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Stability analysis of multi span rotor system
and its application*
(part 2, Experiment for rotor system having two bearings
and sensitivity analysis)

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This paper describes an experiment of the part 1 and also a sensitivity analysis of a rotor system. In the experiment, energy levels of two bearings in the rotor system are measured and the values obtained are compared with the theoretical results in the part 1. Next the concept of the sensitivity is introduced and a method to improve the rotor/bearing system is studied from the sensitivity analysis and the energy levels.

1. Introduction

In a series of reports qualitative evaluation is proposed on the stability of a multi-rotor system using vibrational energy level of each bearing and also on the stability of each bearing for application to the design and the diagnosis of rotor systems. The first report showed the theoretical background and gave some examples of applications.⁽¹⁾ This one, i.e., the second report, purports to verify the first report experimentally. That is, the stability of a rotor/two bearing system is experimentally evaluated and the results are compared with the theoretical ones. Then the concept of stability sensitivity is introduced in order to make a decision i.e. which of the bearings should be improved effectively to stabilize the system. That is, the sensitivity of real part of eigenvalue with respect to bearing eccentricity is defined in a normalized form and the values are calculated for some cases. Then the sensitivity is discussed by using the proposed stability concept.

2. Experimental Apparatus and Procedure

Fig.1 shows schematically the experimental apparatus of a rotor system. The rotor is coupled with a variable speed motor (speed range = 540~5400rpm) by a flexible coupling.

Span length of the test rotor is 1000mm. The rotor whose disk is mounted at the center of the shaft is designated as the model I, and the rotor whose disk is mounted at the quarter of the shaft as shown in Fig.2 is designated as the model II.

Weight of the disk is 70kg and balance holes are bored on both sides of the disk.

Test bearings are all journal bearings. Its inner diameter and clearance are made same size for all test bearings and two widths are given, i.e. (width/diameter) = (L/D) = 1.0 and 0.5. As shown in Fig.1, the bearings are respectively called B.G.1 and B.G.2 for simplicity.

Turbine oil #90 is used for lubrication and it is pumped by a gear pump and supplied from the top of the bearing at pressure 2kg/cm².

Vibration is measured by an inductive type pick up and its signal is sent to the data recorder and visi-graph through a D.C. amplifier. Rotating speed is measured by a pulse counter. Frequency distribution is analyzed by a Fourier analyzer and amplitude ratio as well as phase difference is obtained by the visi-graph. Temperature of the lubrication oil is measured at the exit of the bearing box by an alcohol thermometer. A block diagram of the measuring procedure is shown in Fig.3.

Four cases are experimented as shown in Table 1. In the first case the model I rotor (symmetric rotor) is supported by different bearings. In the cases from the

1. Electric motor 2. Kopps Variator
3. Flexible coupling 4. Journal bearing (B.G.2)
5. Journal bearing 6. Rotor shaft (B.G.1)
7. Rotor disk 8. Balance holes
9. Ring spring 10. Pulse generating
11. Base 12. Lubrication oil pipe
- A Probe for amplitude measurement
- B Probe for rotating speed measurement

Fig.1 Rotor system

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second to the fourth, the model II rotor is supported by different or the same bearings, of which details are shown in Table 1. In the table, surface pressure $P = (\text{bearing load/projected bearing area } (L \times D))$ and radial clearance $C = 50 \mu\text{m}$.

The test was done by the following procedure;

- (1) The rotating speed is very slowly increased as the vibration behavior of rotor keeps steady state. Near the onset speed of instability, the rotating speed is increased until the amplitude due to instability (about one period per two revolutions) becomes almost equal to the amplitude due to unbalance, then the rotating speed is quickly decreased. Thus the onset speed of instability is defined as the speed when the amplitude due to instability becomes larger than that due to unbalance.
- (2) From the above data, the amplitude ratio in x and y directions, rotating speed, whirling frequency and phase angle between the vibrations in x and y directions are obtained.
- (3) From the above values and measured oil temperature, Sommerfeld number is calculated and eight bearing coefficients k_{ij} and b_{ij} ($i, j = x, y$) are calculated by using the infinitely short bearing theory. Then the energy of each bearing is obtained. The results are shown in Fig.4.

3. Experimental Results

The onset speed of instability ω_c , whirling speed Ω and experimental and theoretical values of vibration energy for each bearing are shown in Table 2, where some experimental conditions are also written. In this table, the theoretical

Fig.2 Shape and size of rotors

Fig.3 Analysis of rotor vibration

values I and II mean energy values calculated by using the onset speeds obtained by theory and experiment, respectively. The difference of whirling speed between experiment and theory is not taken into account for calculating the theoretical value I of Table 1, so there are some discrepancies between theoretical values and experimental values. On the other hand, the theoretical value II considers the difference of whirling speed between experiment and theory, therefore it is better to compare the theoretical value II with experimental results. According to the results of the theoretical value II, experimental value of the onset speed of instability is higher than theoretical one. This is because of the neglect of gyromoment and the width of disk. The signs of energy of each bearing in theoretical result II and experimental result coincide with each other completely, and it is known that the bearing which makes the system unstable can be easily identified. Let's take the model II-C as a basic model, and consider making the rotor system more stable. In the model II-C instability occurs first at B.G.2. For the synthesis of the rotor system, L/D of B.G.1 of model II-C is changed to 0.5, which is called model II-B, in order to have a high eccentricity of the journal.

