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Stress Measurement Using Slip-initiation

Phenomenon in Iron Foil*

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By applying a slip-initiation phenomenon by fatigue in an iron foil attached to a specimen, the method of obtaining cyclic stresses acting upon the specimen at high temperatures is studied basically. Effects of elastic moduli of basic materials, speeds of cyclic loads and test temperatures on the critical stress of slip-initiation in the iron foil are examined. The critical stress of slip-initiation is dominated by cyclic shearing strain on the surface of basic material and becomes lower with a decrease of cyclic speed. As an application of the method, stress-concentration factors in notched specimens are obtained under various cyclic speeds and temperatures. The results are in good agreement with those obtained previously. It is concluded that the present method can be applied favourably for stress measurement at high temperatures up to 430°C.

1. Introduction

The authors⁽¹⁾ confirmed that concentrated stresses in notched specimens made of pure iron can be measured accurately by utilizing the property of primary slip lines initiated by fatigue in the material. In the previous paper⁽²⁾, it was found that this method could be applied not only to models but also to machine parts in the range from room temperature to 250°C, by observing slip lines initiated in an iron foil attached to objects. In the copper and nickel electroplating methods of stress analysis, the upper limit of applied temperature is restricted to about 250°C^(3,4) for occurrence of recrystallization in the deposited metals. Furthermore, when the methods are applied at a small number of cycles, there are such problems that grown grains in the deposits are too fine to observe, and proper stresses become so high as to crack the deposited layer and they affect the strength of measured members. Accordingly, a number of cycles more than 1×10^6 is usually adopted in the electroplating methods and it takes relatively long time to obtain the test results. On the other hand, it is considered that the upper limit of applicable temperature in the method of iron foil mentioned above⁽²⁾ may be raised to a higher temperature than 250°C by selecting an appropriate adhesive. According to the previous results⁽²⁾, the region of primary slip is mainly dominated by the grain size of iron foil and is little affected by the number of cycles. Furthermore, the critical stress of slip-initiation

is much lower than the proper stress in the electroplating method and is easily varied by changing the grain size of iron foil. Consequently, it is considered that the present method can be used for a quick measurement under much lower critical stresses than those of the electroplating method and then may be applied favourably to the stress measurement in machine parts in service.

In this paper, from the above view point, a basic research on the method of iron foil is performed with a view to extending the range of applicable temperatures and accuracy of the quick measurement with a small number of cycles. Effects of adhesives, elastic moduli of basic materials, speeds of cyclic load and test temperatures on the critical stress of slip-initiation in iron foil are examined. The stress-concentration factors in notched specimens are obtained at several temperatures below 430°C and in the range of small numbers of cycles below 3.6×10^4 . The results are compared with those obtained previously.

2. Experimental procedure

2.1 Test specimen and testing machine

Rods of aluminum alloy 2017-T4 ($\phi 20$ mm) and drawn rods of carbon steel SK-3 ($\phi 19$ mm) annealed after heating at 850°C for one hour were used as the specimens. Figure 1 shows a tapered calibration specimen. Figure 2 and Table 1 show shapes and dimensions of circular shafts with a circumferential groove made of SK-3. Table 2 shows elastic moduli of the materials. An Ono-type fatigue testing machine (10 kgf-m) was

Fig. 1 Tapered calibration specimen

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Table 1 Dimensions of shafts with a circumferential groove

Specimen	D mm	d mm	t mm	r mm	r/d	t/r
(a)	12.9	12.0	0.45	3.0	0.25	0.15
(b)	17.2	16.0	0.60	4.0		
(c)	13.6	12.0	0.80	3.0	0.25	0.27
(d)	18.1	16.0	1.05	4.0		
(e)	12.9	12.0	0.45	6.0	0.50	0.08
(f)	17.2	16.0	0.60	8.0		
(g)	13.6	12.0	0.80	6.0	0.50	0.14
(h)	18.2	16.0	1.10	8.0

Fig. 2 Circular shaft with a circumferential groove

used for rotary bending tests, and various cyclic speeds were obtained by a direct current motor attached to the testing machine. Tests at high temperatures were carried out in an electric furnace with a program temperature controller and the test temperature was measured by a thermo-couple of alumel-chromel.

