreement with exact solutions solved by Eckert for the case of an isothermal wall. Values of $(d\theta/d\eta)_{\eta=0}$ calculated in this study by the Runge-Kutta method are indicated in Tables 1-4.

4. Experimental results and discussion

4.1 The distribution of velocities and the corresponding Euler number at the outer edge of the boundary layer

Plots of the velocity distribution at the outer edge of the boundary layer for constant values of the parameter (H/B) are presented in Fig.2. A comparison between calculated values and measured values is shown in Fig.2. Measured values are computed from the pressure distribution by using the equation of Bernoulli. The experimental values for $H/B=19$ are in agreement with the calculated values for $H/B=5$. From Fig.2 it can be seen that the inclination of velocity in the vicinity of $x/B=0$ is directly proportional to x/B . This is characteristic of a flow in the stagnation region. Beyond this region the

Table 1 Values of $-(d\theta/d\eta)_{\eta=0}$
for $\beta = -0.19$, $f''(0) = 0.0857$

r	P_r	0.7	1.0	2.0	5.0	10.0
-0.25		0.218 1 ₈	0.244 1 ₆	0.301 3 ₁	0.393 9 ₆	0.481 2 ₆
0.0		0.327 8 ₉	0.365 0 ₄	0.447 8 ₆	0.584 8 ₃	0.715 8 ₀
0.25		0.393 9 ₅	0.437 8 ₀	0.536 2 ₆	0.700 5 ₇	0.858 6 ₈
0.5		0.442 0 ₆	0.490 8 ₈	0.600 9 ₉	0.785 7 ₁	0.964 0 ₇
1		0.512 7 ₁	0.569 0 ₂	0.696 7 ₀	0.912 1 ₃	1.120 9 ₈
2		0.608 8 ₈	0.675 7 ₅	0.828 1 ₀	1.086 5 ₇	1.338 0 ₇
3		0.678 2 ₁	0.752 8 ₈	0.923 4 ₄	1.213 5 ₉	1.496 4 ₈
4		0.733 8 ₂	0.814 8 ₅	1.000 2 ₁	1.316 1 ₀	1.624 4 ₇

Table 2 Values of $-(d\theta/d\eta)_{\eta=0}$
for $\beta = 0.0$, $f''(0) = 0.4696$

r	P_r	0.7	1.0	2.0	5.0	10.0
-0.25		0.278 0 ₄	0.316 8 ₈	0.405 5 ₆	0.556 3 ₃	0.703 6 ₁
0.0		0.413 9 ₃	0.469 6 ₂	0.597 2 ₆	0.815 5 ₉	1.029 7 ₈
0.25		0.504 7 ₄	0.571 4 ₃	0.724 6 ₆	0.987 6 ₃	1.246 1 ₂
0.5		0.574 0 ₄	0.649 1 ₀	0.821 8 ₇	1.118 9 ₃	1.411 2 ₇
1		0.679 3 ₂	0.767 1 ₆	0.969 7 ₄	1.318 8 ₃	1.662 7 ₇
2		0.827 1 ₇	0.933 1 ₂	1.177 9 ₅	1.600 6 ₂	2.017 4 ₆
3		0.935 9 ₂	1.055 3 ₀	1.331 4 ₂	1.808 5 ₃	2.279 2 ₄
4		1.024 1 ₁	1.154 4 ₄	1.456 0 ₄	1.977 4 ₂	2.491 9 ₆

Table 3 Values of $-(d\theta/d\eta)_{\eta=0}$
for $\beta = 1.0$, $f''(0) = 1.2326$

r	P_r	0.7	1.0	2.0	5.0	10.0
-0.75		0.193 2 ₀	0.224 7 ₅	0.298 7 ₃	0.427 8 ₀	0.555 3 ₈
-0.5		0.322 3 ₄	0.373 0 ₀	0.491 2 ₂	0.696 4 ₃	0.898 8 ₈
-0.25		0.418 7 ₉	0.482 9 ₆	0.632 2 ₄	0.890 8 ₁	1.145 6 ₈
0.0		0.495 8 ₈	0.570 4 ₃	0.743 7 ₂	1.043 4 ₃	1.338 8 ₂
0.25		0.560 3 ₂	0.643 4 ₄	0.836 2 ₈	1.169 6 ₈	1.498 2 ₁
0.5		0.615 9 ₀	0.708 2 ₆	0.915 7 ₇	1.277 8 ₄	1.634 6 ₄
1		0.708 9 ₈	0.811 3 ₀	1.048 3 ₆	1.457 9 ₃	1.861 5 ₈
2		0.851 9 ₅	0.972 3 ₉	1.251 2 ₈	1.733 1 ₀	2.208 0 ₉
3		0.962 6 ₇	1.097 0 ₃	1.408 1 ₂	1.945 6 ₄	2.475 6 ₆
4		1.054 5 ₃	1.200 4 ₁	1.538 1 ₆	2.121 8 ₅	2.697 4 ₈