Case	$\tau:\tau$	BG. No.	Bearing				
			B.w. L(mm)	D.v.c. $\mu(\text{cp})$	B.l. F(kg)	S.p. p(kg)	R.c. C(μ)
I	1:1	BG.1	40	22	35	2.19	50
		BG.2	20	33	35	4.38	
II-A	3:1	BG.1	40	23	52.5	3.28	50
		BG.2	20	30	17.5	2.19	
II-B	3:1	BG.1	20	32	52.5	6.56	50
		BG.2	40	35	17.5	1.09	
II-C	3:1	BG.1	40	27	52.5	3.28	50
		BG.2	40	35	17.5	1.09	

B.w.; Bearing width

D.v.c.; Dynamic viscous coef.

B.l.; Bearing load

S.p.; Surface pressure

R.c.; Radial clearance

Table 1 Experimental condition

Fig.4 Flow chart of energy calculation

Comparing these two models, onset speed of instability of model II-B is higher than that of model II-C, and absolute value of energy of B.G.1 decreases. Therefore the high eccentricity of B.G.1 improves the stability of the rotor system.

On the other hand, L/D of B.G.2 of model II-C is changed to 0.5 in model II-A. The onset speed of instability becomes higher, but it is not so much improved as in the model II-B. In the model II-B, B.G.2 becomes first unstable from an energy viewpoint of the bearings.

Fig.5 shows an energy diagram which is calculated by the method in the first report. The point at which the sum of energies of B.G.1 and B.G.2 becomes zero gives the critical speed of stability. Energy values $\bar{E}$ shown in Figs.5 are non-dimensional values obtained by the following equation;

$$\bar{E} = E \frac{\pi F_0}{C} \equiv \frac{E}{A} \quad \dots\dots\dots(1)$$

where C is radial clearance and F static load of bearing.

Fig.6 shows frequency response curves. There are two peaks in the figure, of which the lower one corresponds to a whirling mode (unstable mode) and the upper one to a synchronous vibration (unbalance mode). Therefore, the bearing which causes instability can be identified from the amplitude of whirling. In these figures the results of all cases except for model II-A agree with the results of Table 2. With regard to model II-A, the bearing which causes instability can not be identified from the above obtained information, because instability occurs in both bearings almost at the same time.

4. Sensitivity Analysis⁽²⁾

A general equation of motion with two bearings is given as Eq.(20) of the first report,

Fig.5 Energy diagram

$$\left. \begin{aligned} m\ddot{x} + k(x - \tau_2 x_1 - \tau_1 x_2) &= 0 \\ \tau_2 k(x - \tau_2 x_1 - \tau_1 x_2) &= P_{x1} \\ \tau_1 k(x - \tau_2 x_1 - \tau_1 x_2) &= P_{x2} \\ m\ddot{y} + k(y - \tau_2 y_1 - \tau_1 y_2) &= 0 \\ \tau_2 k(y - \tau_2 y_1 - \tau_1 y_2) &= P_{y1} \\ \tau_1 k(y - \tau_2 y_1 - \tau_1 y_2) &= P_{y2} \end{aligned} \right\} \dots\dots\dots(2)$$

where,

$$\left. \begin{aligned} P_{xn} &= n b_{xx} \dot{x}_n + n b_{xy} \dot{y}_n + n k_{xx} x_n + n k_{xy} y_n \\ P_{yn} &= n b_{yx} \dot{x}_n + n b_{yy} \dot{y}_n + n k_{yx} x_n + n k_{yy} y_n \end{aligned} \right\} \dots\dots(3)$$

As the bearing coefficients are determined from the eccentricity of journal, matrix G_n of characteristic equation described in the first report includes the eccentricity of the journal as well as the product of mass, shaft stiffness and so on. Thus the eigenvalue of the characteristic equation can be written as,

$$\lambda_j = g(a_i, \varepsilon_n) \quad \dots\dots\dots(4)$$

where λ_j is the j th eigenvalue, a_i is the product of mass, shaft stiffness and so on, and ε_n eccentricity of the n th bearing.

The sensitivity of $\lambda_j(\varepsilon_n)$ is defined by

$$S_{\varepsilon_n} \lambda_j = \frac{\partial \lambda_j}{\partial \varepsilon_n} \frac{\varepsilon_n}{\lambda_j} \quad \dots\dots\dots(5)$$

Fig.6 Transient frequency analysis

for small change of ϵ_n . Eq.(5) states that the sensitivity of λ_j with respect to eccentricity ϵ_n is the percentage change in λ_j divided by that percentage change in ϵ_n which has caused the change in λ_j to occur.