2.2 Iron foil and its adhesion

A round bar ($\phi 42\text{mm}$) of stainless steel was plated over 120mm in length in a high temperature bath mainly composed of ferrous chloride, and a sheet of iron foil was made by tearing off the deposits from the bar. Composition of plating solution, plating condition and surface finishing of deposits are the same as those in the previous paper⁽²⁾. The iron foil thus obtained was annealed after heating at 760°C for one hour in a vacuum furnace, and then it was divided into a number of small pieces of rectangular type ($3 \times 20\text{mm}$) and square type ($3 \times 3\text{mm}$) for the tests. Here, thickness and mean grain size of the iron foil were about $10\mu\text{m}$ and 0.03mm , respectively. Two sheets of the rectangular type foil were used for the tapered specimen shown in Fig. 1 and were attached symmetrically along the axis. Six sheets of the square type foil were used for the bottom of groove shown in Fig. 2 and were attached at about regular intervals. For the purpose of preventing oxidation of these foils at high temperatures, a constant quantity of chromium⁽¹⁾ was vacuum evaporated on them. An instant adhesive agent (Aron α), and two

Table 2 Elastic moduli of materials

Material of specimen	2017-T4	SK-3
Young's modulus E kgf/mm ²	6.85×10^3	2.13×10^4
Modulus of rigidity G kgf/mm ²	2.56×10^3	8.19×10^3
Poisson's ratio	0.34	0.30

kinds of heat hardening type cements made of epoxy resin (XS-20)⁽²⁾ and silicon (SC-22) were used as adhesives for the iron foil in the tests at room temperature, at high temperatures below 260°C and above 300°C , respectively. Here, heating conditions of the cement made of silicon were 3 stages of one hour at 80°C , 2 hours at 130°C and 2 hours at 230°C , giving a pressure to the cemented foil. Then, after the pressure was removed at room temperature, the cemented layer was cured by an additional heating at $290 \pm 10^\circ\text{C}$ for 2 hours.

2.3 Items of the experiment

First, rotary bending tests were performed by use of the tapered calibration specimens shown in Fig. 1, and the relation between the critical stress σ_p of slip-initiation in the iron foil attached to them and the number of cycles N was obtained. Namely, effects of adhesives, shearing modulus G of specimen, cyclic speed v and test temperature T on the above relation were investigated. Here, the occurrence of slip lines in the iron foil was judged by using an optical microscope of 200 magnifications. The critical location where slip lines just began to occur in the specimen was observed and the diameter at the location was measured. Then, the value of σ_p was calculated by using the formula of beam theory⁽²⁾.

Effects of difference in adhesives on σ_p were examined by use of SK-3 at $v=3600\text{rpm}$ and $T=20^\circ\text{C}$, which was selected as a common applicable temperature for the three kinds of adhesives. Effects of G of the specimen on σ_p were inspected by use of two materials, namely 2017-T4 and SK-3, at $v=3600\text{rpm}$ and $T=30^\circ\text{C}$. Effects of cyclic speeds on σ_p were examined by use of SK-3 for various speeds in the range of $v=100 \sim 5500\text{rpm}$ at $T=30^\circ\text{C}$. Effects of test temperatures on σ_p were investigated by use of SK-3 at various temperatures in the range of $T=30 \sim 430^\circ\text{C}$ for both $v=100\text{rpm}$ and 3600rpm .

Next, as an example of stress measurements using the present method, the stress-concentration factors of circular shafts with a circumferential groove shown in Fig. 2 were obtained at 430°C (constant), at fluctuating temperatures in the range of $170 \sim 260^\circ\text{C}$, and at small number of cycles in the range of $N=(0.5 \sim 3.6) \times 10^4$.

3. Experimental results and discussions

3.1 Effects of adhesives on the critical stress

Figure 3 shows the σ_p - N relations ob-

Fig. 3 Effect of adhesives on σ_p - N relation (SK-3)

tained individually by use of the three kinds of adhesives. It is found that the values of σ_p are almost the same at a fixed N regardless of the adhesives used and the relation shown in the figure is expressed by one curve within a scatter of $\pm 3\%$. Accordingly, it can be said that the slip-initiation in the foil is almost unaffected by a difference in the adhesives adopted in this experiment.