Table 4 Values of $-(d\theta/d\eta)_{\eta=0}$
for $\beta = 1.6$, $f''(0) = 1.5215$

r	P_r	0.7	1.0	2.0	5.0	10.0
-1.5		0.283 4 ₀	0.329 6 ₈	0.438 5 ₁	0.628 9 ₄	0.817 5 ₅
-1.0		0.375 5 ₁	0.435 3 ₅	0.575 4 ₉	0.819 7 ₅	1.061 2 ₄
-0.5		0.450 7 ₈	0.521 3 ₀	0.686 0 ₀	0.972 3 ₅	1.255 1 ₇
0.0		0.514 5 ₄	0.593 8 ₆	0.778 7 ₇	1.099 7 ₇	1.416 6 ₀
0.5		0.569 9 ₇	0.656 8 ₀	0.858 9 ₄	1.209 4 ₇	1.555 3 ₂
1		0.619 1 ₃	0.712 5 ₁	0.929 7 ₂	1.306 0 ₈	1.677 3 ₂
2		0.703 7 ₄	0.808 2 ₆	1.051 0 ₂	1.471 2 ₅	1.885 6 ₆
3		0.775 3 ₇	0.889 1 ₇	1.153 2 ₈	1.610 2 ₃	2.060 8 ₀
4		0.837 8 ₅	0.959 6 ₉	1.242 2 ₈	1.731 0 ₅	2.212 9 ₉

rate of change of velocity gradually decreases up to about $x/B=2.0$ after which it approaches zero asymptotically. However, in case of $H/B \leq 0.25$, the rate of change of velocity increases rapidly and

Fig.2 Velocity distribution at the outer edge of the boundary layer

Fig.3 Values of H/B and H_0/B against k

Fig.4 Distribution of the parameters

the asymptotic value is almost reached in the neighborhood of the jet exit. A comparison between calculated value of $H/B=0.1$ and result of experiments by Akfirat⁽¹⁰⁾ is shown in Fig.2. The two show a similar distribution qualitatively. The values of H/B and H_0/B are given in Fig.3 against the modulus k of the elliptic integral. According to Strand's theoretical analysis⁽⁶⁾, the relation between H_0/B and k is expressed as,

$$\frac{H_0}{B} = \frac{\pi}{4[K + \ln[k'/(1-k)]]} \quad \dots\dots(18)$$

As is evident from Fig.3, the value of k is nearly equal to zero in the range of $H/B \geq 3$. Consequently, the flow in this range is characterized by a specific velocity distribution. Such a velocity distribution is expressed in stagnation flow zone by Eq.(1) in which the Euler number m takes a value of unity and the velocity gradient is

$$C = (u_0/B)k'[K + \ln[k'/(1-k)]]/2 = (\pi/4)(u_0/B).$$

Beyond this zone the rate of change of velocity gradually approaches zero as x/B increases. The local value of Euler number at a given position along the plate may be found directly from the velocity distribution by employing Eq.(7). In Fig.4 a distribution of the parameter $2m/(m+1)$, which has an initial value of 1.0 at the stagnation point and describes a particular flow condition, is illustrated because it is convenient for the subsequent calculation to introduce such a parameter.

4.2 Stagnation point heat transfer

In the neighborhood of the stagnation point, Euler number m takes a value of unity. Accordingly the inclination of velocity at the stagnation point may be expressed as

$$(u_{max}/u_0)/(x/B) = C(B/u_0) = \{(u_{max}/u_0)/(x/B)\}_{x=0}$$

and the following equation is derived from Eq.(7)

$$\left(\frac{u_{max}/u_0}{x/B}\right)_{x=0} = \frac{k'}{2} \left\{ K + \ln \frac{k'}{1-k} \right\} \quad \dots\dots(19)$$

On this basis, the following relationship equation may be used to determine the heat transfer coefficient at the stagnation point.