Eigenvalue is generally described in complex number as,

$$\lambda_j = \gamma_j + i\omega_j \dots (6)$$

As the real part of the eigenvalue is important for stability, the sensitivity of the real part will be discussed, that is,

$$S_{\epsilon_n \gamma_j} = \frac{\partial \gamma_j}{\partial \epsilon_n} \frac{\epsilon_n}{\gamma_j} \dots (7)$$

Then the sensitivity can be written in an approximate form as

$$S_{\epsilon_n \gamma_j} = \frac{\Delta \gamma_j}{\Delta \epsilon_n} \frac{\epsilon_n}{\gamma_j} = \frac{g(\epsilon_n + (1/2)\Delta \epsilon_n) - g(\epsilon_n - (1/2)\Delta \epsilon_n)}{\Delta \epsilon_n} \frac{\epsilon_n}{\gamma_j} \dots (8)$$

Sensitivity is calculated for four kinds of rotor models shown in Table 1. The results are shown in Fig.7. Fig.7(a) shows the result for model I, in which sensitivities of B.G.1 and B.G.2 are plotted by full line and dotted line, respectively. As the real part of the eigenvalue becomes zero at the onset speed of stability, sensitivity becomes infinite. Absolute value of the sensitivity of B.G.1 is larger than that of B.G.2 near the critical speed. This fact means that the influence of B.G.1 upon the stability is larger than that of the other. Also it is considered that a changing of the eccentricity in high sensitivity bearing is more effective for improving the stability of the system.

Rotor model	Onset speed of stability			Energy level of each bearing			
		Ex.	Th.	B.	Ex.	Th.I	Th.II
I	ω_c	57	44	BG.1	7.65 E 4	-8.14 E 5	4.02 E 4
	Ω	26	23	BG.2	-6.08 E 4	-4.76 E 4	-8.07 E 4
II-A	ω_c	66	54	BG.1	8.92 E 4	-1.52 E 5	1.48 E 4
	Ω	33	27	BG.2	-1.35 E 4	-3.82 E 4	-1.42 E 3
II-B	ω_c	67	52	BG.1	-3.86 E 4	-3.73 E 5	-1.72 E 3
	Ω	33	27	BG.2	1.30 E 4	-4.91 E 5	2.71 E 3
II-C	ω_c	64	53	BG.1	-8.92 E 3	-5.04 E 5	-5.20 E 3
	Ω	33	27	BG.2	1.11 E 5	-8.16 E 5	3.05 E 3

Ex; Experiment Th; Theory
B; Bearing

Table 2 Results of experiments and theoretical calculations

5. Discussions

In this section experimental and theoretical results for stability of rotor supported by two bearings are discussed.

In Table 2, the whirling speed Ω obtained by the experiment is, about 12% for model I and about 18% for model II, higher than that of the theory. The reason is as follows: (1) There are some differences between theory and experiment in evaluation of coupled stiffness between shaft and disk. (2) The gyromoment of disk is neglected in model II, in which the gyro effect is to be considered large, so the difference between theory and experiment in model II is larger than that in model I.

The onset speed of instability ω_c is also different between theory and experiment. The reason is that there are many damping sources in the experiment, but the damping only at bearings is considered in the theory. Therefore the sum of damping forces in theory is smaller than that in experiment.

With regard to the vibration energy of bearing, experimental values II agree qualitatively well with theoretical values and these results are coincident with the frequency-analysis-results shown in Fig.6. But there are some discrepancies quantitatively, because the energy is proportional to square of amplitude, and the measured amplitude and phase angle contain errors.

The bearing which causes the instability is a high surface pressure bearing for model I and model II-B and C, but it is a low surface pressure bearing for model II-A. In model II-A, the difference between both surface pressures is not so much, and from the view point of frequency analysis, the unstable degrees of both bearings are similar, so it is likely that some other effects are acting on the stability of the system in this case. Generally speaking, instability occurs from a higher surface

Fig.7 Sensitivities of rotor model

pressure bearing in the two bearing rotor system.

It is concluded as shown in Fig.6 that by making a frequency analysis of each bearing, the bearing which causes the instability can be identified. So the unstable bearing can be known by searching for a bearing in which the vibration is higher than in the other.

6. Conclusions

The evaluation method of stability of bearings by using vibration energy concept, which is proposed in the first report, was experimentally confirmed by using a simple two bearing rotor system and a new concept of sensitivity of stability is proposed.

Vibration energies of bearings obtained by experiment agree qualitatively well with those by theory and these results are coincident with the frequency-analysis-result. It is known from this result that the bearing which causes instability can be identified by calculating the vibration energy of each bearing.

Sensitivity of each bearing is calculated and discussed. Then it is known that sensitivity of the real part of eigenvalue dependent on the eccentricity of bearing is useful in obtaining the information on how to improve bearings and how to design the optimum rotor system.

References

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