3.2 Effects of the elastic modulus of specimen on the slip-initiation

Figure 4 shows the σ_p - N curves obtained by use of the specimens made of two materials. Here, σ_p is the nominal stress acting

Fig. 4 σ_p - N curves for 2 kinds of material

Fig. 5 γ_p - N curve of iron foil

Fig. 6 Effects of cyclic speed on σ_p - N curve (SK-3)

Fig. 7 Relation between critical stress and mean strain rate (SK-3)

on the surface of specimen. It is found from the figure that the values of σ_p at the same N vary markedly with the modulus of elasticity of the basic material even if the iron foil of same quality is used. This is due to the fact that the magnitude of cyclic strains varies with the material even if the cyclic stresses acting on the specimens are identical. Using the cyclic critical shearing strain $\gamma_p = \sigma_p / 2G$ produced in both specimens, however, Fig. 4 is reduced to Fig. 5, which shows that the γ_p - N relations essentially agree with each other. Accordingly, as the strain in the iron foil attached is considered to be nearly equal to that on the surface of specimen, the critical strain γ_p concerned with the slip-occurrence becomes a proper value of the iron foil used, and depends on the composition of plating solution, plating condition, heat treatment of the foil, and the number of strain cycles.

3.3 Effects of cyclic speeds on the critical stress

Figure 6 shows the σ_p - N curves obtained at various cyclic speeds. It is found from the figure that the value of σ_p at each N reduces with a decrease in ν . In order to make clear this tendency, the relation between σ_p at $N = 2 \times 10^5$, where the value of σ_p at each ν becomes nearly constant, and the mean strain rate $4n\sigma_p/E$ (where $n = \nu/60$) per a quarter cycle is shown in Fig. 7. It is found from the figure that σ_p reduces to about a half when the mean strain rate decreases to one hundredth. This phenomenon owing to the strain rate is analogous to those observed at the yield point and fatigue limit in carbon steel.

3.4 Effects of test temperatures on the σ_p - N curve

Figure 8(a),(b) shows the σ_p - N curves obtained at various test temperatures from 30°C to 430°C for $\nu = 100$ rpm and 3600 rpm, respectively. Slip lines occurring at an assigned temperature were observed at room temperature, by stopping the test at intervals of appropriate numbers of cycles. After the slip lines had been observed, the specimen was held at the assigned temperature and under no load for one hour as the duration of test interruption. The upper limit of test temperature was restricted to about 400°C because a change in quality of the adhesive layer occurred at 460°C and then the test was difficult to complete. Figure 9 shows an example of the relation between σ_p at $N = 1 \times 10^5$ and T obtained by using the results in Fig. 8. Figure 10 shows an example of appearances of the slip lines produced by cyclic stresses somewhat higher than the critical stresses of slip-initiation. It is clear from Figs. 8 and 9 that in the case of 100 rpm the value of σ_p becomes minimum at about 100°C and increases gradually with a rise of temperature, while in the case of 3600 rpm these phenomena appear in the higher temperature range. This tendency has been found at the stage of fatigue limit in steel where micro

cracks initiate and stop propagating⁽⁵⁾⁽⁶⁾. Accordingly, the results obtained in this experiment indicate that the above properties have already begun to appear at the stage of slip-initiation.

It is clear from Fig. 8 that when the

(a) $v=100\text{rpm}$

(b) $v=3600\text{rpm}$

Curves at 140°C and 300°C, and those at 170°C and 260°C respectively coincide with each other.

Fig. 8 Effects of test temperature on σ_p - N curve (SK-3)