$$\frac{\alpha_0 B}{\lambda} \sqrt{u_0 B/\nu} = - \left(\frac{d\theta}{d\eta} \right)_{\eta=0} \sqrt{\frac{u_{max}/u_0}{x/B}} \\ = - \left(\frac{d\theta}{d\eta} \right)_{\eta=0} \sqrt{\frac{k'}{2} \left\{ K + \ln \frac{k'}{1-k} \right\}} \quad \dots\dots(20)$$

where α_0 is heat transfer coefficient at the stagnation point and values of $-(d\theta/d\eta)_{\eta=0}$ can be approximated in a Prandtl number range from 0.7 to 10 within ± 0.9 per cent

Fig.5 Heat transfer coefficient at the stagnation point

by the following equation for a specific case in which the value γ is equal to zero.

$$-(d\theta/d\eta)_{\eta=0} = 0.570 P_r^{0.47}$$

This relation is in good agreement with evaluation of Goldstein⁽¹¹⁾ and Eckert.⁽⁶⁾ The value of $(N_{uB}/\sqrt{Re_B})P_r^{-0.47}$ as a function of H/B given by Eq.(20) is plotted in Fig.5 by solid line for $\gamma=0$ and dotted lines for $\gamma \neq 0$. It is worth noting that there is a maximum point of heat transfer coefficient in the neighborhood of $H/B=0.7$.

4.3 Local heat transfer coefficient

The relationship of local heat transfer coefficients for a uniform heat flux surface is plotted in Fig.6 in a Prandtl number range from 0.7 to 10 with accuracy higher than ± 1.3 %. The following heat transfer relationship equation expressed by Eq.(21), given by Eckert for a uniform temperature surface is plotted in Fig.7.

$$N_{uB} = 0.56 \frac{(\beta + 0.2)^{0.11}}{\sqrt{2-\beta}} P_r^{0.85+0.02\beta} \sqrt{Re_B} \quad \dots\dots(21)$$

This equation can be rewritten as,

$$(N_{uB}/\sqrt{Re_B})P_r^{-(0.85+0.02\beta)} \\ = 0.56 \frac{(\beta + 0.2)^{0.11}}{\sqrt{2-\beta}} \sqrt{\frac{u_{max}/u_0}{x/B}} \quad \dots\dots(22)$$

Fig.6 Heat transfer coefficients for a uniform heat flux surface

Fig.7 Heat transfer coefficients for a uniform temperature surface

If we compare Fig.6 and Fig.7, it is clear that heat transfer coefficients at the stagnation point coincide with each other. As the distance from the stagnation point increases, the local heat transfer coefficient both decreases and approaches respective value of it asymptotically in uniform parallel flow zone along a flat

plate. In an H/B range from 0.1 to 0.25, there is a place where local heat transfer coefficients have a peak. This peak appears at a point of about half nozzle width from the stagnation point for a uniform temperature surface as shown in Fig.7 and moves a little from that point to the stream for a uniform heat flux surface as shown in Fig.6. Such an abrupt rise in heat transfer coefficients following the stagnation point is the result of flow acceleration.

4.4 Estimation of temperature gradient at wall

Levy⁽⁴⁾ showed the relationship equation and temperature gradient at the wall as expressed by Eq.(23) and Eq.(24) respectively, which were required in determining local heat transfer coefficients.

$$N_{\text{us}}/\sqrt{R_{\text{eB}}} = 0.57 \frac{1}{\sqrt{2-\beta}} (\beta+0.205)^{0.104} \times \{(2-\beta)\gamma+1\}^{0.87+0.06\beta} P_r^n \quad \text{.....(23)}$$

$$-\left(\frac{d\theta}{d\eta}\right)_{\eta=0} = 0.57 (\beta+0.205)^{0.104} \{(2-\beta)\gamma+1\}^{0.87+0.06\beta} P_r^n \quad \text{.....(24)}$$

where the exponent n of the Prandtl number is 0.367 for $\beta=1.6$, 0.355 for $\beta=1.0$, 0.327 for $\beta=0$ and 0.254 for $\beta=-0.199$ in a Prandtl number range from 0.7 to 10. The term $\sqrt{2-\beta}$ on the righthand side of Eq.(23) is omitted in his paper, but it may be a misprint. In comparison with Levy's numerical solution, the error due to this approximate equation (24) is ± 5 per cent for $\gamma \geq 0$, $\beta=1.0$ and 1.6, and is ± 6 per cent for $\gamma \geq 0$, $\beta=0$, but the error exceeds ± 10 per cent for negative γ values. Following is an approximate equation of temperature gradient at wall, which the authors estimated from Tables 1-4 with a sufficient accuracy.