Fig. 9 Effects of temperature on critical stress (SK-3)

test temperature rises to the range from 350°C to 400°C, the σ_p - N curves become nearly horizontal regardless of the number of cycles, but the curves at temperatures higher than 400°C fall with an increase in N . An increase in σ_p with a rise of temperature is due to the effect of strain aging. It is presumed, however, that a fall of the σ_p - N curve with an increase in N at high temperatures over 400°C is due to an additional effect of relieving the strain aging. In order to clarify the above phenomenon, effects of test interruptions of various durations on the σ_p - N curve were examined at $T=410^\circ\text{C}$, 430°C and $v=3600\text{rpm}$. The results are shown in Fig. 11. In the figure, the results for no interruption are obtained in such manner that each specimen has only one retained time for one hour at an assigned temperature before starting the test, and the measured value is obtained by using an individual specimen. Broken lines in the figure show the results in Fig. 8(b), the duration of test interruption of which is one hour. From these results, it is found that when the specimens are held in the test interruptions of long duration during the test at the same temperature, the inclination of σ_p - N curve becomes steep, namely, the value of σ_p at the same N lowers. This may be due to the following reasons. When the critical location of slip-initiation in the tapered portion has been determined by cyclic stresses of a certain magnitude, fine slip lines which are impossible to observe by an optical microscope of 200 magnifications may have occurred at the part a little thicker than the critical location. So, it is considered that if the specimen thus tested is held at high temperature above 400°C for a long time, the part having the latent slip lines falls into a state of overaging. Consequently, it can be said that when this part is subjected to additional cyclic stresses in the following test, observable slip lines can occur easily in the part, resulting in a decrease in σ_p , as compared with the parts held in the test interruptions of short duration. From a similar investigation at 380°C with the test interruptions of 15 hours, it was ascertained that the results were in good agreement with those shown in Fig. 8(b) and effects of the test interruption on the σ_p - N relation could not be recognized. Accordingly, it may be said that if the present method is utilized to measure cyclic stresses in machine parts at a constant temperature below 400°C, the critical stress is unaffected by the test interruption accompanied with heating-cooling for the observation of slip lines in the iron foil.

3.5 Measurement of the stress-concentration factor in a grooved shaft

The stress-concentration factors k in shafts with a circumferential groove shown in Fig. 2 were obtained under rotary bending with various conditions.

(1) The results obtained at a constant temperature of 430°C for $v=3600\text{rpm}$ and $N=5 \times 10^5$ are shown in Table 3. The value of σ_p in the case of no interruption shown in

Fig. 11 is adopted in the table. Using several specimens with a kind of geometrical form of the groove, the tests were carried out by decreasing a bending moment M successively, and observing slip lines produced in six sheets of the square type foil attached to the bottom of groove, the nominal stress $\sigma_d = 32M/\pi d^3$ was obtained for each M . Here, judgement of the occurrence of slip lines was made as follows: When slip lines appeared in the greater part of six foils, it was judged "thick" and when they appeared

in less than the half, it was judged "thin". Thus, the standard stress σ_0 was determined by the average value of the nominal stresses corresponding to "thin" and "none". Figure 12 shows an appearance of the slip lines produced thinly. As the stress at the bottom of groove attains σ_p when the specimen is subjected to the standard stress σ_0 , the stress-concentration factor is given by $k = \sigma_p/\sigma_0$. The results obtained previously^{7,8,9} are also shown in the table, and the present results are in good agreement with

Fig. 10 Appearance of slip lines in iron foil (arrow: axial direction)

Table 3 Measured values and calculated ones of stress-concentration factor ($T=430^{\circ}\text{C}$, $N=5 \times 10^5$)

Specimen	Critical stress σ_p kgf/mm ²	Nominal stress σ_d kgf/mm ²	Occurrence of slip lines	Standard stress σ_o kgf/mm ²	Stress-concentration factor $k=\sigma_p/\sigma_o$	Previous results
(e)	16.7	14.7 14.1 13.5	thin thin none	13.8	1.21	1.24 ⁽⁷⁾ 1.26 ⁽⁸⁾ 1.24 ⁽⁹⁾
(g)	16.7	15.3 15.0 14.2 13.2	thick thin thin none	13.7	1.22	1.25 ⁽⁷⁾ 1.28 ⁽⁸⁾ 1.26 ⁽⁹⁾
(a)	16.7	12.9 11.9 11.8	thick thin none	11.9	1.41	1.41 ⁽⁷⁾ 1.40 ⁽⁸⁾ 1.44 ⁽⁹⁾
(c)	16.7	14.7 12.0 11.5	thick thin none	11.8	1.42	1.44 ⁽⁷⁾ 1.47 ⁽⁸⁾ 1.47 ⁽⁹⁾