$$-(d\theta/d\eta)_{\eta=0} = 0.757(\beta+0.2)^{0.1} \{[(2-\beta)\gamma+1]^a - 0.26[(2-\beta)\gamma+1]P_r^b\} \quad \text{.....(25)}$$

$$a = 0.628 + 0.025\beta - 0.003(1-\beta)(2-\beta)(1-\gamma) \\ b = 0.34 + 0.03\beta - 0.004\gamma - 0.02\beta(\beta-1)$$

This equation is approximated with accuracy higher than ± 2 per cent in a Prandtl number range from 0.7 to 10, in a β range from 0 to 1.6 and in a γ range from 0 to 4 (when $\beta=1.0$, γ from -0.25 to 4 and $\beta=1.6$, γ from -0.5 to 4). In this case, the relationship equation of heat transfer is expressed as,

$$N_{\text{us}}/\sqrt{R_{\text{eB}}} = 0.757(\beta+0.2)^{0.1} \{[(2-\beta)\gamma+1]^a - 0.26[(2-\beta)\gamma+1]P_r^b\} \frac{1}{\sqrt{2-\beta}} \sqrt{\frac{\mu_{\text{max}}/\mu_0}{x/B}} \quad \text{.....(26)}$$

Under the uniform heat flux condition, temperature gradient at wall is approximated within ± 1.3 per cent by Eq.(27).

$$-(d\theta/d\eta)_{\eta=0} = 0.757(\beta+0.2)^{0.1} \{[(2-\beta)\gamma+1]^a - 0.26(2-\beta)P_r^b\} \quad \text{.....(27)}$$

$$a = 0.615 + 0.07\beta, \quad b = 0.34 + 0.036\beta$$

Consequently, the relationship equation of heat transfer becomes,

$$N_{\text{us}}/\sqrt{R_{\text{eB}}} = 0.757(\beta+0.2)^{0.1} \{[(2-\beta)\gamma+1]^a$$

$$- 0.26(2-\beta)P_r^b\} \frac{1}{\sqrt{2-\beta}} \sqrt{\frac{\mu_{\text{max}}/\mu_0}{x/B}} \quad \text{.....(28)}$$

5. Comparison of the experimental results with calculated results

5.1 Experimental apparatus and procedure

Figure 8 shows the experimental apparatus for a two-dimensional water jet. The slit nozzle of the knife edge type and the overflow tank were designed to maintain the two-dimensional character of the laminar jet. Tap water was poured into the bottom of the overflow tank which had a large capacity for the quantity of liquid spouted from the nozzle. A wire netting was put on the surface of a liquid in order to prevent the growth of waves formed on it. A platinum foil of thickness 50 μm was heated directly by alternating current and worked as a heated surface. This heated surface was movable in the horizontal direction by means of a lead-screw and a spring, but the movable distance in the vertical direction, that is, the distance from the outlet of the jet nozzle to the heated surface was restricted to 15 mm. Figure 9 shows details of the test section, heat transfer

Fig.8 Experimental apparatus

Fig.9 Details of the test section

surface and thermocouple location. Two conducting copper blocks were connected by a platinum foil of thickness $50\text{ }\mu\text{m}$, and the horizontal square central part of this foil was $10\text{ mm} \times 10\text{ mm}$. The whole size of this platinum foil was $14\text{ mm} \times 20\text{ mm}$. The both edges of the foil which were tucked in 2 mm were separated by a distance 1.0 mm for insulation. The both sides of the foil were fixed on the conducting copper blocks by diffusion-bonding method to reduce the contact electrical resistance. The a.c. power input was controlled with a 10-KVA adjustable transformer and supplied to the electrodes through another transformer of low voltage and high current. The temperature was measured by alumel-chromel thermocouples of $50\text{ }\mu\text{m}$ dia. attached to the back side of the foil by spot welding. The surface temperature of the foil was determined taking into account the thickness of the foil, the thermal conductivity of it and the heat flux across the foil per unit time and area. The platinum wires of $50\text{ }\mu\text{m}$ dia. used as voltage taps were attached in the same way as the thermocouples. The thermocouples and voltage taps were electrically insulated by a Teflon coating. The back side of platinum foil, particularly the slit part was coated with Teflon to prevent both entry of the jet water into the back side and heat loss from the same side. The back side was stiffened with epoxy resin to prevent a swelling of the platinum foil induced by heat expansion. The distance between the voltage taps attached to the central part of the heated surface was about 1.0 mm . The voltage was measured by a highly sensitive a.c. mV meter. The electric current was measured through the shunt-resistor with capacity of 200 ampere, 60 mV. The experiments were carried out under the following condition,

$u_0 = 1.0\text{ m/sec}$, $H/B = 16$, $Re_B = 940$,
 $q_w = 6 \times 10^4 \sim 10^5\text{ kcal/m}^2\text{h}$ and $T_w - T_f = 5 \sim 10\text{ K}$.
 We took account of the variation of properties with temperature by evaluating the properties at the film temperature defined by $(1/2)(T_w + T_f)$.

5.2 A comparison between theoretical and experimental results

Figure 10 shows a comparison between the values calculated with Eq.(28) and the results of the present experiments indicated by open circles. In the same figure, another relationship equation of heat transfer under the uniform heat flux condition, which was derived by Levy, is shown by dash-dotted line, that is, substituting the equation $\gamma = (1-\beta)/(2-\beta)$ into Eq.(23),

$$N_{uB}/\sqrt{Re_B} = 0.57(\beta + 0.205)^{0.106}(2-\beta)^{0.37+0.04\beta} \times \frac{1}{\sqrt{2-\beta}} \sqrt{\frac{u_{max}x/u_0}{x/B}} Pr^n \quad \text{.....(29)}$$

where the exponent n of the Prandtl number, $n = 0.327 + 0.028\beta$ for $0 \leq \beta \leq 1$. Furthermore, the following empirical equation, $N_{uB}/\sqrt{Re_B} = 0.75Pr^{1/8}$, which was derived by McMurray et al.⁽¹⁾, is shown by a broken line in Fig.10 and rewritten as

Fig.10 Comparison between theoretical and experimental results

$$N_{uB}/\sqrt{Re_B} = 0.75Pr^{1/8} \sqrt{\frac{u_{max}x/u_0}{x/B}} \quad \text{.....(30)}$$

This equation ought to agree inherently with Eq.(22). However, the above Eq.(30) has a coefficient of 0.75, while Eq.(22) has a coefficient of 0.57. Thermal boundary layer is liable to receive a disturbance because the thickness of it is the least at the stagnation point. Consequently, it is likely that an increase of heat transfer coefficients at the stagnation point is induced by a turbulence inherent in jets. In our previous work carried out for a range of $B = 10\text{ mm}$, $u_0 = 1.1 \sim 15.3\text{ m/sec}$, the heat transfer coefficients at the stagnation point of a two-dimensional water jet increased to $1.7 \sim 1.8$ times that of the laminar water jet.⁽⁸⁾

Conclusions

- (1) The velocity distributions at the outer edge of the boundary layer computed by using Strand's analytical theory are in reasonably good agreement with the experimental values for $H/B > 5$. The calculated value for $H/B = 0.1$ and result of experiments by Akfirat⁽¹⁰⁾ show a similar velocity distribution qualitatively.
- (2) At the stagnation point, there is a maximum point of heat transfer coefficient in the neighborhood of $H/B = 0.7$.
- (3) In comparison of local heat transfer coefficients for a uniform heat flux surface and for a uniform temperature surface, heat transfer coefficients at the stagnation point coincide with each other. As the distance from the stagnation point increases, the local heat transfer coefficient both decreases and approaches respective value of it asymptotically in uniform parallel flow zone along a flat plate. In an H/B range from 0.1 to 0.25, there is a place where local heat transfer coefficients have a peak. This peak appears at the point of about half nozzle width from the stagnation point for a uniform temperature surface and moves a little from that point to the stream for a uniform heat flux surface.
- (4) Experimental results for a uniform heat flux surface agree well with the theoretical results.

(5) An approximate equation of temperature gradient at wall was obtained with accuracy higher than ± 2 per cent in a Prandtl number range from 0.7 to 10, in a β range from 0 to 1.6 and in a γ range from 0 to 4 (when $\beta=1.0$, γ from -0.25 to 4 and $\beta=1.6$, γ from -0.5 to 4).

Acknowledgements

The authors wish to thank K.Sakamoto, R.Kurihara and N.Tadokoro for their help in manufacture of the experimental apparatus.

It should be noted that the large-capacity computer of Tokyo University was used to calculate the numerical values in this paper.

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