Table 4 Measured values and calculated ones of stress-concentration factor ($T=170\sim 260^{\circ}\text{C}$, $N=3 \times 10^6$)

Specimen	Nominal stress σ_d kgf/mm ²	Occurrence of slip lines	Standard stress σ_o kgf/mm ²	Temperature $^{\circ}\text{C}$	Critical stress σ_p kgf/mm ²	Stress-concentration factor $k=\sigma_p/\sigma_o$	Previous results
(f)	3.10	thin	2.92	170	4.25	1.46	1.24 ⁽⁷⁾
	2.98	thin		200	3.80	1.30	1.26 ⁽⁸⁾
	2.85	none		260	4.15	1.42	1.24 ⁽⁹⁾
(h)	3.15	thick	2.91	170	4.25	1.46	1.25 ⁽⁷⁾
	2.97	thin		200	3.80	1.31	1.28 ⁽⁸⁾
	2.85	none		260	4.15	1.43	1.26 ⁽⁹⁾
(b)	2.85	thick	2.68	170	4.25	1.59	1.41 ⁽⁷⁾
	2.73	thin		200	3.80	1.42	1.40 ⁽⁸⁾
	2.62	none		260	4.15	1.55	1.44 ⁽⁹⁾
(d)	2.75	thick	2.56	170	4.25	1.66	1.44 ⁽⁷⁾
	2.60	thin		200	3.80	1.48	1.47 ⁽⁸⁾
	2.52	none		260	4.15	1.62	1.47 ⁽⁹⁾

Table 5 Measured values and calculated ones of stress-concentration factor ($T=30^{\circ}\text{C}$, $v=100\text{rpm}$)

Specimen	Standard stress σ_o kgf/mm ²	N	Occurrence of slip lines	Critical value σ_s kgf/mm ²	Critical stress σ_p kgf/mm ²	Stress-concentration factor $k=\sigma_p/\sigma_o$	Previous results
(f)	4.83	5×10^3 15	none thin	6.00 5.60	5.80	1.20	1.24 ⁽⁷⁾ 1.26 ⁽⁸⁾ 1.24 ⁽⁹⁾
	4.56	13 20	none thin	5.65 5.55	5.60	1.23	
(h)	4.80	5 10	none thin	6.00 5.75	5.88	1.23	1.25 ⁽⁷⁾ 1.28 ⁽⁸⁾ 1.26 ⁽⁹⁾
	4.71	6 10	none thin	5.93 5.75	5.84	1.24	
(b)	4.09	8 20	none thin	5.82 5.55	5.69	1.39	1.41 ⁽⁷⁾ 1.40 ⁽⁸⁾ 1.44 ⁽⁹⁾
	4.11	15 36	none thin	5.60 5.50	5.55	1.35	
(d)	4.00	8 22	none thin	5.82 5.54	5.68	1.42	1.44 ⁽⁷⁾ 1.47 ⁽⁸⁾ 1.47 ⁽⁹⁾
	4.01	8 20	none thin	5.82 5.55	5.69	1.42	

those values.

(2) Table 4 shows the results obtained in the case that the fluctuating test temperature shown in Fig. 13 is repeated periodically at $v=3600\text{rpm}$ and up to $N=3\times 10^6$. The results shown in Fig. 8(b) are adopted as the critical stresses σ_p and the standard stresses σ_o are determined in the same manner as in the above example. Figure 14 shows an appearance of the slip lines produced thinly at the bottom of groove. It is found from Table 4 that though the k -values based on the σ_p -value at 200°C are

Fig. 11 Effects of test interruption on σ_p - N curve (SK-3)

Specimen (g)
 $\sigma_d=14.2\text{kgf/mm}^2$

Fig. 12 Appearance of slip lines at bottom of groove (arrow: axial direction)

Fig. 13 Temperature cycle

in good agreement with the previous results^{(7) (8) (9)}, those based on the σ_p -values at 170°C and 260°C are higher than the previous results. Namely, when the fluctuating temperature shown in Fig. 13 is repeated periodically, valid results are obtained by using the lowest critical stress in the temperature range. The reasons may be considered as follows. Figure 15 shows a schematic illustration of the σ_p - N curves obtained in the range of $170\sim 260^\circ\text{C}$, referring to the results in Fig. 8(b). Each curve becomes nearly horizontal at a number of

Specimen (d)
 $\sigma_d=2.60\text{kgf/mm}^2$

Fig. 14 Appearance of slip lines at bottom of groove (arrow: axial direction)

Fig. 15 Schematic illustration for σ_p - N curve

Specimen (f)
 $\sigma_o=4.56\text{kgf/mm}^2$, $N=20\times 10^3$

Fig. 16 Appearance of slip lines at bottom of groove (arrow: axial direction)

cycles more than $N_1 (=4 \times 10^5)$ and shows separately the lowest critical stress σ_{pmin} of slip-initiation at the temperature concerned. Especially, the lowest critical stress σ_{p2min} at T_2 is the minimum value in the range of $T_1 \sim T_3$. Now, let us consider the case when σ_{p2min} acting on the bottom of groove is repeated by a number of cycles more than N_1 under the fluctuating temperature shown in Fig. 13. If the temperature is different from T_2 , σ_{pmin} is higher than σ_{p2min} . So, it is presumed that σ_{p2min} contributes little to the slip-initiation at the above temperature, but it promotes the slip-occurrence markedly when the temperature attains T_2 . In Table 4, though the k -values are obtained by using the σ_p -values at $N_t = 3 \times 10^6$, it is considered from the valid results that the σ_p -value at about 200°C corresponds to σ_{p2min} . In the test under the fluctuating temperature, the number of cycles repeated at the temperature near T_2 is much smaller than N_t . It is certain, however, that the number of cycles in the range of $T_2 \pm 10^\circ\text{C}$ is above N_1 before the test is completed. Moreover, it is considered that the σ_p -value at 200°C and N_t yields valid results as shown in the table because it becomes about constant from N_1 to N_t in the above temperature range.

(3) Table 5 shows the results obtained at a constant temperature 30°C for a low cyclic speed $v=100\text{rpm}$ and low numbers of cycles in the range of $N=(5\sim36) \times 10^3$. Figure 16 shows an appearance of slip lines produced thinly at the bottom of groove. In this case, using one specimen and increasing the number of cycles N successively under a constant bending moment, N_1 at which slip lines are about to appear and N_2 at which they begin to occur thinly in the iron foils attached to the bottom of groove are obtained. Using the σ_p - N curve shown in Fig. 6, the critical values σ_s corresponding to N_1 and N_2 are determined. Taking an average of these values, the critical stress σ_p at the bottom of groove is obtained. The stress-concentration factor k based on the standard stress σ_0 at the minimum cross section is obtained. As shown in the table, the results obtained are in good agreement with the previous results⁽⁷⁾⁽⁸⁾⁽⁹⁾. Accordingly, it is found that the present method is applicable under much smaller number of cycles than those in the usual electroplating method with sufficient accuracy.

4. Conclusions

The primary slip lines initiated by fatigue in the iron foil attached to specimens are applied for measurement of cyclic stresses acting on them at high tempera-

tures. Effects of adhesives, elastic moduli of specimens, cyclic speeds, and test temperatures on the critical stress of slip-initiation are investigated. As an application of the present method, the stress-concentration factors in grooved shafts are obtained under various conditions and accuracy of the method is discussed. The following conclusions are drawn from the results obtained.

(1) The critical stress σ_p of slip-initiation in the iron foil is dominated by the maximum cyclic shearing strain on the surface of basic material.

(2) σ_p depends on the cyclic speed and it reduces with a decrease in strain rate. When the cyclic speed decreases, the σ_p - N relation indicated at high temperatures shifts relatively to the lower temperature range.

(3) In the test at a constant temperature below 400°C , σ_p is unaffected by the test interruption accompanied with heating-cooling. In the test above 400°C , however, hardening owing to strain aging in the iron foil is relieved remarkably during the test interruption and this leads to a decrease in σ_p .

(4) The present method can be appropriately applied not only at room temperature but also in the high temperature range up to about 430°C , if the iron foil is prevented from oxidation by evaporating chromium.

(5) The present method can be utilized at much smaller numbers of cycles and at lower stress amplitudes than those in the usual electroplating method.

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